P$_4$ - Decomposition in Boolean Function Graph B($\overline{Kp}$, L(G), NINC) of a graph

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Abstract

For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph B($\overline{Kp}$, L(G), NINC) of G is a graph with vertex set V(G)∪E(G) and two vertices in B($\overline{Kp}$, L(G), NINC) are adjacent if and only if they correspond to two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by B$_2$(G). In this paper, P$_4$-decomposition of Boolean Function Graph B($\overline{Kp}$, L(G), NINC) of some standard graphs and corona graphs are obtained.

Keywords: Boolean Function graph, Edge Domination Number, Decomposition

1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A graph with p vertices and q edges is denoted by G(p, q). A subset F ⊆ E(G) is called an edge dominating set of G, if every edge not in F is adjacent to some edge in F. The edge domination number γ′(G) of G is the minimum cardinality taken over all edge dominating sets of G. The corona G$_1$o G$_2$ of two graphs G$_1$ and G$_2$ is defined as the graph obtained by taking one copy of G$_1$ (which has p$_1$ vertices) and p$_1$ copies of G$_2$, and then joining the ith vertex of G$_1$ to every vertex of in the ith copy of G$_2$. For any graph G, GoK$_1$ is denoted by G$^+$.

A decomposition of a graph G is a family of edge-disjoint subgraphs{G$_1$, G$_2$, …, G$_k$} such that E(G) = E(G$_1$) ∪E(G$_2$) ∪ … ∪E(G$_k$). If each G$_i$ is isomorphic to H, for some subgraph H of G, then the decomposition is called a H-decomposition of G. In particular, a P$_4$-decomposition of a graph G is a partition of the edge set of G into paths of length 3. In this case, G is said to be P$_4$-decomposable.   Several authors studied various types of decomposition by imposing conditions on G$_i$ in the decomposition. Heinrich, Liu and Yu[3] proved that a connected 4-regular graph admits a P$_4$-decomposition if and only if |E(G)| ≡ 0(mod 3). Sunil Kumar[10] proved that a complete r- partite graph is P$_4$-decomposable if and only if its size is a multiple of 3. P.Chithra devi and J. Paulraj Joseph [1] gave a necessary and sufficient condition for the decomposition of the total graph of standard graphs and corona of graphs into paths on three edges. Janakiraman et al., introduced the concept of Boolean function graphs [4 - 6]. For a real x, ⌊x⌋ denotes the greatest integer less than or equal to x. For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph B($\overline{Kp}$, L(G), NINC) of G is a graph with vertex set V(G)∪E(G) and two vertices in B($\overline{Kp}$, L(G), NINC) are adjacent if and only if they correspond to two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by B$_2$(G).

In this paper, P$_4$ - decomposition of Boolean Function Graph B($\overline{Kp}$, L(G), NINC) of some standard graphs are obtained.

2. Prior Results

Observation 2.1. [4]
  Let G be a graph with p vertices and q edges.
1. L(G) is an induced subgraph of $B_2(G)$ and the subgraph of $B_2(G)$ induced by vertices of G in $B_2(G)$ is totally disconnected.
2. Number of vertices in $B_2(G)$ is p + q and if $d_i = \deg_G(v_i)$, $v_i \in V(G)$, then the number of edges in $B_2(G)$ is $q(p - 3) + \frac{1}{2}\sum_{1 \le i \le p} d_i^2$.
3. The degree of a vertex of G in $B_2G)$ is q and the degree of a vertex e' of L(G) in $B_2(G)$ is $\deg_{L(G)}(e') + p - 2$. Also if $d^*(e')$ is the degree of a vertex e' of L(G) in $B_2(G)$, then $0 \le d^*(e') \le p + q - 3$.

3. Main Results

  In the following, P_4- decomposition of $B_2(P_n)$, $B_2(C_n)$, $B_2(K_{1,n})$ and corona graphs are found.
Theorem 3.1

  (i) For $n \ge 6$ and $n \equiv 0 \pmod 3$, the graph $B_2(P_n) - (2n - 3)K_2$ is P_{4-} decomposable .
  (ii) For $n \ge 7$ and $n \equiv 1 \pmod 3$, the graph $B_2(P_n) - 2K_2$ is P_{4-} decomposable.
  (iii) For $n \ge 5$ and $n \equiv 2 \pmod 3$, the graph $B_2(P_n) - (n+1)K_2$ is P_{4-} decomposable.

