Complementary Tree Domination in Jahangir Graph $J_{2,m}$

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Abstract

A set $D$ of a graph $G = (V,E)$ is a dominating set if every vertex in $V - D$ is adjacent to some vertex in $D$. The domination number $\gamma(G)$ of $G$ is the minimum cardinality of a dominating set. A dominating set $D$ is called a complementary tree dominating set if the induced sub graph $< V - D >$ is a tree. The minimum cardinality of a complementary tree dominating set is called the complementary tree domination number of $G$ and is denoted by $\gamma_{ctd}(G)$. In this paper, results on complementary tree domination number $\gamma_{ctd}$, total complementary tree domination number $\gamma_{tctd}$ and connected complementary tree $\gamma_{cc_{td}}$ in Jahangir graph $J_{2,m}$ are found.

Keywords: Complementary tree domination number, total, connected complementary tree domination number, Jahangir graphs.

1 Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph $G$, let $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. The concept of domination was first studied by Ore [6]. A set $D \subseteq V$ is said to a dominating set of $G$, if every vertex in $V - D$ is adjacent to some vertex in $D$. The minimum cardinality of a dominating set is called the domination number of $G$ and is denoted by $\gamma(G)$. The concept of complementary tree domination was introduced by S. Muthammal M. Bhanumathi and P. Vidhya in [5]. A dominating set $D \subseteq V$ is called a complementary tree dominating (ctd) set, if the sub graph $< V - D >$ is a tree. The minimum cardinality of a complementary tree dominating set is called the complementary tree domination number of $G$ and is denoted by $\gamma_{ctd}(G)$.

A dominating set $D_t$ is called a total complementary tree dominating set of every vertex $v \in V$ is adjacent to an element of $D_t$ and $< V - D_t >$ is a tree. The minimum cardinality of a total complementary tree dominating set (tctd) is called the total complementary tree domination number of $G$ and is denoted by $\gamma_{tctd}(G)$.

A dominating set $D_c$ is called complementary tree dominating set ($cc_{td}$) if the induced sub graph $< D_c >$ is connected. The connected complementary tree domination number $\gamma_{cc_{td}}(G)$ is the minimum cardinality of $cc_{td}$ set. In this paper, complementary tree domination number($\gamma_{ctd}$), total and connected complementary tree domination number in Jahangir graphs are found.

Definition 1 Jahangir graphs $J_{n,m}$ for $m \geq 3$, is a graph on $nm + 1$ vertices i.e., a graph consisting of a cycle $C_{nm}$ with one additional vertex which is adjacent to $m$ vertices of $C_{nm}$ distance $n$ to each other on $C_{nm}$. 
Example 1 Figure 1 shows Jahangir graph $J_{2,8}$. It appears on Jahangir's tomb in his mausoleum. It lies in 5 kilometer north-west of Lahore, Pakistan across the river Ravi [1].

![Figure 1: J_{2,8}](image)

Example 2

In Figure 2 $\gamma_{ctd}(J_{2,m}) = 2$ where $D = \{3,6\}$.

2 Complementary Tree Domination Number, Total and Connected Domination Number of $J_{2,m}$

In this section, we study $\gamma_{ctd}$, $\gamma_{ctd}$, and $\gamma_{ctd}$ in Jahangir graphs $J_{2,m}$.

Remark 1 Let $v_{2m+1}$ be the label of the center vertex and $v_1$, $v_2$, ..., $v_{2m}$ be the label of the vertices that incident clockwise on cycle $C_{2m}$ so that $\deg(v_1) = 3$.

Theorem 2.1 For $m \geq 3$ the $\gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil$

Proof. Let $v_{2m+1}$ be the label of the center vertex and $v_1$, $v_2$, ..., $v_{2m}$ be the label of the vertices that incident clockwise on cycle $C_{2m}$ so that $\deg(v_1) = 3$. Let $D$ be a minimum ctd set of $J_{2,m}$. Therefore $|D| = \gamma_{ctd}(J_{2,m})$.

Case (i) $2m = 0 \pmod{3}$

Let $D = \{v_3, v_6, v_9, ..., v_{3i}\}$ where $i = 1,2, ..., m$ Then $D$ is a minimum ctd-set of $J_{2,m}$ and $3i = 2m$. Since $v_3$ dominates $v_2$ and $v_4$, $v_6$ dominates $v_5$ and $v_7$ etc and $v_{3i}$ dominates $v_{3i-1}$ and $v_1$ and odd label vertices dominated by the central vertex $v_{2m+1}$. Also $<V - D \geq T$, Where $T$ is a tree.
Therefore \( |D| = \gamma_{ctd}(G) = \left\lceil \frac{2m}{3} \right\rceil \)

Let \( D = \{v_1, v_4, \ldots, v_{3i-2}\} \) is a minimum ctd-set of \( J_{2,m} \).

