



## An Insight into SU(5)-The Prototype GUT

Neelu Mahajan, Samandeep Sharma\*

Assistant Professor

Department of Physics,

Goswami Ganesh Dutta Sanatan Dharma College, Chandigarh, India.

Corresponding Author: samandeep.sharma@ggdsd.ac.in

**Abstract:** The Standard Model (SM) of particle physics, based on the gauge group  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  accounts for the basic constituents of matter, in three generations of quarks, three generations of leptons and the gauge bosons. Despite providing a successful explanation of the fundamental particles and their interactions to a remarkable level of precision, the SM suffers from few shortcomings which urge us to look for physics beyond the SM (BSM). Amongst the various BSM approaches proposed over the last few decades, the ones based upon Grand Unified Theories (GUTs) have turned out to be quite noteworthy. Numerous models based upon GUTs such as SU(5), SU(6),  $E_6$ , SO(10) etc. have been discussed in literature, however ideas based on SU(5) continue to be especially interesting. This can be attributed to the fact that SU(5) being the prototype GUT incorporates all the essential features of grand unification without the need of complicated algebra. In the present paper, beginning with the urge to look for grand unified theories, we intend to lay out the fundamentals of SU(5) GUT. Particular emphasis would be laid on its theoretical framework and generation of fermion masses in it. Shortcomings of SU(5) and a roadmap to more ambitious theories would also be sketched.

**Keywords -** BSM Physics, Grand Unified Theories, SU(5) GUT.

### I. INTRODUCTION

From the earliest times, man's dream has been to comprehend the complexity of nature in terms of as few unifying concepts as possible. History of unification in Physics dates back to 11th century when physicists Al Biruni and (later on) Galileo declared that the laws of Physics discovered here on earth apply equally to the phenomena occurring elsewhere in the universe. This faith in the unity of nature acted as a stepping stone for the idea of unification. The next step in this direction was by Newton who some 300 years ago unified the terrestrial gravitation of Galileo with the celestial gravitation of Kepler and gave birth to the concept of universal gravitation. In 19th century, attempts towards unification of the till then two distinct forces of nature - electricity and magnetism were begun by Faraday and Ampere who showed that magnetic forces are produced by electric charges in motion. The work of Faraday culminated with Maxwell who showed that a manifestation of the unification of electricity and magnetism must mean the production by accelerating electric charges of electromagnetic radiation. In the present day context, Glashow, Weinberg and Salam unified electromagnetic and weak interactions using the gauge group  $SU(2)_L \times U(1)_Y$  along with the idea of spontaneous symmetry breaking. This was further unified with the strong interactions [1] to form the "Standard Model (SM) of Particle Physics" [2] using the gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  which is spontaneously broken to  $SU(3)_C \times U(1)_{em}$ . The SM successfully describes or atleast is consistent with all known facts of elementary particle physics. Despite of its tremendous success, the SM is quite unsatisfactory, since it builds on many assumptions and leaves many fundamental questions unanswered, and thus is likely to be an incomplete theory. To this end, several beyond the Standard Model (BSM) approaches have been proposed in literature. Amongst these approaches, the ideas based upon grand unification, especially SU(5) grand unified theory would be our subject of interest in the present paper. However, before going into the details of beyond standard model theories, we would like to discuss some of the essentials of SM.

The paper is organized as follows. Section II introduces the SM along with various fields in it. The shortcomings of the SM urging us to look for theories beyond the Standard Model (BSM) have been discussed in Section III. Section IV describes basics of grand unified theories (GUTs) with particular emphasis on SU(5) GUT. Along with the description of incorporating the SM fermion fields in SU(5) matter multiplets and gauge bosons in SU(5), the spontaneous symmetry breaking and the fermion mass generation mechanism have also been discussed. A very famous model based upon SU(5) GUT, viz., the Georgi Jarlskog Model has been discussed in Section V. Despite being quite a successful theory, SU(5) GUT suffers from certain shortcomings, which have been discussed briefly in Section VI. Lastly, Section VII summarizes our conclusions.

