Performance Analysis of MIXTURE GRINDER: A Markovian Approach

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Abstract: The main aim of this paper is to analyze the performance of the mixture grinder. Mixture grinder is a very important appliance whichis used in every household. Authors developed the model of the grinder by taking the main components of the grinder into their consideration. Failure and repair rates are assumed to be constants. With the help of transition state diagram, chapman Kolmogorov differential equations are developed. Laplace transformation is used for solving the differential equation. Grinder main performance indicator like Reliability, MTTF, are determined explicitly. Sensitivity analysis is also performed for MTTF and Reliability.
Keywords: Mixture grinder, Laplace transformation, Availability, Reliability

1 Introduction

In this age of technology, many machines have been made to make human life easier. Mixture grinder is the most important kitchen appliance. This is used for different purposes like grinding tomatoes, onions and spices etc. It is also used for making juice or shakes at home. One can also churn curd in it and it can also be used for making cold coffee. It has a very compact design and in the kitchen, it occupies very less space. Nayak and Swamy [1] proposed the mathematical modelling for the universal motorof the mixer grinder which runs on both AC and DC supply. In their research, they found that the efficiency of DC operating condition is good in comparison with AC operating condition of the universal motor.Babu and Gopinath [2] presented the design procedure of the Switch reluctant motor. Switch reluctant motor is more efficient than a universal motor. They gave the step by step procedure for designing switch reluctant motor. It was also verified using Maxwell equations. Gholase and Fernandes[3] preferred the use of BLDC motor because its cost is quite low as compared to universal motor. Besides this for modelling any engineering system, the researcher should have good knowledge of the basic concepts. All important concepts are given by [4], [5],[6] in their book respectively.

The existing literature ignores the reliability, mean time to failure modelling of the mixer grinder.

Above authors didn't also perform the sensitivity analysis of the reliability and MTTFthe problem considered in this paper is quite different from the problems considered by the researchers in their papers. This study focuses on the performance analysis of mixture grinder through reliability approach. The main purpose of this aim is to find the performance measure of reliability.

1.1 Problem statement

It has been observed that mixture grinder of some companies are not that much reliable as compared to the mixer grinder of well established companies. They frequently fail when they are operated. Therefore, authors analyze the performance of the mixture grinder with the help of reliability approach.

2Description of the system

1) **Control panel:** control panel is actually the switch board. Which controls the rotation of the motor. Basically, there are three/four switches in the assembly. One switch just stops the rotation of the motor. Other three switches are used for the rotation of the machine. If the control panel fails the whole system fails

2) **Motor:** Motor is the major component of the mixture grinder. The motor has stator and rotor. It needs 230-240V to run properly. Generally mixture motor rotates 18000 times in a minute. If the motor fails the whole system fails

3) **Cutting Assembly:** The basic purpose is to grind the spices, churn curd, or making juice. Due to failure of this component the whole system fails.

4) **Coupler:** Coupler is a device that connects two things especially mechanical components or systems. Coupler is used to connect the mixture jar with the machine. It should be done with proper care if not done then coupler may degrade and after degradation they still work. With the passage of time couplers also fail. Once they fail the whole system fails.

2.1Assumption of the system

Performance analysis of the system is based on the following assumptions.

a) Initially, all the components of the mixture grinder are in good working condition.
b) Repairman is called only on the failure of any component of the mixture grinder
c) Failures of components statistically independent and only one failure can occur at a time
d) Coupler of the mixture degrades after some time.
e) Failure rates and repair rates of the system assumed to be constants and follow exponential distribution.
f) Machine doesn't stop its working in degraded state.

Table 1(a): Notations

t	Time variable
s	Laplace variable
$\lambda_i (i = 1, 2, 3, 4)$	Failure rate of Control panel, Motor, cutting assembly, coupler respectively
$\mu_i \ (i = 1, 2, 3, 4)$	Repair rate of Control panel, Motor, cutting assembly, coupler respectively
$P_i(t)\,(i = 0,1,\ldots 8)$	Probability that the system being in the state S_i
$\overline{P}_i(s)\,(i = 0,1,\ldots 8)$	Laplace transform of $P_i(t)$

