Few identities connected with intuitionistic fuzzy sets

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ABSTRACT:

Intuitionistic fuzzy sets are characterized by two functions expressing the degree of membership and the degree of non-membership respectively. In this paper, we have proved few identities connected with intuitionistic fuzzy sets based on this operation. (Denoted by $\cup$, $\cap$, , $+$, $\cdot$, @, $\$$, $\#$, $*$,).

Keywords: Intuitionistic fuzzy sets, Operations, Identities.

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1. INTRODUCTION:

Fuzzy set was introduced by L. A. Zadeh in 1965 as an extension of the classical notion of set. Fuzzy set was introduced by L. A. Zadeh[7] in 1965 as an extension of the classical notion of set. Fuzzy set deals only membership function. Intuitionistic Fuzzy Sets was introduced by K. T. Atanassov[1] in 1986 as an extension of fuzzy set. Intuitionistic fuzzy sets deals membership function and non-membership function. A lot of operations are introduced and proved over the intuitionistic fuzzy sets by many reasecher. K. T. Atanassov (1994) [2,3] proposed new operations defined over the intuitionistic fuzzy sets. De, Supriya Kumar, Ranjit Biswas, and Akhil Ranjan Roy (2000, 2001) [4,5] proposed some operations on intuitionistic fuzzy sets and also proposed an application of intuitionistic fuzzy sets in medical diagnosis. Riecan, B., K. T. Atanassov. (2006) [9] proposed n-extraction operation over intuitionistic fuzzy sets. Riecan, B., K. T. Atanassov. (2010) [10] proposed Operation division by n over intuitionistic fuzzy sets. Vasilev. T (2008) [11] proposed Four equalities connected with intuitionistic fuzzy sets. Verma, R. K and Sharma, B. D (2011) [12] proposed Intuitionistic fuzzy sets: Some new results. Parvathi, Rangasamy, Beloslav Riecan, and Atanassov K T (2012) [8] proposed Properties of some operations defined over intuitionistic fuzzy sets. In

2014 Ezhilmaran. D and Sudharsan. S [6] introduced two new operator defined over intuitionistic fuzzy sets. In the present communication, we have proved few identities connected with intuitionistic fuzzy sets based on this operation (Denoted by ∪, ∩, ¬, +, ·, @, \$, ≠, ∗, →).This paper proceeds as follows: In section II, some basic definitions related to fuzzy set, intuitionistic fuzzy sets (IFSs) and operation on intuitionistic fuzzy sets are presented. In section III, few identities connected with intuitionistic fuzzy sets based on this operation. (Denoted by ∪ , ∩, ¬, +, ·, @, \$, ≠, ∗, →) are proved. In section IV, Conclusion are given.

2. PRELIMINARIES

In section 2 some basic definitions related to fuzzy set, intuitionistic fuzzy sets (IFSs) and operation on intuitionistic fuzzy sets are discused.

2.1 Fuzzy Set[7]

Let a set X be fixed. A fuzzy set A in X is an object having the form $A = \{[x, M_A(x)] \mid x \in X\}$, where the functions $M_A : X \to [0, 1]$ define the degree of membership of the element $x \in X$ to the set A which is a subset of X, respectively, and for every $x \in X: 0 \leq M_A(x) \leq 1$

2.2 Intuitionistic Fuzzy Set[1,2,3]

Let a set X be fixed. An Intuitionistic fuzzy set A in X is an object having the form $A = \{[x, M_A(x), N_A(x)] \mid x \in X\}$, where the functions $M_A : X \to [0, 1]$ and $N_A : X \to [0, 1]$ define the degree of membership and the degree of non- membership of the element $x \in X$ to the set A which is a subset of X, respectively, and for every $x \in X: 0 \leq M_A(x) + N_A(x) \leq 1$.

2.3 Operations on Intuitionistic Fuzzy Sets[1,2,3,6,8,9]

Let A and B be two intuitionistic fuzzy sets on the universe X,

Where $A = \{[x, M_A(x), N_A(x)] \mid x \in X\}$ & $B = \{[x, M_B(x), N_B(x)] \mid x \in X\}$

$$A \cup B = \{[x, \max(M_A(x), M_B(x)), \min(N_A(x), N_B(x))] \mid x \in X\}$$

$$A \cap B = \{[x, \min(M_A(x), M_B(x)), \max(N_A(x), N_B(x))] \mid x \in X\}$$

$$A \subseteq B \text{ iff } (\forall x \in X)\left(M_A(x) \leq M_B(x) \,\&\, N_A(x) \geq N_B(x)\right)$$

$$A = B \text{ iff } (\forall x \in X)\left(M_A(x) = M_B(x) \,\&\, N_A(x) = N_B(x)\right)$$

$$\neg A = \{[x, N_A(x), M_A(x)] \mid x \in X\}$$

$$A + B = \{[x, M_A(x) + M_B(x) - M_A(x)M_B(x), N_A(x)N_B(x)] \mid x \in X\}$$

$$A \cdot B = \{[x, M_A(x)M_B(x), N_A(x) + N_B(x) - N_A(x)N_B(x)] \mid x \in X\}$$