### II. THE STANDARD MODEL

The SM is a quantum field theory that is based on the gauge symmetry  $SU(3)_C \times SU(2)_L \times U(1)_Y$  ( $\equiv G_{SM}$ ). This gauge group includes the symmetry group of strong interactions,  $SU(3)_C$ , and the symmetry group of electroweak interactions,  $SU(2)_L \times U(1)_Y$ . The sign  $\times$  means the direct product of the three subgroups i.e., the three subgroups commute. The model contains three types of fields:

## 2.1 The Matter Fields

The matter field of SM are the leptons and quarks, carrying spin 1/2. They are classified as left handed isospin doublets and right handed isosinglets, moreover, quarks are color triplets. This symmetry pattern is realized in the first, second and third generations of the fermions in identical form. The quantum numbers of the matter fields under the symmetry group  $(SU(3)_c \times SU(2)_L \times U(1)_Y)$  can be given as-

$$\text{Quarks: } q_L^A(3, 2, 1/3) + u^c A_L(\bar{3}, 1, -4/3) + d^c A_L(3, 1, 2/3) \quad (1)$$

$$\text{Leptons: } l_L^A(1, 2, -1) + e^c A_L(1, 1, 2), \quad (2)$$

where  $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ ,  $l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$  are the left handed quark and lepton doublets respectively. Here  $A=1, 2, 3$  is the family index and the superscript  $c$  denotes charge conjugation. The different isospin assignment to the left handed and right handed fields allows for maximal parity violation in the weak interactions. Given the assignments of electric charge, hypercharge and isospin, three color degrees of freedom are needed in quark sector to cancel anomalies and to render the gauge field theory renormalizable. The same symmetry pattern is needed in each of the three generations to suppress the flavor changing neutral current interactions to the level excluded by experimental analysis. Moreover, atleast three generations must be realized in nature to incorporate CP violation in the Standard Model.

## 2.2 The Gauge Fields-The Force Carriers

There are twelve spin one boson fields which belong to the adjoint i.e. the  $(1, 8) + (3, 1) + (1, 1)$  representation of  $SU(2)_L \times SU(3)_c$ . The first eight are the gluons which mediate the strong interactions between quarks and the last four are  $(W^+, W^-, Z_0, \gamma)$ , the vector bosons of the electroweak theory. The remarkable point is that the gauge fields are purely geometrical objects. Their number and properties are uniquely determined by the gauge structure of the theory.

## 2.3 The Higgs Field - 'The GOD's Particle'

Gauge symmetry does not allow any mass term for the gauge bosons or fermions in the SM. The fact that the weak gauge bosons are massive particles i.e.  $M_{W^\pm}, M_Z \neq 0$  indicated that  $SU(2)_L \times U(1)_Y$  is NOT a symmetry of the vacuum. In contrast, the photon being massless reflects that  $U(1)_{em}$  is a good symmetry of the vacuum. Therefore, the symmetry breaking pattern in the SM should be:

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}$$

The above pattern is implemented in the SM by means of the so called 'Higgs Mechanism' which provides proper masses to the  $W^\pm$  and  $Z$  gauge bosons and to the fermions, and leaves as a consequence the prediction of a new particle- The Higgs boson, which is given by

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, Y(\phi)=1, V(\phi)=-\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \quad (3)$$

## III. WHY BEYOND STANDARD MODEL?

The SM is a mathematically consistent renormalizable field theory which predicts or at least is consistent with all the experimental facts. Major successes of the SM include the prediction of existence as well as the form of weak neutral currents; prediction of existence and masses of charm quark,  $W$  and  $Z$  boson; providing a very close estimate to the value of the magnetic moment of electron and many more. In spite of these extraordinary successes, we can still not be satisfied with the theory. The main reason is that there are conceptual difficulties with the SM, which seems to indicate that there is new physics beyond it. The main reasons for considering new physics beyond SM are-

1. Too many free parameters: There are 19 free parameters in SM, if neutrinos are massless. These include- 3 gauge coupling constants, 9 charged fermion masses (3 leptons, 6 quarks), 2 parameters of Higgs potential ( $\mu^2$  and  $\lambda$ ), 3 mixing angles and 1 phase in the quark mixing matrix, the vacuum parameter of QCD, i.e.  $\theta_{QCD}$ . If neutrinos are taken to be massive, several additional parameters (viz. 3 neutrino masses, 3 mixing angles and 1 phase of the lepton mixing matrix) have to be included and total number of free parameters of SM increases to 26.