Table 1(b):State description of the system

S_0	All the components of the mixture grinder are in good working condition
S_1	State in which the control panel fails and system fails completely
S_2	State in which motor fails and system fails completely
S_3	State in which cutting assembly fails completely
S_4	State in which coupler degrades and is in a degraded state
S_5	State in which control panel fails after the degradation of coupler
S_6	State in which motor fails after the degradation of coupler
S_7	State in which cutting assembly fails after the degradation of coupler
S_8	State in which the coupler fails completely

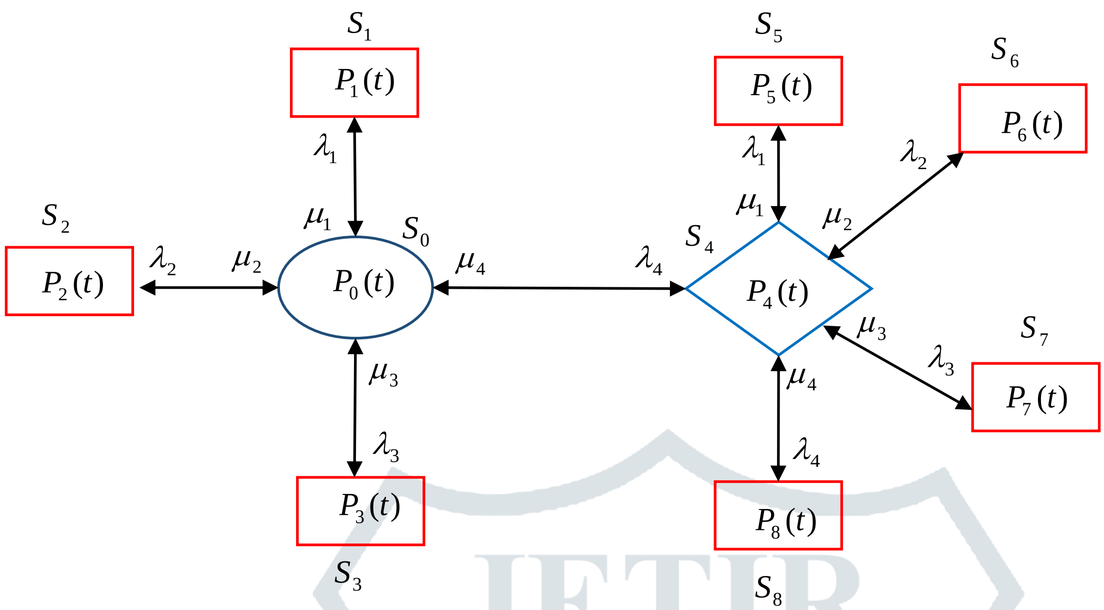


Figure 1: State transition diagram

3 Mathematical formulation of the system

To determine reliability and MTTF of the mixer grinder by considering its four major components control panel, motor, cutting assembly and coupler mathematical formulation of the model is carried out. From the transition state diagram the probabilities of the transition from one state to another state at time $t+\Delta t$ are obtained and then letting $\Delta t \to 0$, the following set of the differential equations is obtained:

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+\lambda_3+\lambda_4\right]P_0(t)=\mu_1\,P_1(t)+\mu_2\,P_2(t)+\mu_3\,P_3(t)+\mu_4\,P_4(t) \quad (1)$$

$$\left[\frac{d}{dt}+\mu_1\right]P_1(t)=\lambda_1\,P_0(t) \quad (2)$$

$$\left[\frac{d}{dt}+\mu_2\right]P_2(t)=\lambda_2\,P_0(t) \quad (3)$$

$$\left[\frac{d}{dt}+\mu_3\right]P_3(t)=\lambda_3\,P_0(t) \quad (4)$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4\right]P_4(t)=\mu_1\,P_5(t)+\mu_2\,P_6(t)+\mu_3\,P_7(t)+\mu_4\,P_8(t)+\lambda_4\,P_0(t) \quad (5)$$

$$\left[\frac{d}{dt}+\mu_1\right]P_5(t)=\lambda_1\,P_4(t) \quad (6)$$

$$\left[\frac{d}{dt}+\mu_2\right]P_6(t)=\lambda_2\,P_4(t) \quad (7)$$

$$\left[\frac{d}{dt}+\mu_3\right]P_7(t)=\lambda_3\,P_4(t) \quad (8)$$

$$\left[\frac{d}{dt}+\mu_4\right]P_8(t)=\lambda_4\,P_4(t) \tag{9}$$

$$P_i(0)=\begin{cases}1 & i=0\\ 0 & i\neq 0\end{cases} \tag{10}$$

Taking inverse Laplace on both sides, we get:

