Performance evaluation of a Liquor Filler system: Reliability Approach

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Abstract

System reliability measure play a significant role in both analyzing of a system in terms of its production as well as designing of the system. The present paper investigate a complex system namely liquor Filler system to analyze some of its important reliability measures to make understand that how the occurrence of different failures affect the performance of the same. For this purpose different component of the Filler system are taken into consideration and on the basis of different failure and repair a mathematical model is developed by the aid of Markov process. To understand the Filler system performance its reliability and MTTF is obtained. Most significant components of the Filler system are obtained by the use of sensitivity analysis. Graphs are used to depict the results more visibly.

Key words: Filler system; Sensitivity analysis; Reliability indices; Markov process;

1. Introduction

System reliability (along with other reliability indices) is the foremost concern for each and every industrial system for their existence in this competitive era. As a well-known fact the overall performance of a structure is depend on the collective performance of each individual unit of the system [1]. Hence one of the necessary things to improve a system performance is to make system components more reliable [2-3]. One of the other means to enhance the performance of a system is to use most appropriate type of redundancy in it [4]. In the past a lot of research work has been done to improve system reliability through mathematical modellingand Markov process [5-8]. Besides these there many other techniques also exist in the literature of system reliability [9-11]. Ram and Kumar [12] used Markov method to analyze a coal handling plant and evaluate the reliability characteristics for the same. Kumar et al. [13] investigated the casting process through mathematical modeling and Markov process for evaluating the reliability, availability and MTTF for the same. In this work author’s performed sensitivity analysis for identifying the critical components of casting process which affects its performance most.

Analyzing the above research work, in the present paper the author investigated liquor Filler system, which is used in many industries, for evaluating its various reliability characteristics. A Filler system is also investigated by Kadiyan et al. [14] in past but in that paper authors restrict their discussion up to availability of the Filler system only. Many other parameters for system reliability e.g. MTTF, reliability and sensitivity analysis are not investigated. Hence to fill this research gap, here authors investigate a Filler system to find these results to get more insight about the system reliability.

2. Problem description

In the present paper liquor Filler system is investigated for finding its important reliability indices. A filler system is a commercial system consists of many components including conveyor, sensor assembly, star wheel, rotary filler, and rotary cork assembly, Conveyer belt speed. These components are inter-connected in mixed configuration. The authors critically analyze the working of considered system, on the basis of different failure and repairs which may occurs during its working, and draw the following state transition diagram (Fig. 1) for the same. The Fig. 1 depicts the various states of the system during its working.

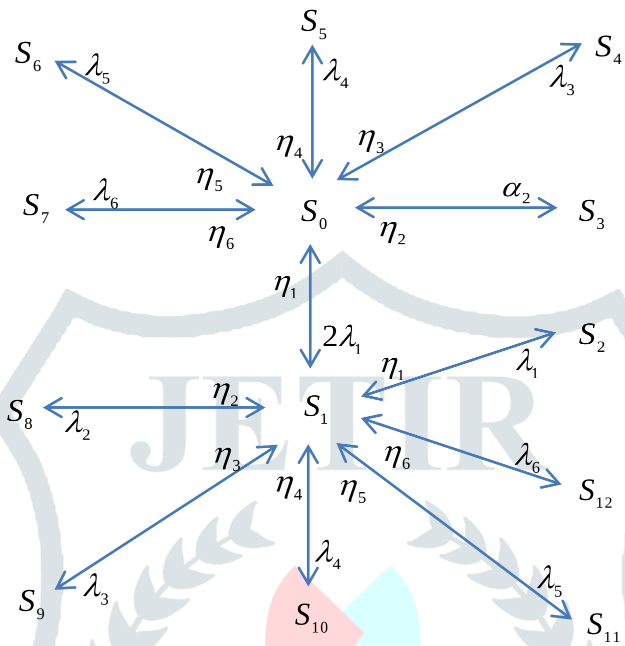

Fig. 1 State Transition diagram for Filler system

3. Assumptions

- No waiting time for maintenance of a failed or degraded unit.
- Initially system is working in perfect state i.e. all components of the Filler system is in good state.
- Average failure and repair of each unit is taken as constant.
- After repair the performance of a component is not degraded.