Proof: Let $V(P_n) = \{ v_1, v_2, \ldots, v_n \}$ and $e_i = (v_i, v_{i+1})$, $i = 1, 2, \ldots, n-1$, be the edges of P_n. Then $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1} \in V(B_2(P_n))$. $B_2(P_n)$ has 2n-1 vertices and $n^2 -2n$ edges. It is to be noted that, in all the sets the suffix i in v_i is integer modulo n, and j in e_j is integer modulo n-1, $v_0 = v_n$ and $e_0 = e_{n-1}$.

Let $F = \bigcup_{i=1}^{n-1} F_i$, where $F_1 = \{(v_1, e_i), i = 2, 3, \ldots, n-1\}$,

$F_i = \{ (v_i, e_j), 1 \le j \le n-1$ and $j \ne i-1, i\}$, $i = 2, 3, \ldots, n-2$.

$F_{n-1} = \{(v_n, e_j), j = 1, 2, \ldots, n-2\}$. $E(B_2(P_n)) = E(L(P_n)) \cup F = E(P_{n-1}) \cup F$.

Case 1. $n \equiv 0 \pmod 3$.

  Then the edge set of $B_2(P_n)$ can be decomposed into$((n^2 - 4n + 3)/3)P_4$ and $(2n - 3)K_2$. The edge set of $(2n - 3)K_2$ is given by the set $\{(v_1, e_{n-1})\} \cup \{ \bigcup_{i=1}^{n-2}(v_i, e_i) (e_i, e_{i+1})\}$. The edge sets of $((n^2 - 4n + 3)/3) P_4$ are given by the edge sets $A^{(1)}$, where $A^{(1)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-3)/3} A_{j,i}^{(1)})$ and $A_{j,i}^{(1)} = \{(v_i, e_{i+j,}), (e_{i+j}, v_{i+3j}), (v_{i+3j}, e_{i+j-1})\}$. Here, $< A^{(1)} > \cong (((n-1) (n-3))/ 3) P_4$.
Therefore, $B_2(P_n) - (2n - 3)K_2$ is P_{4-} decomposable .
Case 2. $n \equiv 1 \pmod 3$.

  Then the edge set of $B_2(P_n)$ can be decomposed into $((n^2 -2n -2)/3)P_4$ and $2K_2$. The edge set of $2K_2$ is given by the edge set $\{(v_1, e_{n-1}), (v_{(2n+1)/3}, e_1)\}$. The edge sets of $((n^2 -2n -2)/3)P_4$ are given by the edge sets $A^{(2)}$ and $A^{(3)}$, where $A^{(2)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-4)/3} A_{j,i}^{(2)})$,where

$A_{j,i}^{(2)} = \{(v_i, e_{i+j,}), (e_{i+j}, v_{i+3j}), (v_{i+3j}, e_{i+j-1})\}$ and $A^{(3)} = \bigcup_{i=1}^{n-2} A_i^{(3)}$,

$A_i^{(3)} = \{(v_n, e_i), (e_i, e_{i+1}), (e_{i+1}, v_{i+(2n+1)/3})\}$. Here, $< A^{(2)} > \cong ((n-1)((n-4))/3) P_4$ and
$< A^{(3)} > \cong (n-2) P_4$. Therefore, $B_1(P_n) -2K_2$ is P_{4-} decomposable.
Case 3. $n \equiv 2 \pmod 3$

  Then the edge set of $B_2(P_n)$ can be decomposed into$((n^2 -3n -1)/3) P_4$ and $(n+1)K_2$.