$$\left[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4\right]\overline{P_0}(s)=1+\mu_1\,\overline{P_1}(s)+\mu_2\,\overline{P_2}(s)+\mu_3\,\overline{P_3}(s)+\mu_4\,\overline{P_4}(s) \tag{11}$$

$$\left[s+\mu_1\right]\overline{P_1}(s)=\lambda_1\,\overline{P_0}(s) \tag{12}$$

$$\left[s+\mu_2\right]\overline{P_2}(s)=\lambda_2\,\overline{P_0}(s) \tag{13}$$

$$\left[s+\mu_3\right]\overline{P_3}(s)=\lambda_3\,\overline{P_0}(s) \tag{14}$$

$$\left[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4\right]\overline{P_4}(s)=\mu_1\,\overline{P_5}(s)+\mu_2\,\overline{P_6}(s)+\mu_3\,\overline{P_7}(s)+\mu_4\,\overline{P_8}(s)+\lambda_4\,\overline{P_0}(s) \tag{15}$$

$$\left[s+\mu_1\right]\overline{P_5}(s)=\lambda_1\,\overline{P_4}(s) \tag{16}$$

$$\left[s+\mu_2\right]\overline{P_6}(s)=\lambda_2\,\overline{P_4}(s) \tag{17}$$

$$\left[s+\mu_3\right]\overline{P_7}(s)=\lambda_3\,\overline{P_4}(s) \tag{18}$$

$$\left[s+\mu_4\right]\overline{P_8}(s)=\lambda_4\,\overline{P_4}(s) \tag{19}$$

On solving equation from (11)-(19), we get:

$$\overline{P_0}(s)=\frac{1}{H_0}$$

$$\overline{P_1}(s)=\frac{\lambda_1}{s+\mu_1}\overline{P_0}(s)$$

$$\overline{P_2}(s)=\frac{\lambda_2}{s+\mu_2}\overline{P_0}(s)$$

$$\overline{P_3}(s)=\frac{\lambda_3}{s+\mu_3}\overline{P_0}(s)$$

$$\overline{P_4}(s)=\frac{\lambda_4}{H_1}\overline{P_0}(s)$$

$$\overline{P_5}(s)=\frac{\lambda_1}{s+\mu_1}\overline{P_4}(s)=\frac{\lambda_1}{(s+\mu_1)}\frac{\lambda_4}{H_1}\overline{P_0}(s)$$

$$\overline{P_6}(s)=\frac{\lambda_2}{s+\mu_2}\overline{P_4}(s)=\frac{\lambda_2}{(s+\mu_2)}\frac{\lambda_4}{H_1}\overline{P_0}(s)$$

$$\overline{P_7}(s)=\frac{\lambda_3}{s+\mu_3}\overline{P_4}(s)=\frac{\lambda_3}{(s+\mu_3)}\frac{\lambda_4}{H_1}\overline{P_0}(s)$$

$$\overline{P_8}(s)=\frac{\lambda_4}{s+\mu_4}\overline{P_4}(s)=\frac{\lambda_4}{(s+\mu_4)}\frac{\lambda_4}{H_1}\overline{P_0}(s)$$

$$H_0=\left[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4-\frac{\mu_1\lambda_1}{s+\mu_1}-\frac{\mu_2\lambda_2}{s+\mu_2}-\frac{\mu_3\lambda_3}{s+\mu_3}-\frac{\mu_4\lambda_4}{H_1}\right]$$

$$H_1=\left[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4-\frac{\mu_1\lambda_1}{s+\mu_1}-\frac{\mu_2\lambda_2}{s+\mu_2}-\frac{\mu_3\lambda_3}{s+\mu_3}-\frac{\mu_4\lambda_4}{s+\mu_4}\right]$$

The system upstate and downstate are given below:

$$P_{up}(s)=P_0(s)+P_4(s) \tag{20}$$

$$P_{down}(s) = P_1(s) + P_2(s) + P_3(s) + P_5(s) + P_6(s) + P_7(s) + P_8(s) \quad (21)$$