4. Notations

$\lambda_1 / \lambda_2 / \lambda_3 /$ $\lambda_4 / \lambda_5 / \lambda_6$	*The failure rate of conveyor/sensor assembly/star wheel/rotary filler/ rotary cork assembly/Conveyer belt speed.*
$\eta_1 / \eta_2 / \eta_3 /$ $\eta_4 / \eta_5 / \eta_6$	*The repair rate of conveyor/sensor assembly/star wheel/rotary filler/ rotary cork assembly/Conveyer belt speed.*
$P_i(t)$; $i = 0,1,2,3,4,5,6,$ $7,8,9,10,11,12$	*The probability the Filler system is being in state* S_i; $i = 0,1,2,3,4,5,6,7,8,9,10,11,12$ *respectively.*
$\overline{P}_i(s)$	*Laplace transformation of* $P_i(t)$.
t / s	*Time scale/Laplace transform variable.*

Table 1 Notations

5. State description

S_0	*The state in which Filler system is working with full efficiency.*
S_1	*The state is which one unit of conveyor is failed.*
S_2	*The state is which conveyor is failed.*
S_3	*The state is which sensor assembly is failed.*
S_4	*The state is which star wheel is failed.*
S_5	*The state is which rotary filler is failed.*
S_6	*The state is which rotary cork assembly is failed.*
S_7	*The state is which Conveyer belt speed is failed.*
S_8	*The state is which one unit of conveyor along with sensor assembly is failed.*
S_9	*The state is which one unit of conveyor along with star wheel is failed.*
S_{10}	*The state is which one unit of conveyor along with rotary filler is failed.*
S_{11}	*The state is which one unit of conveyor along with rotary cork assembly is failed.*
S_{12}	*The state is which one unit of conveyor along with Conveyer belt speed is failed.*

Table 2State description

6. Mathematical formulation

The following set of differential equation is generated on the basis of Fig. 1.

$$\left(\frac{\partial}{\partial t}+2\lambda_1+\sum_{i=2}^{6}\lambda_i\right)P_0(t)=\eta_1P_1(t)+\eta_2P_3(t)+\eta_3P_4(t)+\eta_4P_5(t)+\eta_5P_6(t)+\eta_6P_7(t)\ (1)$$

$$\left(\frac{\partial}{\partial t}+\eta_1+\sum_{i=1}^{6}\lambda_i\right)P_1(t)=2\lambda_1P_0(t)+\eta_1P_2(t)+\eta_2P_8(t)+\eta_3P_9(t)+\eta_4P_{10}(t)+\eta_5P_{11}(t)+\eta_6P_{12}(t)\ (2)$$

$$\left(\frac{\partial}{\partial t}+\eta_1\right)P_2(t)=\lambda_1P_1(t)\ (3)$$

$$\left(\frac{\partial}{\partial t}+\eta_2\right)P_3(t)=\lambda_2P_0(t)\ (4)$$

$$\left(\frac{\partial}{\partial t}+\eta_3\right)P_4(t)=\lambda_3P_0(t)\ (5)$$

$$\left(\frac{\partial}{\partial t}+\eta_4\right)P_5(t)=\lambda_4P_0(t)\ (6)$$

$$\left(\frac{\partial}{\partial t}+\eta_5\right)P_6(t)=\lambda_5P_0(t)\ (7)$$

$$\left(\frac{\partial}{\partial t}+\eta_6\right)P_7(t)=\lambda_6P_0(t)\ (8)$$

$$\left(\frac{\partial}{\partial t}+\eta_2\right)P_8(t)=\lambda_2P_1(t)\ (9)$$

$$\left(\frac{\partial}{\partial t}+\eta_3\right)P_9(t)=\lambda_3 P_1(t)\,(10)$$

$$\left(\frac{\partial}{\partial t}+\eta_4\right)P_{10}(t)=\lambda_4 P_1(t)\,(11)$$

$$\left(\frac{\partial}{\partial t}+\eta_5\right)P_{11}(t)=\lambda_5 P_1(t)\,(12)$$

$$\left(\frac{\partial}{\partial t}+\eta_6\right)P_{12}(t)=\lambda_6 P_1(t)\,(13)$$