The edge set of $(n+1)K_2$ is given by the set $\{(v_{n-4}, e_1), (v_1, e_{n-1})\} \cup (\bigcup_{i=1}^{n-1} \{(v_i, e_{i+1}) \})$. The edge

sets of $((n^2 -3n -1)/3) P_4$ are given by the edge sets $A^{(4)}$ and $A^{(5)}$, where

$A^{(4)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-5)/3} A_{j,i}^{(4)})$ and $A_{j,i}^{(4)} = \{(v_i, e_{i+j+1}), (e_{i+j+1}, v_{i+2j+2}), (v_{i+2j+2}, e_{i+j-1})\}$ and

$A^{(5)} = \bigcup_{i=1}^{n-2} A_i^{(5)}$, $A_i^{(5)} = \{(v_n, e_i), (e_i, e_{i+1}),(e_{i+1}, v_{i+(n-4)})\}$. Here, $< A^{(4)} > \cong ((n-1)((n-5))/3)\, P_4$ and $< A^{(5)} > \cong (n-2)\, P_4$. Therefore, $B_2(P_n) - (n+1)K_2$ is P_4 – decomposable.

Theorem 3.2

(i)   For $n \geq 6$ and $n \equiv 0 \pmod 3$, the graph $B_2(C_n) - 2nK_2$ is P_4 – decomposable .
(ii)   For $n \geq 4$ and $n \equiv 1 \pmod 3$, the graph $B_2(C_n)$ is P_4 – decomposable.
(iii)   For $n \geq 5$ and $n \equiv 2 \pmod 3$, the graph $B_2(C_n) - nK_2$ is P_4 – decomposable.

Proof: Let $V(C_n) = \{ v_1, v_2, \dots, v_n\}$ and $e_i = (v_i, v_{i+1})$, $i = 1, 2, \dots, n$-1, be the edges of C_n. Then $v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{n-1}$, $e_n \in V(B_2(C_n))$. $B_2(C_n)$ has 2n vertices and n^2 - n edges. It is to be noted that, in all the sets the suffices i in v_j and j in e_j is integer modulo n, $v_0 = v_n$ and $e_0 = e_n$. Let $F_n = \bigcup_{j=1}^{n} F_j$, where $F_j = \{(v_i , e_{i+j}), i = 1, 2, \dots, n-1\}$.
 $E(B_2(C_n))= E(L(C_n)) \cup F = E(C_n) \cup F$.
Case 1. $n \equiv 0 \pmod 3$.

    Then the edge set of $B_2(C_n)$ can be decomposed into$((n^2$ - $3n)/3)$ P_4's and $2nK_2$. The edge set of $2n\, K_2$ is given by the set $\{\bigcup_{i=1}^{n}(v_{i,}\, e_{i+1}) \cup (e_i, e_{i+1}\}$. The edge sets of $((n^2$ - $3n)/3)P_4$ are given by the edge sets $B^{(1)}$, where $B^{(1)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-3)/3} B_{j,i}^{(1)})$, where $B_{j,i}^{(1)} = \{(v_i, e_{i+j+1,}),$ $(e_{i+j+1}, v_{i+3j+1}), (v_{i+3j+1}, e_{i+j})\}$. Here, $< B^{(1)} > \cong ((n)(n-3))/3)P_4$. Therefore, $B_2(C_n) - 2nK_2$ is P_4 – decomposable .

Case 2. $n \equiv 1 \pmod 3$.

    Then the edge set of $B_2(C_n)$ can be decomposed into $((n^2$ - $n)$ /3$)$ P_4 . The edge sets $((n^2$ - $n)/3)P_4$ are given by the edge sets $B^{(2)}$ and $B^{(3)}$, where $B^{(2)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-4)/3} B_{j,i}^{(2)})$, where $B_{j,i}^{(2)} = \{(e_{i+j+1}, v_i), (\, v_{i,}\, e_{i+2j+2}), (e_{i+2j+2,}\, v_{i+j+1})\}$ and $B^{(3)} = \bigcup_{i=1}^{n} B_i^{(3)}$, $B_i^{(3)}= \{(e_{i+1}, v_i),$ $(v_i, e_{i+2}), (e_{i+2}, e_{i+1})\}$. Here, $< B^{(2)} > \cong ((n(n-4))/3)\, P_4$ and $< B^{(3)} > \cong nP_4$. Therefore, $B_2(C_n)$ is P_4 – decomposable.