4Performance measures of Mixer grinder

4.1 Reliability of Mixer grinder

Reliability of a product or equipment is the probability that the product will perform its task for a specified period of time when operated under stated condition. If the product gives the same performance every time whenever it is used then it is called reliable product. Hence reliability is a function of time. It is generally denoted by $R(t)$. It is the probability that the product cannot fail before the time period $'t'$.To obtain the reliability of the system set all repair rate equal to zero in equation (20) and failure rate equal to $\lambda_1 = 0.25/yr$, $\lambda_2 = 0.33/yr$, $\lambda_3 = 0.2/yr$, $\lambda_4 = 0.66/yr$ take inverse Laplace of equation (20) one can get the expression for the reliability

$$R(t) = 0.02000\, e^{-1.44000\,t}(50 + 33t) \quad (22)$$

Now varying time $'t'$ from 0 to 10 one can easily obtain the Table (2) and Figure (2)

Table 2. Reliability of the system

Time (t) (In years)	Reliability R(t)
0	1.00000
1	0.39330
2	0.13023
3	0.03963
4	0.01147
5	0.00321
6	0.00087
7	0.00023
8	0.00006
9	0.00001
10	0.00000

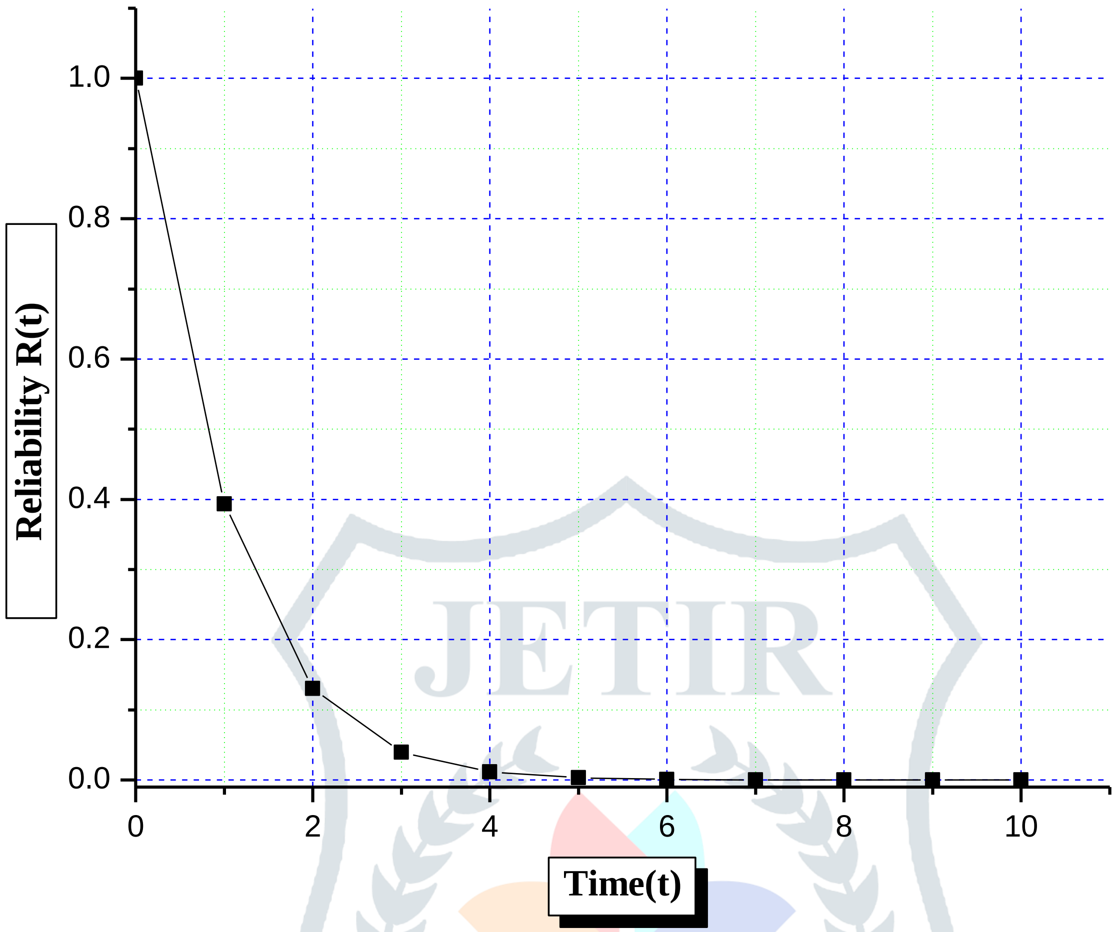


Figure 2. Reliability of the system w.r.t time

4.2 MTTF

Mean time to failure is defined as the time to the first failure of the system. To obtain the expression for the MTTF in equation (20) set all repair rates equal to zero and take limit $s \to 0$ One can obtain the explicit express for the MTTF of the system.