Initial condition $P_0(0)=1, P_i(0)=0 : i=1,2,3,4,5,6,7,8,9,10,11,12; (14)$

The above set of equations 1-13 can be re-written as

$$\left(s+2\lambda_1+\sum_{i=2}^{6}\lambda_i\right)\overline{P}_0(s)=\eta_1\overline{P}_1(s)+\eta_2\overline{P}_3(s)+\eta_3\overline{P}_4(s)+\eta_4\overline{P}_5(s)+\eta_5\overline{P}_6(s)+\eta_6\overline{P}_7(s)\,(15)$$

$$\left(s+\eta_1+\sum_{i=1}^{6}\lambda_i\right)\overline{P}_1(s)=2\lambda_1\overline{P}_0(s)+\eta_1\overline{P}_2(s)+\eta_2\overline{P}_8(s)+\eta_3\overline{P}_9(s)+\eta_4\overline{P}_{10}(s)+\eta_5\overline{P}_{11}(s)+\eta_6\overline{P}_{12}(s)\,(16)$$

$$(s+\eta_1)\overline{P}_2(s)=\lambda_1\overline{P}_1(s)\,(17)$$

$$(s+\eta_2)\overline{P}_3(s)=\lambda_2\overline{P}_0(s)\,(18)$$

$$(s+\eta_3)\overline{P}_4(s)=\lambda_3\overline{P}_0(s)\,(19)$$

$$(s+\eta_4)\overline{P}_5(s)=\lambda_4\overline{P}_0(s)\,(20)$$

$$(s+\eta_5)\overline{P}_6(s)=\lambda_5\overline{P}_0(s)\,(21)$$

$$(s+\eta_6)\overline{P}_7(s)=\lambda_6\overline{P}_0(s)\,(22)$$

$$(s+\eta_2)\overline{P}_8(s)=\lambda_2\overline{P}_1(s)\,(23)$$

$$(s+\eta_3)\overline{P}_9(s)=\lambda_3\overline{P}_1(s)\,(24)$$

$$(s+\eta_4)\overline{P}_{10}(s)=\lambda_4\overline{P}_1(s)\,(25)$$

$$(s+\eta_5)\overline{P}_{11}(s)=\lambda_5\overline{P}_1(s)\,(26)$$

$$(s+\eta_6)\overline{P}_{12}(s)=\lambda_6\overline{P}_1(s)\,(27)$$

$$\overline{P}_0(0)=1, \overline{P}_i(0)=0 : i=1,2,3,4,5,6,7,8,9,10,11,12; (28)$$

The time dependent probability of different states will be obtained by solving the above set of equations as follows

$$\overline{P}_0(s)=\frac{1}{\left\{\begin{array}{l} s+2\lambda_1+\sum_{i=2}^{6}\lambda_i-\frac{\eta_2\lambda_2}{(s+\beta_2)}-\frac{\eta_3\lambda_3}{(s+\beta_3)}-\frac{\eta_4\lambda_4}{(s+\beta_4)}-\frac{\eta_5\lambda_5}{(s+\beta_5)}-\frac{\eta_6\lambda_6}{(s+\beta_6)} \\ -\frac{2\lambda_1\eta_1}{\left[s+\eta_1+\sum_{i=1}^{6}\lambda_i-\frac{\eta_1\lambda_1}{(s+\beta_1)}-\frac{\eta_2\lambda_2}{(s+\beta_2)}-\frac{\eta_3\lambda_3}{(s+\beta_3)}-\frac{\eta_4\lambda_4}{(s+\beta_4)}-\frac{\eta_5\lambda_5}{(s+\beta_5)}-\frac{\eta_6\lambda_6}{(s+\beta_6)}\right]}\end{array}\right\}} \quad (29)$$