2. Gauge problem: The SM is a complicated direct product of three subgroups with separate gauge couplings. There is no explanation for why only the electroweak part is chiral (parity violating). Similarly, the SM incorporates but does not explain charge quantization, another fundamental fact of nature, i.e. why all particles have charges which are multiples of  $e/3$ .

3. Fermion problem: All matter under ordinary terrestrial conditions can be constructed out of the fermions ( $\nu_e, e^-, u, d$ ) of the first family. Yet we know from laboratory studies that there are ( $\geq 3$ ) families. SM does not provide any explanation for the purpose of the second and third families, which are merely heavier repetitions of the first family. Furthermore, there is no explanation or prediction for the fermion masses which vary over atleast five orders of magnitude. Also, the neutrinos have zero mass in the SM. The convincing evidence for neutrino mass from recent neutrino oscillation experiments imply that one would certainly need an extension of SM.

4. Non-inclusion of gravity: Gravitational interactions are not fundamentally unified with the other interactions in the SM, although it is possible to graft on classical general relativity by hand. However, the attempts to quantize it yield a nonrenormalizable theory.

5. Absence of dark matter candidate: Astronomical observations tell us that the ordinary baryonic matter is only a tiny fraction of the energy of the universe. There are observations that point towards the fact that a kind of nonluminous matter is out there that is actually much more abundant than the ordinary baryonic matter. A natural explanation is that dark matter consists of a new kind of particles that are stable, massive at electroweak scale and weakly interacting. However, no such particle is present within the framework of SM which is consistent with these properties. The SM thus lacks a viable dark matter candidate and hence any explanation for the existence of dark matter in the Universe.

The above discussion clearly brings about the fact that despite being tremendously successful in explaining numerous phenomena, the SM is far from being a complete theory. This led particle theorists to develop and study various extensions [3-7] of the SM such

as Grand Unified Theories (GUTs), technicolor theories, supersymmetric theories, string theory etc. Out of various beyond Standard Model (BSM) approaches as listed above, the ones based on GUTs have turned out to be quite noteworthy. Various GUT models based upon symmetries such as SU(5), SU(6), SO(10), E<sub>6</sub> etc. have been proposed in literature. However, the models based upon SU(5) GUT can be considered especially interesting owing to their simple algebra. In the following sections, we would present a brief introduction to the SU(5) based GUTs with special emphasis on the fermion mass generation mechanisms therein.

**IV. GRAND UNIFIED THEORIES**

The basic idea in a GUT [8] is that if gauge group of SM is embedded in a larger underlying group G, then the additional symmetries may restrict some of the features that were arbitrary in the SM. The hypothesis of grand unification states that gauge group of SM is the remnant of a larger simple or semisimple group G, which is spontaneously broken at very high energies. The scheme looks like-

$$G \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em} \tag{4}$$

where the breaking of G may be a multistage one at one (or several) characteristic mass scale. The first attempt in this direction was made by Pati and Salam [9], who unified the quarks and leptons within the group SU(2)<sub>L</sub> × SU(2)<sub>R</sub> × SU(4)<sub>c</sub> by extending the color gauge group to include the leptons. A more ambitious approach was taken independently in the same year by Georgi and Glashow who proposed the rank 4 simple group SU(5) as the grand unification group. Another minimal possibility (which contains the Pati Salam group) is SO(10) first proposed by Georgi [10] and Fritzsch and Minkowski [11]. In this section, we will discuss the essentials of SU(5) based GUT models in somewhat detail.

**4.1 SU(5) GUT**

As we know the SM has four diagonal generators corresponding to T<sub>3</sub> and T<sub>8</sub> of color, T<sub>3</sub> of weak isospin and Y, therefore, any group G ⊃ SU(3)<sub>c</sub> × SU(2)<sub>L</sub> × U(1)<sub>Y</sub> must be large enough to contain these four diagonal generators, i.e., it must be at least of rank 4. It was argued by Howard Georgi and Sheldon Glashow [12] in 1974 that SU(5) has the most desirable properties to form a group G to incorporate SM as a subgroup.