$$MTTF = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} + \frac{\lambda_4}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)^2} \quad (23)$$

When all the four components of the system operate simultaneously the time to the first failure of the system is given below w.r.t variation in failure rates. In the equation (23) set failure rates $\lambda_1 = 0.25/yr$, $\lambda_2 = 0.33/yr$, $\lambda_3 = 0.2/yr$, $\lambda_4 = 0.66/yr$ and one by one vary each failure rate from 0.1 to 0.9, one can easily obtain the Table 3 and Figure 3

Table3. MTTF

Variation in failure rates	λ_1	λ_2	λ_3	λ_4
0.1	1.17180	1.27723	1.11383	1.26549
0.2	1.06102	1.14795	1.01273	1.22865
0.3	0.96842	1.04119	0.92764	1.18312
0.4	0.88999	0.95171	0.85514	1.13473

0.5	0.82280	0.87573	0.79270	1.08642
0.6	0.76464	0.81050	0.73842	1.03969
0.7	0.71386	0.75394	0.69082	0.99525
0.8	0.66917	0.70447	0.64878	0.95337
0.9	0.62956	0.66087	0.61140	0.91411

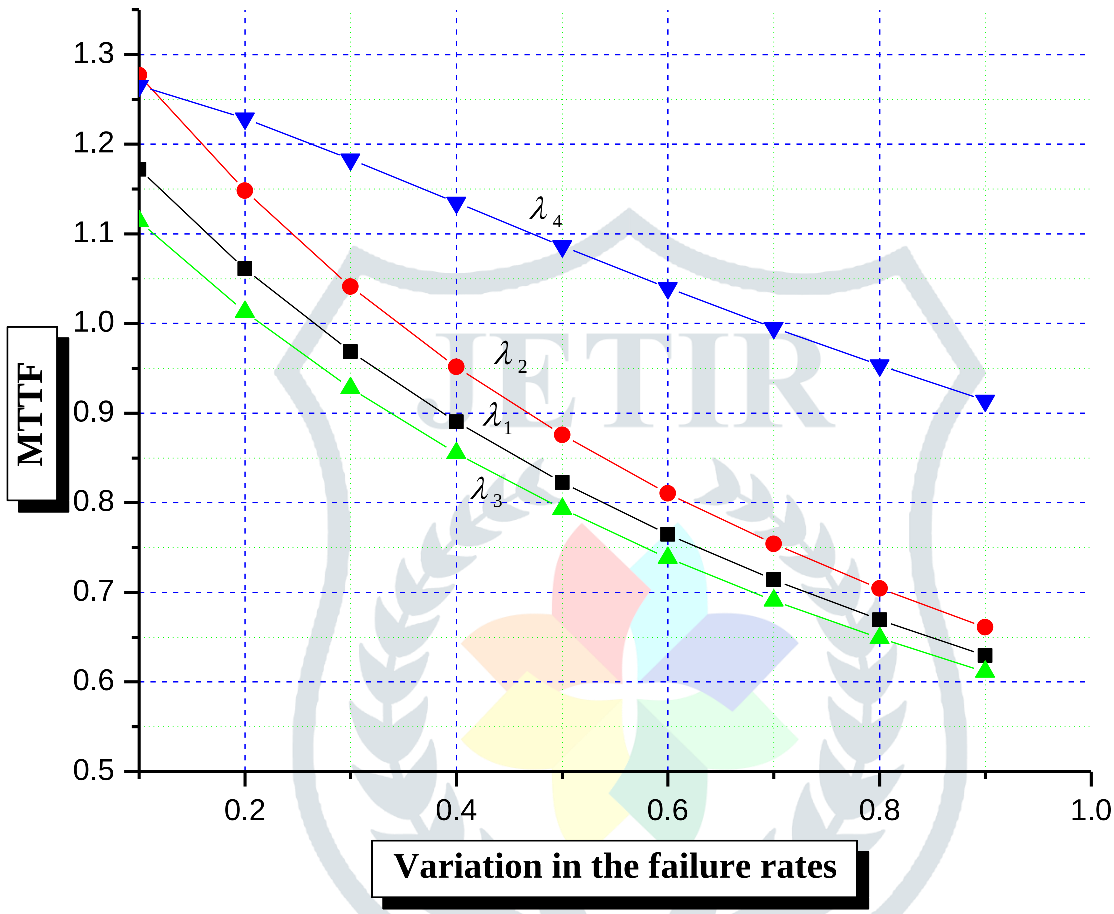


Figure 3: MTTF w.r.t. variation in failure rates

4.3 Sensitivity analysis of MTTF

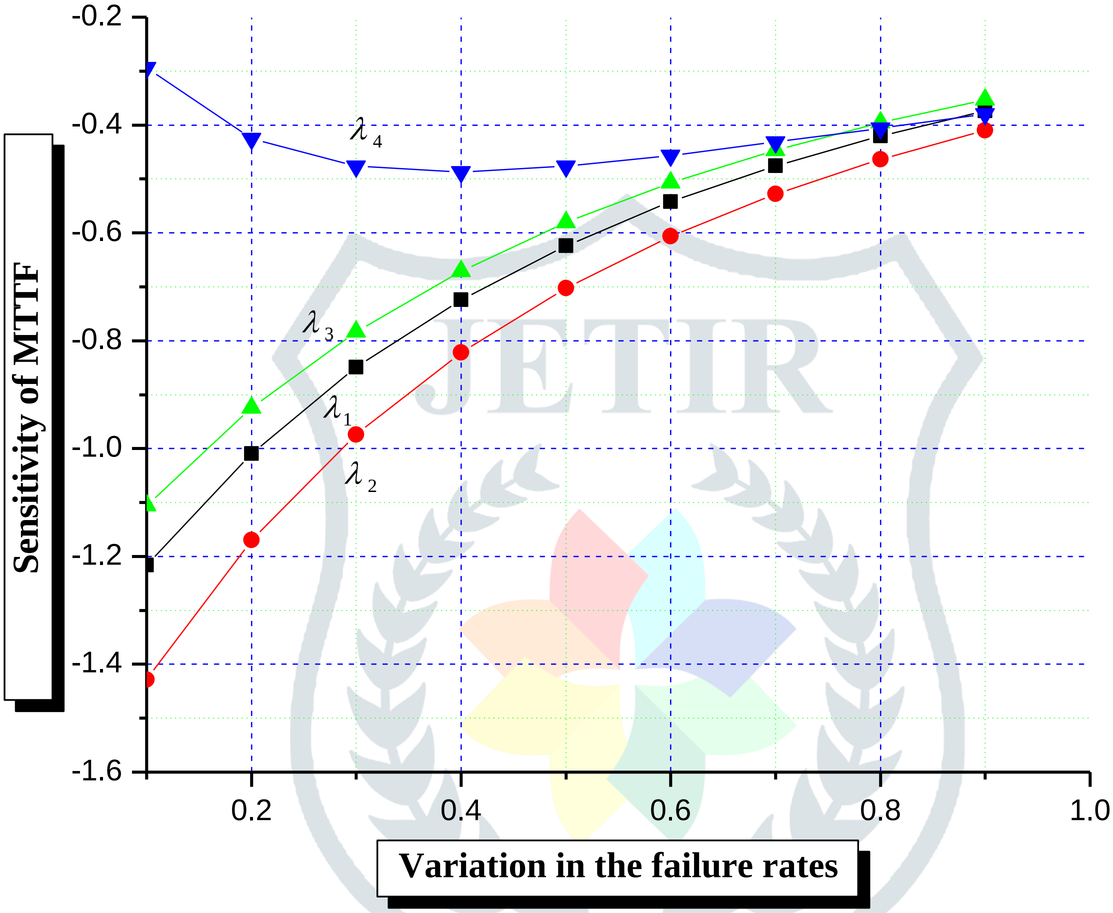
Authors performed the sensitivity analysis of the MTTF with respect to variation in the failure rates. For this, differentiate MTTF w.r.t. $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and set failure rate $\lambda_1 = 0.25/yr$, $\lambda_2 = 0.33/yr$, $\lambda_3 = 0.2/yr$, $\lambda_4 = 0.66/yr$. One by one vary each failure rate from 0.1 to 0.9