Case 3. $n \equiv 2 \pmod 3$

        Then the edge set of $B_2(C_n)$ can be decomposed into$((n^2$ -$2n)/3)$ P_4 and nK_2 . The edge set of nK_2 is given by the set $\bigcup_{i=1}^{n} \{(e_i, e_{i+1})\})$. The edge sets of $((n^2$ -$2n)/3)P_4$ are given by the edge set $B^{(4)}$, where $B^{(4)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-2)/3} B_{j,i}^{(4)})$ and $B_{j,i}^{(4)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, v_{i+3j+1})\}$. Here, $< B^{(4)} > \cong ((n^2 - 2n)/3)P_4$.
Therefore, $B_2(C_n) - nK_2$ is P_4 – decomposable.
Theorem 3.3
    For $n \geq 3$, the graph $B_2(K_{1,n})$ is P_4 – decomposable .

Proof: Let $V(K_{1,n}) = \{v, v_1, v_2, \dots, v_n \}$, where v is the central vertex and $e_i = (v, v_i)$, $i = 1, 2, \dots, n$ be the edges of $K_{1,n}$. Then $v, v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n \in V(B_2(K_{1,n}))$. $B_2(K_{1,n})$ has $2n+1$ vertices and $(3n^2 -3n))/2$ edges. In all the sets defined below, the suffices are integers modulo n and $v_o = v_n$, $e_o = e_n$.
 Let $F = \bigcup_{j=1}^{n} F_j$, where $F_i = \{ (v_i, e_j)$, for all j, $1 \leq j \leq n$ and $j \neq i \}$.
$E(B_1(K_{1, n}))= E(L(K_{1, n})) \cup F = E(K_n) \cup F$.

Case 1. n is even

Then the edge set of $B_2(K_{1,\,n})$ can be decomposed into$((n^2-n)/2)$ P_4 .
Let $C^{(1)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-2)/2} C_{j,i}^{(1)})$, where $C_{j,i}^{(1)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_{i+j})\}$ and
$C^{(2)} = \bigcup_{i=1}^{n/2} C_i^{(2)}$, where $C_i^{(2)} = \{(v_i, e_{n+i-1}), (e_{n+i-1}, e_{n/2+i-1}), (e_{n/2+i-1}, v_{n/2+2-i})\}$.
Here, $< C^{(1)} > \cong (n(n-2))/2\ P_4$, $< C^{(2)} > \cong (n/2)\ P_4$. Hence, $B_2(K_{1,n})$ is P_4 – decomposable.
Case 2. n is odd.

The edge set of $((n^2 - n)/2)$ P_4 is given by the set $C^{(3)}$, where

$C^{(3)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-1)/2} C_{j,i}^{(3)})$, where $C_{j,i}^{(3)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_{i+j})\}$,

$< C^{(3)} > \cong n\ ((n-1)/2)P_4$. Therefore, $B_2(K_{1,\,n})$ is P_4 – decomposable .

In the following, P_4 – decomposition of Boolean function graph of corona of P_n, C_n and $K_{1,n}$ are found.

Theorem 3.4

(i) For $n \geq 3$ and $n \equiv 0$ (mod 3), the graph $B_2(P_n{}^+) - (n+1)K_2$ is P_4 – decomposable .
(ii) For $n \geq 4$ and $n \equiv 1$ (mod 3), the graph $B_2(P_n{}^+) - 2nK_2$ is P_4 – decomposable .
(iii) For $n \geq 5$ and $n \equiv 2$ (mod 3), the graph $B_2(P_n{}^+) - (3n-1)K_2$ is P_4 – decomposable .
Proof: Let $V(P_n{}^+) = \{ v_1, v_2, …, v_n , u_1, u_2, …, u_n\}$, where $v_1, v_2, …, v_n$ are the vertices of P_n in $P_n{}^+$ and $u_1, u_2, …, u_n$ are the pendant vertices of $P_n{}^+$ and $e_i = (v_i, v_{i+1})$, i = 1, 2, …, n-1and $f_i = (v_i, u_i)$, i =1,2, …, n be the edges of $P_n{}^+$. Then $v_1, v_2, …, v_n, u_1, u_2, …, u_n$, $e_1, e_2, …, e_{n-1}, f_1, f_2, …, f_n \in V(B_1(P_n{}^+))$. $L(P_n{}^+)$ is an induced subgraph of $B_2(P_n{}^+)$. $B_2(P_n{}^+)$ has 4n-1 vertices and $4n^2 - 3n - 2$ edges.
Let $F_k = \{(v_i, e_j) /$ for all i, $1\leq i \leq n$, $j \equiv (i+k)$ mod (n-1) and $e_0 = e_{n-1}\}$ and $F = \bigcup_{k=1}^{n-3} F_k$.