**4.1.1 Embedding fermions and gauge bosons of SM into SU(5) representations**

Each generation of SM consists of 15 left handed (LH) fermionic fields which can be shown as:

$$\begin{aligned} (u_a, d_a)_L &: (3, 2); (v_e, e^-)_L : (1, 2); \\ u^{ca}_L, d^{ca}_L &: (3, 1); \\ e^+_L &: (1, 1) \end{aligned} \tag{5}$$

where the superscript c denotes charge conjugation and the numbers in the parenthesis represent (SU(3), SU(2)) quantum numbers. The simplest realization of the model incorporates the 15 LH fields in the representations-

$$\begin{aligned} \psi_i &: 5 = (3, 1, -1/3) + (1, 2, 1/2) \\ \psi^i &: \bar{5} = (\bar{3}, 1, 1/3) + (1, 2, -1/2) \\ \chi_{ij} &: 5 \otimes_A 5 = 10 = (\bar{3}, 1, -2/3) + (3, 2, 1/6) + (1, 1, 1) \end{aligned} \tag{6}$$

The above equations represent the (SU(3)<sub>c</sub>, SU(2)<sub>L</sub>, U(1)<sub>Y</sub>) quantum numbers of the fundamental, conjugate and 2 index antisymmetric representations of SU(5) respectively. We know that the representation R of the unified group should be free of anomaly for consistency. In any SU(N), N > 3, all complex representations have anomaly, so one considers two representations R<sub>1</sub> + R<sub>2</sub> with equal and opposite anomalies. It was found that the 5 pllet and the 10 pllet of SU(5) contribute equally to the anomaly. Thus, the SM fields of a generation may be accommodated in the anomaly free  $\bar{5} + 10$  representation of SU(5). To be more explicit, (d<sub>1</sub><sup>c</sup>, d<sub>2</sub><sup>c</sup>, d<sub>3</sub><sup>c</sup>, e<sup>-</sup>, -v<sub>e</sub>) and

$$\begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ & 0 & u_1^c & -u_2 & -d_2 \\ & & 0 & -u_3 & -d_3 \\ & & & 0 & -e^c \\ & & & & 0 \end{pmatrix} \tag{7}$$

respectively represent the  $\bar{5}$  and 10 dimensional representations of SU(5). The gauge bosons belong to the adjoint (regular) representation of SU(5) with dimension 5<sup>2</sup> - 1 = 24 which decomposes as:

$$24 = (8, 1, 0) + (1, 3, 0) + (1, 1, 0) + (3, \bar{2}, -5/6) + (\bar{3}, 2, 5/6) \tag{8}$$

where the first three terms represent the usual SM bosons, while the next two terms represent the 12 new gauge bosons of SU(5), which carry both flavor and color and thus can mediate baryon number and lepton number violating processes.

**4.1.2 Spontaneous Symmetry Breaking in SU(5)**

The choice of Higgs scalar and the components that acquire vacuum expectation value (VEV) determine the phenomenology in any GUT. Conventionally, symmetry breaking of SU(5) gauge group takes place in two steps: first, via a large VEV (10<sup>15</sup> GeV) to the G<sub>SM</sub> and then via a much smaller VEV from G<sub>SM</sub> to SU(3)<sub>c</sub> × U(1)<sub>em</sub>. The first stage of symmetry breaking does not change the rank of the group, thus, can be achieved by a Higgs scalar in the adjoint representation Σ(24). The most general form for the scalar potential for the field Σ which acquires a VEV can be written as,

$$V(\Sigma) = -1/2 m_\Sigma^2 \text{Tr} \Sigma^2 + a/4 \text{Tr}(\Sigma^2)^2 + 1/2 b \text{Tr} \Sigma^4 \tag{9}$$

where m<sub>Σ</sub><sup>2</sup> > 0. VEV of Σ should commute with the generators of SU(3)<sub>c</sub>, SU(2)<sub>L</sub>, U(1)<sub>Y</sub> so that the SM gauge bosons remain massless at this stage of symmetry breaking while the remaining gauge bosons of SU(5) i.e. X, Y become superheavy with