One can easily obtain the following Table (4) and Figure(4)

Table 4. Sensitivity of the MTTF

Variation in failure rates	$\frac{\partial(MTTF)}{\partial\lambda_1}$	$\frac{\partial(MTTF)}{\partial\lambda_2}$	$\frac{\partial(MTTF)}{\partial\lambda_3}$	$\frac{\partial(MTTF)}{\partial\lambda_4}$
0.1	-1.21582	-1.42811	-1.10552	-0.29348
0.2	-1.00907	-1.16988	-0.92431	-0.42499
0.3	-0.84946	-0.97387	-0.78307	-0.47629
0.4	-0.72393	-0.82196	-0.67105	-0.48690
0.5	-0.62360	-0.70208	-0.58086	-0.47683

0.6	-0.54225	-0.60597	-0.50726	-0.45660
0.7	-0.47546	-0.52784	-0.44649	-0.43185
0.8	-0.42001	-0.46355	-0.39577	-0.40564
0.9	-0.37352	-0.41006	-0.35304	-0.37961



4.4 Sensitivity of Reliability

Authors performed the sensitivity analysis of the reliability with respect to time. For this, differentiate $R(t)$ w.r.t. $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and set failure rate $\lambda_1 = 0.25/yr, \lambda_2 = 0.33/yr, \lambda_3 = 0.2/yr, \lambda_4 = 0.66/yr$. One by one vary time from 0 to 10 in these derivatives,One can easily obtain the following Table (5) and Figure(5)

Table 5:Sensitivity of the reliability

Time (t) In years	$\frac{\partial(R(t))}{\partial\lambda_1}$	$\frac{\partial(R(t))}{\partial\lambda_2}$	$\frac{\partial(R(t))}{\partial\lambda_3}$	$\frac{\partial(R(t))}{\partial\lambda_4}$
0	0	0	0	0
1	-0.39330	-0.39330	-0.39330	-0.15637
2	-0.26046	-0.26046	-0.26046	-0.14819
3	-0.11890	-0.11890	-0.11890	-0.07900

4	-0.04588	-0.04588	-0.04588	-0.03327
5	-0.01605	-0.01605	-0.01605	-0.01231
6	-0.00526	-0.00526	-0.00526	-0.00420
7	-0.00164	-0.00164	-0.00164	-0.00135
8	-0.00049	-0.00049	-0.00049	-0.00041
9	-0.00014-	-0.00014-	-0.00014-	-0.00012
10	-0.00004	-0.00004	-0.00004	-0.00003