$$\overline{P}_1(s)=\frac{2\lambda_1}{\left[s+\eta_1+\sum_{i=1}^{6}\lambda_i-\frac{\eta_1\lambda_1}{(s+\beta_1)}-\frac{\eta_2\lambda_2}{(s+\beta_2)}-\frac{\eta_3\lambda_3}{(s+\beta_3)}-\frac{\eta_4\lambda_4}{(s+\beta_4)}-\frac{\eta_5\lambda_5}{(s+\beta_5)}-\frac{\eta_6\lambda_6}{(s+\beta_6)}\right]}\overline{P}_0(s)$$

(30)

$$\overline{P}_2(s)=\frac{\lambda_1}{(s+\eta_1)}\overline{P}_1(s) \quad (31)$$

$$\overline{P}_3(s)=\frac{\lambda_2}{(s+\eta_2)}\overline{P}_0(s) \quad (32)$$

$$\overline{P}_4(s)=\frac{\lambda_3}{(s+\eta_3)}\overline{P}_0(s) \quad (33)$$

$$\overline{P}_5(s)=\frac{\lambda_4}{(s+\eta_4)}\overline{P}_0(s) \quad (34)$$

$$\overline{P}_6(s)=\frac{\lambda_5}{(s+\eta_5)}\overline{P}_0(s) \quad (35)$$

$$\overline{P}_7(s)=\frac{\lambda_6}{(s+\eta_6)}\overline{P}_0(s) \quad (36)$$

$$\overline{P}_8(s)=\frac{\lambda_2}{(s+\eta_2)}\overline{P}_1(s) \quad (37)$$

$$\overline{P}_9(s)=\frac{\lambda_3}{(s+\eta_3)}\overline{P}_1(s) \quad (38)$$

$$\overline{P}_{10}(s)=\frac{\lambda_4}{(s+\eta_4)}\overline{P}_1(s) \quad (39)$$

$$\overline{P}_{11}(s)=\frac{\lambda_5}{(s+\eta_5)}\overline{P}_1(s) \quad (40)$$

$$\overline{P}_{12}(s)=\frac{\lambda_6}{(s+\eta_6)}\overline{P}_1(s) \quad (41)$$

Keeping the above state probabilities, the up (working) and down (failed) states of the considered Filler system is given as following.

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s) \quad (42)$$

$$\overline{P}_{down}(s) = \overline{P}_2(s) + \overline{P}_3(s) + \overline{P}_4(s) + \overline{P}_5(s) + \overline{P}_6(s) + \overline{P}_7(s) + \overline{P}_8(s) + \overline{P}_9(s) + \overline{P}_{10}(s) + \overline{P}_{11}(s) + \overline{P}_{12}(s) \quad (43)$$

Also it is trivial to say the following

$$\overline{P}_{up}(s) + \overline{P}_{down}(s) = \frac{1}{s} \text{ i.e. } P_{up}(t) + P_{down}(t) = 1 \quad (44)$$

7. Numerical computation

7.1. Reliability

The reliability of the considered Filler system will be obtained by using inverse Laplace transformation in equation (42) as follows

$$R(t) = \begin{cases} \exp\left(-\left(2\lambda_1 + \lambda_2 + +\lambda_3 + +\lambda_4 + \lambda_5 + \lambda_6\right)t\right) \\ +4\exp\left(-\left(\frac{3}{2}\lambda_1 + \lambda_2 + +\lambda_3 + +\lambda_4 + +\lambda_5 + \lambda_6\right)t\right) + \sin\left(\frac{1}{2}\lambda_1 t\right) \end{cases} \quad (45)$$