mass  $M_{X,Y} \approx M_U$ . Thus,  $\langle \Sigma \rangle$  is chosen to be proportional to the unit matrix in the subspace of  $SU(3)$  and  $SU(2) \times U(1)$ . Hence, one can see that the expected symmetry breaking pattern requires the 24 plet Higgs scalar to acquire a VEV-  

$$\langle \Sigma \rangle = v_0 [\text{diag}(1, 1, 1, -3/2, -3/2)]$$

where  $v_0 = (2 m_\Sigma^2 / (15a + 7b))^{-1/2}$ . The next stage of symmetry breaking requires a SM Higgs doublet, which can now belong to a 5 plet of  $SU(5)$ ,  $\phi(5)$ , whose potential can be written as

$$V(\phi) = -1/2 m_\phi^2 \phi^\dagger \phi + 1/4 (\phi^\dagger \phi)^2, m_\phi^2 > 0$$

ensures a VEV of this field given by-

$$\langle \phi \rangle = (0, 0, 0, 0, 1) v / \sqrt{5}$$

At this stage, the  $W^\pm$  and  $Z$  bosons acquire mass of the order  $M_W$  while the gluons and the photons still remain massless.

#### 4.1.3 Fermion Masses in $SU(5)$

Knowing the fact that each generation of fermions is contained in  $\bar{5} + 10$  representation of  $SU(5)$ , the dimensionality of the scalars that can form Yukawa couplings to generate fermion masses can be inferred from the product  $(\bar{5} + 10) \times (\bar{5} + 10)$  or equivalently from the decomposition of the Kronecker product of fermion irreducible representations (irrep):

$$\begin{aligned} \bar{5} \times \bar{5} &= \bar{10} + \bar{15} \\ \bar{5} \times 10 &= 5 + \bar{45} \\ 10 \times 10 &= \bar{5} + 45 + 50 \end{aligned}$$

the possible Higgs representations are the conjugates of those found in the products above i.e. the Higgs can be a 5, 10, 15, 45 or a 50 plet one. However, as we know  $SU(3)_c \times U(1)_{em}$  is the remnant symmetry of SM, so our Higgs should be color and charge neutral. Out of the above 5 irreps 10, 15, 50 do not have a neutral color singlet component and thus can not contribute to fermion masses. So, we can introduce only a 5 and 45 dimensional Higgs scalar.

Considering the minimal model with 5 dimensional Higgs  $\phi_i$ , the Yukawa couplings in the lagrangian can be written as:

$$V(\Sigma) = -\frac{1}{2} m_\Sigma^2 \text{Tr} \Sigma^2 + \frac{a}{4} \text{Tr}(\Sigma^2)^2 + \frac{1}{2} b \text{Tr} \Sigma^4$$

Here we have suppressed the generation indices and the charge conjugation matrices.  $i, j, k, l, m$  are  $SU(5)$  indices.  $y^{(D)}$  and  $y^{(U)}$  are the Yukawa coupling matrices in flavor space. When  $\phi$  develops a VEV, it will generate down quark and charged lepton masses through the term proportional to  $y^{(D)}$  and up quark masses through the term proportional to  $y^{(U)}$ .  $y^{(U)}$  is necessarily symmetric due to the form of the index contractions. These  $\psi$  and  $\chi$  states which appear in the original gauge invariant lagrangian are generally not the physical mass eigen states and the coupling matrices  $y^{(D)}$  and  $y^{(U)}$  are not diagonal in this basis. We can diagonalize these in another basis by using biunitary transformations such that e.g.  $y^{(D)'} = U^T y^{(D)} V$  is diagonal. Since down sector quarks and charged leptons get masses from the same yukawa coupling  $y^{(D)}$ , we find the following mass relations:

$$\begin{aligned} m_d = m_e &= M_{11}^D = y_{11}^{(D)'} \langle \phi \rangle > \\ m_s = m_\mu &= M_{22}^D = y_{22}^{(D)'} \langle \phi \rangle > \\ m_b = m_\tau &= M_{33}^D = y_{33}^{(D)'} \langle \phi \rangle > \end{aligned}$$