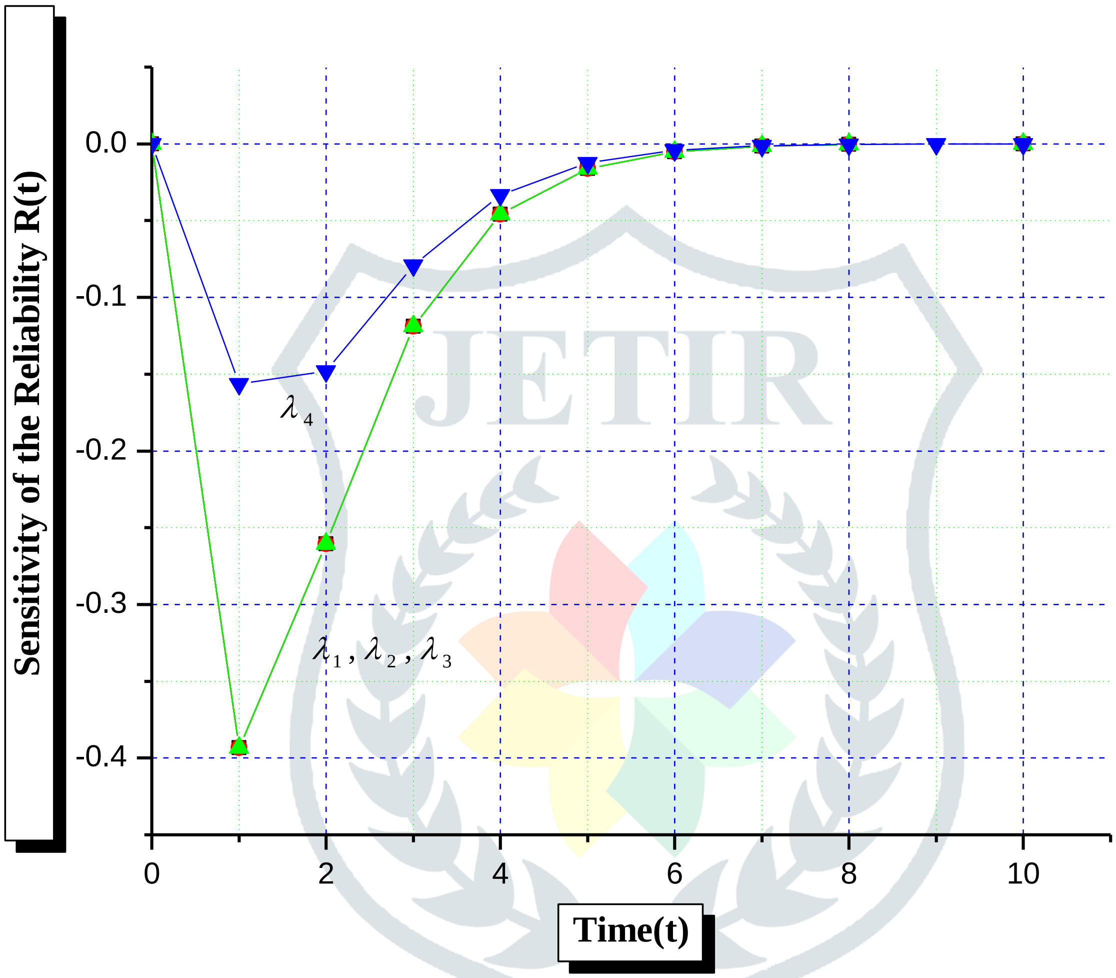


Figure 5: Sensitivity of the Reliability w.r.t. Time

5 Results and discussion

In this paper, the authors analyzed the performance of the mixer grinder through reliability approach. For this analysis only four major components of the mixture grinder like Control panel, Motor, Cutting assembly, coupler was taken into consideration. This present analysis was conducted by setting failure rates $\lambda_1 = 0.25/yr, \lambda_2 = 0.33/yr, \lambda_3 = 0.2/yr, \lambda_4 = 0.66/yr$. For these value of the parameters, the above graphs and figures were obtained. By examining the above tables and graphs the following results were obtained.

(a) From Table 2 and Figure 2, one can observe that in one year of operation, reliability decreases very rapidly. At time t=1 reliability is 0.39300 and at time t=2 reliability is 0.13023.After 2 year reliability decreases and once it crosses five years reliability is 0.00321 which is quite low. In 8th and 9th year reliability is almost zero. At t=10 reliability is zero. This clearly indicates that the average life of mixture grinder is 4 to 5 years. After this time period, it is not proper to use the same mixture grinder without repair or replacement.

(b) From Table 3 and Figure 3, one can observe that MTTF of the mixture grinder decreases with an increase in failure rates of the each component. But for the variation in the failure rate of coupler MTTF is more and it is less for the cutting assembly. It indicates that increase in the failure rate of coupler doesn't affect the system MTTF much.

(c) From Table 4 and Figure 4, one can observe that System MTTF is highly sensitive with respect to the failure rates of the motor. As failure rate of the motor increases, it affects the mixture grinder MTTF very much. MTTF is very less sensitive when the failure rate of coupler is increased.

(d) From Table 5 and Figure 5, one can observe that sensitivity of reliability with respect to time. System reliability is the most sensitive with respect to non-functioning of the control panel, motor and cutting assembly.

6 Conclusion

In the present paper, authors analyzed the performance of the mixer grinder by taking its four major components like control panel, motor, cutting assembly and coupler into their consideration. With the help of Markov model state transition diagram was drawn. Kolmogorov differential equations were developed and solved with the help of Laplace transformation. Explicit expression of system reliability, MTTF were obtained. Sensitivity analysis on system MTTF and reliability were also carried out. Overall from the above result and discussion it is found that control panel, motor, cutting assembly all effect the system reliability and MTTF is sensitive w.r.t. increase in the failure rate of the Motor. So, it is suggested to the Mixture grinder company that pay more attention to the performance of these components so that overall performance of the mixer grinder could be improved. It is expected that this result would be of great help for the mixer grinder companies.

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