Let $H_k = \{(v_i, f_j) (u_i, f_j) /$ for all i, $1\leq i \leq n$, $j \equiv (i+k)$ mod (n) and $f_0 = f_n\}$ and $H = \bigcup_{k=1}^{n-1} H_k$ and $\left| H \right| = 2n(n-1)$.

Let $J_k = \{(u_k, e_j) /$ for all i, $1\leq i \leq n-1\}$, $J = \bigcup_{k=1}^{n} J_k$ and $\left| J \right| = n(n-1)$.

$E(B_2(P_n{}^+)) = L(P_n{}^+) \cup (F \cup H \cup J)$

In all the sets the suffix j in e_j are integers modulo n-1 and suffices k in u_k, v_k and f_k, are integers modulo n, $u_0 = u_n, v_0 = v_n , f_0 = f_n, e_0 = e_{n-1}$.

 Case 1. $n \equiv 0$ (mod 3).

Then the edge set of $B_2(P_n{}^+)$ can be decomposed into$((4n^2-4n-3)/3)$ P_4 and $(n+1)$ K_2.
The edges set $(n+1)K_2$ is given by the set $\{\bigcup_{i=1}^{n} \{ (e_i, f_{i+1})\} \cup \{(v_1, e_{n-1}), (e_1, f_1)\}$. The edge sets of $((4n^2-4n-3)/3)$ P_4's are given by the edge sets $D^{(1)}$, $D^{(2)}$, and $D^{(3)}$, where

$D^{(1)} = \bigcup_{i=1}^{n} \bigcup_{j=1}^{(n-1)} D_{j,i}^{(1)})$, where $D_{j,i}^{(1)} = \{(v_i, f_{i+j}), (f_{i+j}, u_i), (u_i, e_{i+j-1})\}$ and
$D^{(2)} = \bigcup_{i=1}^{n-1} \bigcup_{j=1}^{(n-3)/3} D_{j,i}^{(2)})$, where $D_{j,i}^{(2)} = \{(e_{i+2j+1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, v_{i+3j+1})\}$ and
$D^{(3)} = \{ (v_n, e_i), (e_i, e_{i+1}), (e_{i+1}, v_{i+(n+4)/3})$ i = 1, 2, …, n-2\} . Here, $< D^{(1)} > \cong n\ (n-1)P_4$, $< D^{(2)} > \cong ((n-1)(n-3)/3)P_4$, $< D^{(3)} > \cong (n-2)P_4$. Therefore, $B_2(P_n{}^+) - (n+1)K_2$ is P_4 – decomposable.
Case 2. $n \equiv 1$(mod 3).

Then the edge set of $B_2(P_n{}^+)$ can be decomposed into $((4n^2-5n-2)/3)P_4$ and $2nK_2$. The edge set of $2nK_2$ is given by the set $\bigcup_{i=1}^{n-1}\{(v_i, e_{i+2}), (e_i, e_{i+1})\}$, $\{(v_1, e_{n-1}), (e_1, f_1)\}$. The

edge sets of $((4n^2 - 5n - 2)/3)$ P_4's are given by the edge sets $D^{(1)}$ and $D^{(3)}$ as in Case 1, and the set $D^{(4)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-4)/3} D_{j,i}^{(4)})$, where $D_{j,i}^{(4)} = \{ (e_{i+2j}, v_i), (v_i, e_{i+2j-1}), (e_{i+2j-1}, v_{i+3j+1})\}$.
Here, $< D^{(4)} > \cong (n-1) ((n-4)/3)$ P_4. Hence, $B_2(P_n{}^+) - 2nK_2$ is P_{4-} decomposable.

Case 3. $n \equiv 2 \pmod 3$.