Up quark masses come from the term involving  $M_U$ , but since there are no right handed neutrinos, no neutrino masses are present in minimal  $SU(5)$ . The relation  $m_b = m_\tau$  is quite compatible with the low energy data-  $m_b(M_Z) \sim 5\text{GeV}$  and  $m_\tau(M_Z) \sim 1.7\text{GeV}$  because the inclusion of radiative corrections changes the ratio  $m_b/m_\tau$  from 1 at  $M_X$  to approximately 3 at  $M_Z$  due to large renormalization of the colored b quark field. However, the same effect applied to the second generation would require  $m_s \sim 3m_\mu$  at energies  $\sim M_Z$  while experimentally  $m_s \sim 100\text{ MeV}$  and  $m_\mu \sim 105\text{ MeV}$ . Thus, a single Higgs 5 plet cannot account for the observed low energy fermion masses. Similarly, exploring the couplings with a 45 dimensional Higgs leads to quark lepton (q-l) mass relations given as:  $m_e = 3m_d$ ,  $m_\mu = 3m_s$ ,  $m_\tau = 3m_b$  at  $M_X$  scale which are again not in agreement with the experimental data.

#### V. THE GEORGI JARLSKOG MODEL

Georgi and Jarlskog chose reasonable values for quark masses at 1 GeV as

$$m_d \approx 5\text{MeV}, m_s \approx 100\text{MeV}, m_b \approx 5\text{GeV}$$

when renormalized at  $\approx 10\text{ GeV}$ . To compare these with the lepton masses, we should decrease all of them by an additional factor of 3 to incorporate the effect of the color  $SU(3)$  interactions in going to the unification scale  $\approx 10^{15}\text{ GeV}$ . This gives

$$m_d \approx 1.7\text{MeV}, m_s \approx 33\text{MeV}, m_b \approx 1.3\text{GeV}$$

renormalized at  $10^{15}\text{ GeV}$ . This is all very rough and many details have been neglected for simplicity, but we have obtained a reasonable set of quark masses which can be compared with the lepton masses

$$m_e \approx 0.51\text{MeV}, m_\mu \approx 106\text{MeV}, m_\tau \approx 1.8\text{GeV}$$

Comparing the above two equations, we find that the quark -lepton mass relations above  $10^{15}\text{ GeV}$  should be the following-

$$m_b = m_\tau, m_\mu = 3m_s, m_d = 3m_e$$

As we have already seen, considering either 5 dimensional Higgs or the 45 dimensional one alone cannot yield the above mass relations. The basic problem being faced here is the fact that we are getting the same q-l mass relations for the three generations. But actually these mass relations should be different for the three generations of quarks and leptons as required by eqn.(19). To overcome this problem, Howard Georgi and C. Jarlskog constructed a model [13] incorporating eqn.(19) in a natural zeroth order mass relation, which is discussed below.

Fermionic fields in this model are: three right handed(RH) SU(5) 5's denoted by  $\psi^a_{jR}$  and three left handed (LH) SU(5) 10's  $\chi^{ab}_{jL} = -\chi^{ba}_{jL}$  with a,b=1,2,3,4,5 being the SU(5) indices and j being the flavor index. To break the SU(5) symmetry and implement the improved q-l mass relations, Georgi and Jarlskog used somewhat complicated Yukawa couplings. The Higgs content of the model is - three 5 plets  $\phi^a_j$ ,  $\langle \phi^a_j \rangle = \mu_j$ ,  $j = 5$  and  $\langle \phi^a_j \rangle = 0$ ,  $j \neq 0$  and one 45 plet Higgs  $F^{ab}_c$  with VEV as defined earlier. In this model, the mass terms for quarks and leptons can be written as

$$-\mathcal{L}_{\text{mass}} = \bar{U}_{jR} M^U_{jk} U_{kl} + \bar{D}_{jR} M^D_{jk} D_{kl} + \bar{l}_{jR} M^l_{jk} l_{kl} + h.c \tag{19}$$