The time dependent reliability of the Filler system will be obtained by putting the values of different failure rates as $\lambda_1 = 0.01$, $\lambda_2 = 0.04$, $\lambda_3 = 0.07$, $\lambda_4 = 0.02$, $\lambda_5 = 0.05$, $\lambda_6 = 0.02$, in (45) as given in equation (46).

$$R(t) = \exp\left(-0.220000000\ t\right) + 4\exp\left(-0.2150000000\ t\right) + \sinh(0.0050000000\ t) \quad (46)$$

Finally the behavior of Filler system reliability w.r.t. to time can be seen in Fig. 2.

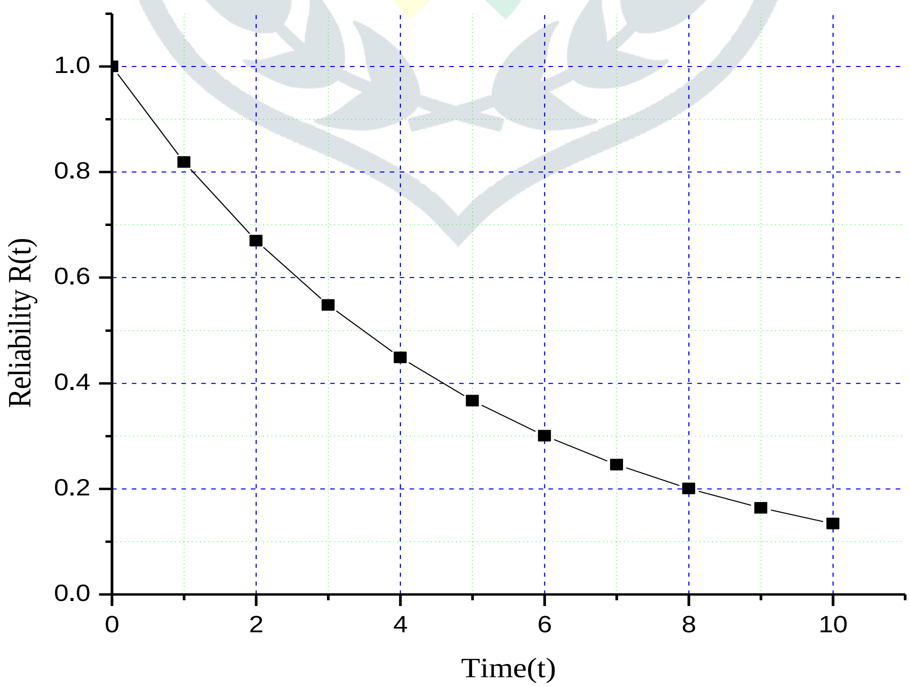

Fig. 2 Filler system Reliability w.r.t. Time unit (t)

7.2.Sensitivity Analysis for Filler system reliability

Basically one of the essential facts for the improvement of system reliability is to understand that how different failure can affects the overall performance of the same. Sensitivity analysis wills address this problem by analyzing the reliability of the system. Here in the present paper author perform the sensitivity analysis for Fillers system reliability i.e. to analyze the impact of different failures on its reliability. Author used equation (45) to do the same. The following Table 3 and Fig. 3 have shown the sensitivity analysis w.r.t. Filler system reliability.