Then the edge set of $B_2(P_n{}^+)$ can be decomposed into $((4n^2 - 6n - 1)/3)P_4$ and $(3n-1)K_2$. The edge set of $(3n-1)K_2$ is given by $\bigcup_{i=1}^{n-1}\{(e_i, f_{i+1}), (v_i, e_{i+1})\}, \{(v_i, e_{i+2}), (e_1, f_1), (v_1, e_{n-1})\}$. The edge sets of $((4n^2 - 6n - 1)/3)P_4$ are given by the edge set $D^{(1)}$ and $D^{(3)}$ as in case1, and the set $D^{(5)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-5)/3} D_{j,i}^{(5)})$, where $D_{j,i}^{(5)} = \{ (e_{i+2j+1}, v_i), (v_i, e_{i+2j+2}), (e_{i+2j+2}, v_{i+3j+3})\}$.
Here, $< D^{(5)} > \cong (n-1) ((n-5)/3)P_4$. Hence, $B_2(P_n{}^+) - (3n-1)K_2$ is P_{4-} decomposable.

Theorem3.5

(i) For $n \geq 3$ and $n \equiv 0 \pmod 3$, the graph $B_2(C_n{}^+) - 2nK_2$ is P_{4-} decomposable .
(ii) For $n \geq 4$ and $n \equiv 1 \pmod 3$, the graph $B_2(C_n{}^+) - 3nK_2$ is P_{4-} decomposable .
(iii) For $n \geq 5$ and $n \equiv 2 \pmod 3$, the graph $B_2(C_n{}^+) - nK_2$ is P_{4-} decomposable .
Proof: Let $V(C_n{}^+) = \{ v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ where $v_1, v_2, \ldots, v_n$ are the vertices of C_n in $C_n{}^+$ and $u_1, u_2, \ldots, u_n$ are the pendant vertices of $C_n{}^+$ and $e_i = (v_i, v_{i+1})$, $i = 1, 2, \ldots, n-1$, $e_n = (v_n, v_1)$ and $f_i = (v_i, u_i)$, $i = 1,2, \ldots, n$ be the edges of $C_n{}^+$. Then $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$, $e_1, e_2, \ldots, e_n, f_1, f_2, \ldots, f_n \in V(B_2(C_n{}^+))$. $L(C_n{}^+)$ is an induced subgraph of $B_2(C_n{}^+)$. $B_2(C_n{}^+)$ has $2n$ vertices and $4n^2 - n$ edges.
Let $F_k = \{(v_i, e_j) /$ for all i, $1 \leq i \leq n$, $j \equiv (i+k) \bmod (n-1)$ and $e_0 = e_n\}$ and $F = \bigcup_{k=1}^{n-3} F_k$.

Let $H_k = \{(v_i, f_j) (u_i, f_j) /$ for all i, $1 \leq i \leq n$, $j \equiv (i+k) \bmod (n)$ and $f_0 = f_n\}$ and $H = \bigcup_{k=1}^{n-1} H_k$ and $|H| = 2n(n-1)$.

Let $J_k = \{(u_k, e_j) /$ for all i, $1 \leq i \leq n-1\}$, $J = \bigcup_{k=1}^{n} J_k$ and $|J| = n(n-1)$.

$E(B_2(C_n{}^+)) = L(C_n{}^+) \cup (F \cup H \cup J)$

In all the sets, the suffices i in v_i, u_i, e_i and f_i are integers modulo n and $v_0 = v_n$, $u_0 = u_n$, $e_0 = e_n$ and $f_0 = f_n$.

 Case 1. $n \equiv 0 \pmod 3$.

Then the edge set of $B_2(C_n{}^+)$ can be decomposed into $(4n^2 - 3n)/3)$ P_4 and $2nK_2$. The edges set $2nK_2$ is given by the set $\{\bigcup_{i=1}^{n} \{ (e_i, f_i), (u_i, e_{n-1+i})\}$. The edge sets of $((4n^2 - 3n)/3)P_4$ are given by the edge sets $K^{(1)}, K^{(2)}$, and $K^{(3)}$, where
$K^{(1)} = \bigcup_{i=1}^{n} (\bigcup_{j=1}^{(n-1)} K_{j,i}^{(1)})$, where $K_{j,i}^{(1)} = \{(v_i, f_{i+j}), (f_{i+j}, u_i), (u_i, e_{i+j-1})\}$.
$K^{(2)} = \{ (v_i, e_{i+1}), (e_{i+1}, e_{i+2}), (e_{i+2}, f_{i+3}), i = 1, 2, \ldots, n\}$

$K^{(3)} = \bigcup_{i=1}^{n} (\bigcup_{j=1}^{(n-3)/3} K_{j,i}^{(3)})$ where $K_{j,i}^{(3)} = \{(e_{i+2j}, v_i), (v_i, e_{i+2j+1}), (e_{i+2j+1}, v_{i+3j+2}\}$.