where  $U_j = (u, c, t)$ ;  $D_j = (d, s, b)$ ;  $l_j = (e^-, \mu^-, \tau^-)$  are the +2/3, -1/3 quark fields and the charged lepton fields respectively with the mass matrices given as-

$$\begin{aligned} M^U &= \begin{pmatrix} 0 & D & 0 \\ D & 0 & F \\ 0 & F & E \end{pmatrix} \\ M^D &= \begin{pmatrix} 0 & A' & 0 \\ A & C' & 0 \\ 0 & 0 & B \end{pmatrix} \\ M^l &= \begin{pmatrix} 0 & A' & 0 \\ A & -3C' & 0 \\ 0 & 0 & B \end{pmatrix} \end{aligned} \tag{20}$$

The factor of -3 in the leptonic mass matrix arises because of the form of VEV of the 45 dimensional Higgs field. Diagonalization of the above mass matrices leads to the desired q-l mass relations given by eqn.(19).

### VI. GOING BEYOND SU(5)

SU(5) undoubtedly can be considered as the simplest GUT. Though a satisfactory explanation of fermion mass generation can be obtained in this theory, still it cannot be considered as a complete and ultimate grand unified theory. This can be attributed to some of the lacunas in this model, which we are briefly going to discuss now. One of the most prominent issues in this model is the absence of gauge coupling unification. Figure 1 shows [14] the evolution of the three gauge coupling constants as a function of energy. A careful look at this figure reveals the fact that these couplings do not meet at a point, even including the errors in the measurements of the coupling constants. This result is valid even when the two loop contributions are included and hence there is no grand unification. This rules out the minimal SU(5) grand unified theory. Another blow to the minimal SU(5) GUT comes from the proton decay constraints. For the minimal SU(5) grand unified theory, the prediction [15] for proton lifetime comes out to be within  $10^{30}$ - $10^{31}$  years. This bound is ruled out by the present experimental data [16] which predicts the proton lifetime to be  $>1.6 \times 10^{33}$  years. However, some extensions such as inclusion of gravity effects or the supersymmetric version of the model may still be consistent. Therefore, it is still worthwhile to study the SU(5) GUT.

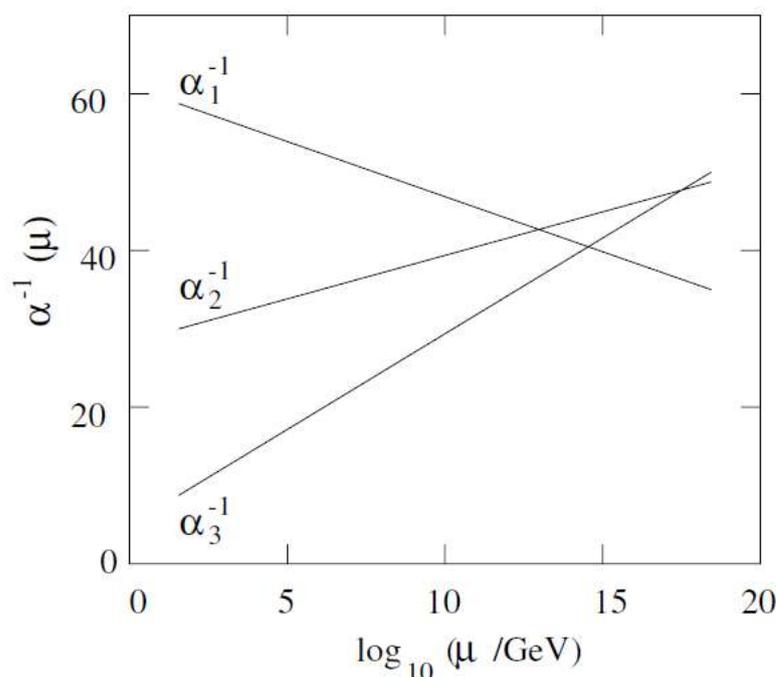


Figure 1: Evolution of gauge coupling constants in SU(5) grand unified theory.