Time unit	Sensitivity of reliability					
	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$	$\frac{\partial R(t)}{\partial \lambda_3}$	$\frac{\partial R(t)}{\partial \lambda_4}$	$\frac{\partial R(t)}{\partial \lambda_5}$	$\frac{\partial R(t)}{\partial \lambda_6}$
0	0	0	0	0	0	0
1	-0.0161308	-0.8186496	-0.8186496	-0.8186496	-0.0161308	-0.8186496
2	-0.0520415	-1.3401144	-1.3401144	-1.3401144	-0.0520415	-1.3401144
3	-0.0944428	-1.6449968	-1.6449968	-1.6449968	-0.0944428	-1.6449968
4	-0.1354208	-1.7945525	-1.7945525	-1.7945525	-0.1354208	-1.7945525
5	-0.1706666	-1.8350220	-1.8350220	-1.8350220	-0.1706666	-1.8350220
6	-0.1982246	-1.8010365	-1.8010365	-1.8010365	-0.1982246	-1.8010365
7	-0.2176213	-1.7182890	-1.7182890	-1.7182890	-0.2176213	-1.7182890
8	-0.2292657	-1.6056247	-1.6056247	-1.6056247	-0.2292657	-1.6056247
9	-0.2340462	-1.4766694	-1.4766694	-1.4766694	-.234046287	-1.4766694
10	-0.2330653	-1.3410969	-1.3410969	-1.3410969	-0.2330653	-1.3410969

Table 3 Filler system reliability w.r.t. different failures and Time unit (t)

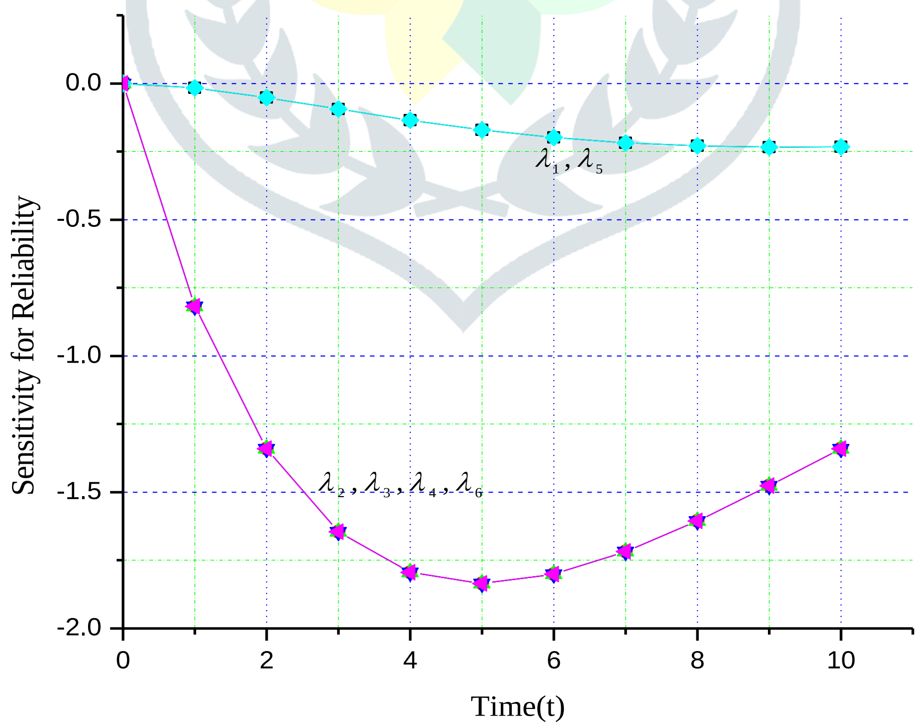

Fig.3 Filler system reliability w.r.t. different failures and Time unit (t)

7.3.Mean Time to failure (MTTF) analysis

The MTTF of the considered Filler system is obtained as given in equation (47).

$$MTTF = \frac{1}{(2\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6)}\left\{1+\frac{2\lambda_1}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6)}\right\} \quad (47)$$

To see the impact of different failures on Filler system's MTTF, author vary all failure rates one by one in equation (47). Table 4 and corresponding Fig. 4 have been obtained by the author for the MTTF of the Fillers system.