Here, $< K^{(1)} > \cong n(n-1)P_4$, $< K^{(2)} > \cong nP_4$ and $< K^{(3)} > \cong ((n(n-3)/3)P_4$.

Therefore, $B_2(C_n{}^+) - 2nK_2$ is P_{4-} decomposable.

Case 2. $n \equiv 1 \pmod 3$.

Then the edge set of $B_2(C_n{}^+)$ can be decomposed into $((4n^2 - 4n)/3)$ P_4's and $3nK_2$. The edge set of $3nK_2$ is given by the sets $\bigcup_{i=1}^{n}(u_i, e_{n-1+i}), (e_i, e_{i+1}), (e_i, f_i)\}$. The edge sets of $((4n^2 - 4n)/3)$ P_4's are given by the edge sets $K^{(1)}$ and $K^{(2)}$ as in Case 1, and the set
$K^{(4)} = \bigcup_{i=1}^{n} \bigcup_{j=1}^{(n-4)/3} K_{j,i}^{(4)})$, where $K_{j,i}^{(4)} = \{ (e_{i+2j+1}, v_i), (v_i, e_{i+2j+2}), (e_{i+2j+2}, v_{i+3j+3})\}$.

Here, $< K^{(4)} > \cong n ((n-4)/3) P_4$. Hence, $B_2(C_n{}^+) - 3nK_2$ is P_4 – decomposable.

Case 3. $n \equiv 2 (\mathrm{mod}\ 3)$.

Then the edge set of $B_2(C_n{}^+)$ can be decomposed into $((4n^2 - 2n)/3) P_4$ and nK_2. The edge set of nK_2 is given by $\{\cup_{i=1}^{n}(u_i,\ e_{n-1+i})\ \}$. The edge sets of $((4n^2 - 2n)/3) P_4$ are given by the edge set $K^{(1)}$ are in case 1, $K^{(5)}$ and $K^{(6)}$, where $K^{(5)} = \cup_{i=1}^{n} \cup_{j=1}^{(n-2)/3} K_{j,i}^{(5)})$ and $K_{j,i}^{(5)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, v_{i+3j+1})\}$, where $K^{(6)} = \{(e_i, f_{i+2}), (f_{i+2}, e_{i+1}), (e_{i+1}, e_{i+2}),$

$i = 1, 2, \ldots, n\}$. Here, $< K^{(5)} > \cong n((n-2)/3)P_4$ and $< K^{(6)} > \cong nP_4$. Hence, $B_2(C_n{}^+) - nK_2$ is P_4 – decomposable.

Theorem3.6

For $n \geq 3$, the graph $B_2(K_{1,n}{}^+) - 2nK_2$ is P_4 – decomposable .

Proof: Let $V(K_{1,n}{}^+) = \{v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, u_{n+1}\}$, where v is the central vertex and $< \{v, v_1, v_2, \ldots, v_n\}> \cong K_{1,n}$ and $u_1, u_2, \ldots, u_n, u_{n+1}$ are the pendant vertices of $K_{1,n}{}^+$ and $e_i = (v, v_i)$, $i = 1, 2, \ldots, n$ and $f = (v, u)$, $f_i = (v_i, u_i)$, $i = 1, 2, \ldots, n$, be the edges of $K_{1,n}{}^+$. Then $v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, f_1, f_2, \ldots, f_n, e_1, e_2, \ldots, e_{n-1}, e_n \in V(B_2(K_{1,n}{}^+))$. $L(K_{1,n}{}^+)$ is an induced subgraph of $B_2(K_{1,n}{}^+)$. $B_2(K_{1,n}{}^+)$ has $4n+3$ vertices and $\frac{1}{2}(9n^2 + 7n)$ edges. In all the sets, the suffices i in v_i, u_i, e_i and f_i are integers modulo n and $v_0 = v_n$, $u_0 = u_n$, $e_0 = e_n$ and $f_0 = f_n$.