## VII. SUMMARY AND CONCLUSIONS

The Standard Model of particle physics gives us quite a satisfactory explanation for the present-day fundamental particles and their interactions. However, owing to some conceptual and fundamental shortcomings in SM, various beyond the SM approaches have been proposed over the past few decades. Out of these BSM approaches, the ideas based upon Grand Unified Theories are especially interesting. Many models of GUTs have been proposed, out of which the SU(5) GUT has been reviewed in the present paper. SU(5) is a rank 4 gauge group, which can incorporate the fermions of one generation of SM into its 5+10 plet representations. In order to provide a satisfactory explanation of fermion masses and mixings within this theory, various models have been proposed. In the present paper, one of the most popular SU(5) GUT models, viz. the Georgi-Jarlskog Model has been reviewed briefly. One finds that despite providing a satisfactory explanation of fermion mass generation mechanisms with the help of relatively simpler GUT algebra, SU(5) suffers from certain shortcomings such as gauge coupling unification and proton decay constraints. Thus, SU(5) can not be considered as a complete GUT and some higher rank gauge group such as SO(10) can be the ultimate step for formulating a unified theory for quark and lepton interactions.

## VIII. ACKNOWLEDGMENT

The authors would like to thank the Principal, Goswami Ganesh Dutta Sanatan Dharma College, Chandigarh for providing the necessary facilities to work.

## REFERENCES

- [1] Fritzsche H., Gell-Mann M. and Minkowski P. 1975. Vectorlike weak currents and new elementary fermions. *Physics Letters* 59B: 256.
- [2] For excellent reviews on the Standard Model see, Donoghue J.F., Golowich E. and Holstein B.R. 1992. *Dynamics of the Standard Model*, Cambridge University Press.
- [3] Weinberg S. 1976, Implications of dynamical symmetry breaking. *Physical Review D* 13: 974-996; Implications of dynamical symmetry breaking: An addendum. *Physical Review D* 19: 1227.
- [4] Susskind L. 1979. Dynamics of spontaneous symmetry breaking in the Weinberg-Salam theory. *Physical Review D* 20: 2619; Eichten E. and Lane K. Dynamical breaking of weak interaction symmetries. *Physics Letters B* 90: 125-130.
- [5] Matrin S. A supersymmetry primer. arXiv:hep-ph/9709356; Kaul R.K. 1982. Gauge hierarchy in a supersymmetric model. *Physics Letters B* 109: 19-24.
- [6] Kaluza T. and Sitzungsber. d. Preuss.(1921). Zum Unitatsproblem in der Physik. *Akad. d. Wiss. Berlin*, 966; Klein O. and Zeitschrift F. 1926. Quantentheorie and funfdimensionale relativitatstheorie. *Physik* 37: 895-906.
- [7] Mukhi S. (1999). *The Theory of Strings: A Detailed Introduction*; Green M.B. et al. *Superstring theory* (1987) Cambridge University Press.
- [8] For reviews see Langacker P. 1981. Grand unified theories and proton decay. *Physics Reports Phys. Rep.* 72 (4): 185-385; Mohapatra R. N. 1986. *Unification and Supersymmetry: The frontiers of quark-lepton physics*. Springer, New York.
- [9] Pati J.C. and Salam A. 1973. Is baryon number conserved? *Physical Review Letters* 31(10): 661-664.
- [10] Georgi H. 1975. *Particles and Fields*, ed. C.E. Carlson, AIP, NY.
- [11] Fritzsche H. and Minkowski P. 1975. Unified interactions of leptons and hadrons. *Annals of Physics* 93: 193-266.
- [12] Georgi H. and Glashow S. 1974. Unity of all elementary particle forces. *Physical Review Letters* 32(8): 438-441.
- [13] Georgi H. and Jarlskog C. 1979. A new quark lepton mass relation in a unified theory. *Physics Letters B* 86(3):297-300.
- [14] Sarkar U. 2008. *Particle and astroparticle physics*. Taylor and Francis group NY London.
- [15] Georgi H., Quinn H. and Weinberg S. 1974. Hierarchy of interactions in unified gauge theories. *Physical Review Letters* 33, 451.
- [16] Hayato Y. et al (Super-Kamiokande Collaboration). 1999. Search for proton decay in  $p \rightarrow \bar{\nu} K^+$  in a large water Cherenkov detector. *Physical Review Letters* 83, 1529.