Variation in Failures	MTTF					
	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0.01	4.978354978	5.847953217	7.083333333	5.238095238	6.209150327	5.238095238
0.03	4.849498328	5.238095238	6.209150327	4.743083004	5.526315789	4.743083004
0.05	4.666666666	4.743083004	5.526315789	4.333333333	4.978354978	4.333333333
0.07	4.466230938	4.333333333	4.978354978	3.988603989	4.528985508	3.988603989
0.09	4.264972776	3.988603989	4.528985508	3.694581281	4.153846154	3.694581281

Table 4 Filler system's MTTF w.t.t. failure rates

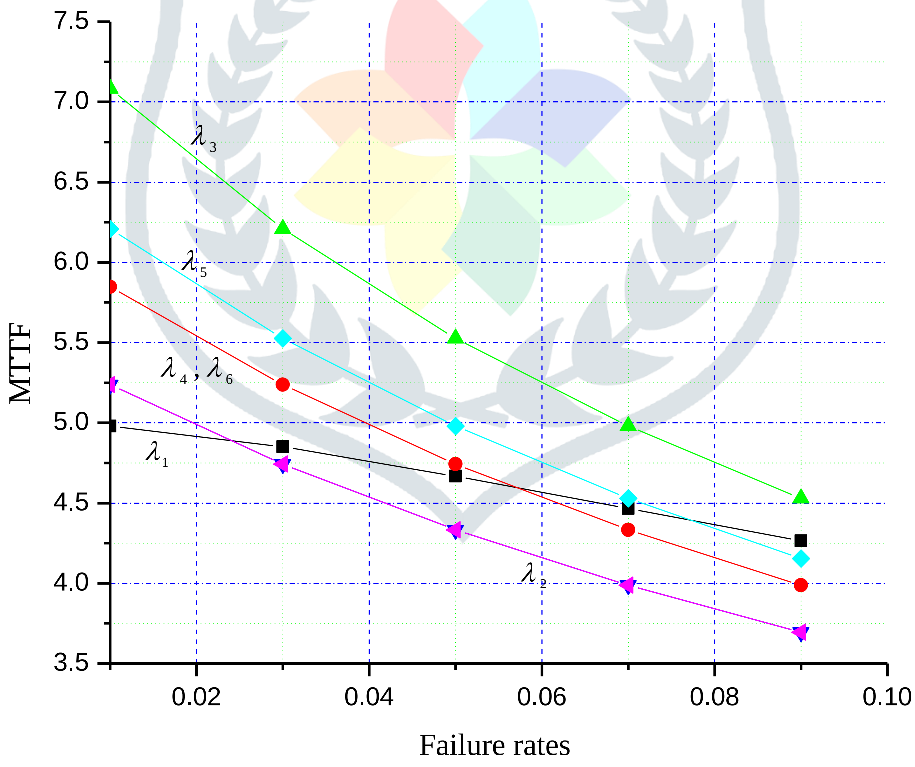

Fig.4 Filler system's MTTF w.t.t. failure rates

8. Result discussion and conclusion.

In the present paper the author investigated a Filler system through mathematical modeling and Markov process for find its different performance measures for the improvement of the same. The reliability of the

Filler system w.r.t. time is shown in Fig. 2. It reflects that the reliability of the Fillers system at five and ten unit of time is 0.3670044145 and 0.1341096982 respectively. The nature of Filler system's MTTF w.r.t. failure rates is given in Fig. 4. It has been observe from this figure that that Filler system's MTTF if highest and lowest w.r.t. failure rate of star wheel andConveyer belt speed, rotary fillerrespectively. The sensitivity of Filler system reliability is shown in Fig. 3. It reflects that the Filler system's reliability is most sensitive with respect to the failure rate of sensor assembly, star wheel, rotary filler and conveyer belt speedand lest sensitive w.r.t. the failure rate of conveyor, rotary cork assembly. Finally it is concluded that the for the improvement in the filler system's reliability more attention should be given to the failures of sensor assembly, star wheel, rotary filler and conveyer belt speedalong with rest of failure, by the maintenance team.

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