If \( 3i - 2 = 2m \), then \( |D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil \)

If \( 3i - 2 \neq 2m \), then \( v_{2m} \) is a isolate vertex in \( <V - D> \) so that \( v_{2m} \in D \).

Therefore \( |D| = \left\lceil \frac{2m}{3} + 1 \right\rceil \)

= \( \left\lceil \frac{2m}{3} \right\rceil + 1 \)

Which contradicts the minimality

Let \( D = \{v_2, v_3, \ldots, v_{3i-1}\} \) is a minimum ctd-set of \( J_{2,m} \).

If \( 3i - 1 = 2m \), then \( |D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil \)

If \( 3i - 2 \neq 2m \), then \( v_{2m} \) is a dominated by \( v_{3i-1} \) and \( v_1 \) is dominated by \( v_2 \)

Therefore \( |D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil \)

Case (ii) \( 2m = 1 \) (mod 3)

Let \( D = \{v_3, v_6, \ldots, v_{3i}\} \) is a minimum ctd-set of \( J_{2,m} \). Hence \( v_4 \) is not dominated by \( D \) so that \( v_{2m} \in D \).

Therefore \( |D| = \left\lceil \frac{2m}{3} + 1 \right\rceil \)

= \( \left\lceil \frac{2m}{3} \right\rceil + 1 \)

Let \( D = \{v_1, v_4, \ldots, v_{3i-2}\} \) is a minimum ctd-set of \( J_{2,m} \).

If \( 3i - 2 = 2m \), then \( D \) is a minimal ctd-set of \( J_{2,m} \) then \( |D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil \)

If \( 3i - 2 \neq 2m \), then \( v_{2m} \) is a isolate in \( <V - D> \), so that \( v_{2m} \in D \).

Therefore \( |D| = \left\lceil \frac{2m}{3} + 1 \right\rceil \)

= \( \left\lceil \frac{2m}{3} \right\rceil + 1 \)
Which contradicts the minimality

Let \( D = \{v_2, v_5, \ldots, v_{3i-1}\} \)

If \( 3i - 1 = 2m \), then \( |D| = \left\lceil \frac{2m}{3} \right\rceil \)

If \( 3i - 2 \neq 2m \), then \( v_{2m} \) is dominated by \( v_{3i-1} \) and \( v_1 \) is dominated by \( v_2 \)

Therefore \( |D| = \left\lceil \frac{2m}{3} \right\rceil \)

**Case (iii)\( 2m = 1 \mod 3 \)**

Let \( D = \{v_1, v_4, \ldots, v_{3i-2}\} \)

If \( 3i - 2 = 2m \) or \( 3i - 2 = 2m - 2 \) then \( |D| = \left\lceil \frac{2m}{3} \right\rceil \)

From the above three cases

\[ \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil, \text{ for } m \geq 3 \]

**Total Complementary tree domination of \( J_{2,m} \)**

**Theorem 2.2** For \( m \geq 3 \), \( \gamma_{ctd}(J_{2,m}) > \gamma_{ctd}(J_{2,m}) \)

**Proof:**

Let \( D_t \) be a minimum tctd set of \( J_{2,m} \).

**Case (i) \( m \) is odd**

Let \( D_i = \{v_1, v_2, v_5, \ldots, v_{2m-1}, v_{2m}\} \)

\[ \therefore |D_i| = m + 1 \]

\[ i.e., \quad \frac{2m}{2} + 1 \]

\[ = \left\lceil \frac{2m}{2} \right\rceil + 1 \]

\[ = \left\lceil \frac{2m}{2} \right\rceil \]

**Case (ii) \( m \) is even**
Let \( D_t = \{v_1, v_2, v_5, v_6, \ldots, v_{2m-1}, v_{2m}\} \)

\[ \therefore |D_t| = m \]

\[ = \left\lfloor \frac{2m}{2} \right\rfloor \]

Which is minimum tctd set of \( J_{2m} \).

\[ \therefore \gamma_{ctd}(J_{2m}) = \left\lfloor \frac{2m}{2} \right\rfloor > \left\lfloor \frac{2m}{3} \right\rfloor \]

\[ = \gamma_{ctd}(J_{2m}) \]

**Connected complementary tree domination of \( J_{2m} \).**

**Theorem 2.3** For \( m \geq 3 \), \( \gamma_{ctd}(J_{2m}) = 2m - 2 \)

**Proof.** We know \( \gamma_{ctd}(C_m) = m - 2 \) for \( m \geq 3 \)

Let \( D_c = \{v_1, \ldots, v_{2m-3}, v_{2m+1}\} \). Where \( D_c \) is the minimum ctd-set of \( C_m \).

Here \( v_{2m} \) is dominate by \( v_1 \), \( v_{2m-3} \) dominate \( v_{2m-2} \) and \( v_{2m-1} \) is dominate by \( v_{2m+1} \).

\[ \therefore |D_c| = 2m - 3 + 1 = 2m - 2 \]

\[ \therefore \gamma_{ctd}(J_{2m}) = 2m - 2 \]

**References:**