Let $F_i = \{(v_i, e_j), (v_i, f_j), (u_i, f_j)\ /\ 1 \leq j \leq n$ and $j \neq i\}$. $F = \cup_{i=1}^{n} F_i$, $|F| = 3n(n-1)$

Let $H_k = \{(u_k, e_j)\ /\ 1 \leq i \leq n\}$, $H = \cup_{k=1}^{n} H_k$ and $|H| = n^2$. Let $J = \{(v, f_j)\ /\ 1 \leq j \leq n\}$.

Let $L = \{(v_i, e), (u_i, e), (u, e_i), (u, f_i)\ /\ 1 \leq i \leq n\ \}$

$E(B_2(K_{1,n}{}^+)) = E(L(K_{1,n}{}^+)) \cup (F \cup H \cup J \cup L\ )$.

Let $M^{(1)} = \cup_{i=1}^{n} M_i^{(1)}$ and $M_i^{(1)} = \{(v, f_{i+1}), (f_{i+1}, u_i), (u_i, e_i)\}$ and $M^{(2)} = \cup_{i=1}^{n}(\cup_{j=1}^{(n-1)} M_{j,i}^{(2)})$ and $M_{j,i}^{(2)} = \{(v_i, f_{i+j-1}), (f_{i+j-1}, u_i), (u_i, e_{i+j})\}$ and $M^{(3)} = \cup_{i=1}^{n} M_i^{(3)}$ and $M_i^{(3)} = \{(v_i, f_i), (f_i, u_{n+1}), (u_{n+1}, e_i)\}$. Here, $< M^{(1)} > \cong nP_4$, $< M^{(2)} > \cong n(n-1)P_4$, $< M^{(3)} > \cong nP_4$. The edge set of $2nK_2$ is given by the set $\{(e_i, f_i), (e_i, f_{i+1}), i = 1, 2, \ldots, n\}$.

Case 1: n is odd

The edge set of $((3n^2 + n)/2)P_4$ are given by the set $M^{(1)} \cup M^{(2)} \cup M^{(3)} \cup M^{(4)}$, where $M^{(4)} = \cup_{i=1}^{n} \cup_{j=1}^{(n-1)/2} M_{j,i}^{(4)})$ and $M_{j,i}^{(4)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_i)\}$.
Here, $< M^{(4)} > \cong n(n-1)P_4$.
Case 1: n is odd

The edge set of $((3n^2 + n)/2)P_4$ are given by the set $M^{(1)} \cup M^{(2)} \cup M^{(3)} \cup M^{(4)}$, where $M^{(4)} = \cup_{i=1}^{n}(\cup_{j=1}^{(n-1)/2} M_{j,i}^{(4)})$, where $M_{j,i}^{(4)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_i)\}$.
Here, $< M^{(4)} > \cong n(n-1)P_4$.
Case 2: n is even

The edge set of $((3n^2 + n)/2)P_4$ are given by the set $M^{(1)} \cup M^{(2)} \cup M^{(3)} \cup M^{(5)} \cup M^{(6)} \cup M^{(7)}$, where $M^{(5)} = \cup_{i=1}^{n}(\cup_{j=1}^{(n-4)/2} M_{j,i}^{(5)})$, where $M_{j,i}^{(5)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_{i+j-1})\}$ and $M^{(6)} = \cup_{i=1}^{n} M_i^{(6)}$, where $M_i^{(6)} = \{(e_{i+(n-3)}, v_i), (v_i, e_{i+(n-2)}), (e_{i+(n-2)}, e_{i+(n-1)})\}$ and $M^{(7)} = \cup_{i=1}^{n/2} M_i^{(7)}$, where $M_i^{(7)} = \{(v_i, e_{i+5}), (e_{i+5}, e_{i+2}), (e_{i+2}, v_{i+3})\}$. Here, $< M^{(5)} > \cong (n(n-4)/2)P_4$, $< M^{(6)} > \cong nP_4$, $< M^{(7)} > \cong (n/2)P_4$. Therefore, $B_2(K_{1,n}{}^+) - 2nK_2$ is P_4 – decomposable.

4. Conclusion

In this paper, P_4-Decomposition of Boolean Function Graph B($\overline{Kp}$, L(G), NINC) of path, cycle, stars and corona graphs are obtained.

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