$$A \,@\, B = \left\{\left[x, \left(\frac{M_A(x) + M_B(x)}{2}, \frac{N_A(x) + N_B(x)}{2}\right)\right] \mid x \in X\right\}$$

$$A \,\$\, B = \left\{\left[x, \left(\sqrt{M_A(x)M_B(x)}, \sqrt{N_A(x)N_B(x)}\right)\right] \mid x \in X\right\}$$

$$A \neq B = \left\{\left[x, \left(\frac{2M_A(x)M_B(x)}{M_A(x) + M_B(x)}, \frac{2N_A(x)N_B(x)}{N_A(x) + N_B(x)}\right)\right] \mid x \in X\right\}$$

$$A * B = \left\{\left[x, \left(\frac{M_A(x) + M_B(x)}{2(M_A(x)M_B(x) + 1)}, \frac{N_A(x) + N_B(x)}{2(N_A(x)N_B(x) + 1)}\right)\right] \mid x \in X\right\}$$

$$A \rightarrow B = \left\{\left[x, \left(\mathrm{Max}(N_A(x), M_B(x)), \mathrm{Min}(M_A(x), N_B(x))\right)\right] \mid x \in X\right\}$$

III. Few identities connected with intuitionistic fuzzy sets

In this section, few identities connected with intuitionistic fuzzy sets based on this operation. [3,4,11,12] (Denoted by ∪ , ∩, ¬, +, ·, @, \$, #, ∗, →) are proved.

Proposition 3.1 Idempotent law

Let $A, B \in$ intuitionistic fuzzy set(X) then,

(i). $(A \cap A) = A$, (ii). $(A \cup A) = A$, (iii). $(A@A) = A$,

(iv). $(A\$A) = A$, (v) $(A \# A) = A$, (vi) $(A \rightarrow A) = A$

Proof (i).

$$A \cap A = \{[x, \min(M_A(x), M_A(x)), \max(N_A(x), N_A(x))] \mid x \in X\}$$
$$= \{[x, M_A(x), N_A(x)] \mid x \in X\}$$
$$= A$$

Hence (i) is proved. Similarly (ii), (iii), (iv), (v) and (vi) can be proved by analogously.

Proposition 3.2 Commutative law

Let $A, B \in$ intuitionistic fuzzy set(X) then,

(i). $(A \cap B) = (B \cap A)$, (ii). $(A \cup B) = (B \cup A)$, (iii). $(A@B) = (B@A)$,

(iv). $(A\$B) = (B\$A)$, (v) $(A \# B) = (B \# A)$, (vi) $(A * B) = (B * A)$,

Proof (i).

$$(A \cap B) = \{[x, \min(M_A(x), M_B(x)), \max(N_A(x), N_B(x))] \mid x \in X\}$$
$$= \{[x, \min(M_B(x), M_A(x)), \max(N_B(x), N_A(x))] \mid x \in X\}$$
$$= (B \cap A)$$

Hence (i) is proved. Similarly (ii), (iii), (iv), (v) and (vi) can be proved by analogously.

Proposition 3.3 Demorgan's law

Let $A, B \in$ intuitionistic fuzzy set(X)then,

(i). $\neg(A \cap B) = \neg A \cup \neg B$, (ii). $\neg(A \cup B) = \neg A \cap \neg B$,

(iii). $\neg(A + B) = \neg A \cdot \neg B$, (ii). $\neg(A \cdot B) = \neg A + \neg B$,

Proof (i). Prove that $\neg(A \cap B) = \neg A \cup \neg B$

$$\neg(A \cap B) = \neg\{[x, \min(M_A(x), M_B(x)), \max(N_A(x), N_B(x))] \mid x \in X\}$$
$$= \{[x, \max(N_A(x), N_B(x)), \min(M_A(x), M_B(x))] \mid x \in X\} \quad \text{.......(1)}$$

$$\neg A \cup \neg B = \{[x, N_A(x), M_A(x)] \mid x \in X\} \cup \{[x, N_B(x), M_B(x)] \mid x \in X\}$$
$$= \{[x, \max(N_A(x), N_B(x)), \min(M_A(x), M_B(x))] \mid x \in X\} \quad \text{.......(2)}$$

From (1) and (2), we get $\neg(A \cap B) = \neg A \cup \neg B$

Hence (i) is proved. Similarly (ii), (iii) & (iv) can be proved by analogously.

Proposition 3.4 Absorption law

Let $A, B \in$ intuitionistic fuzzy set(X)then,

(i). $A \cup (A \cap B) = A$, (ii). $A \cap (A \cup B) = A$

Proof(i). Prove that $A \cup (A \cap B) = A$

$$A \cup (A \cap B) = \{[x, M_A(x), N_A(x)] \mid x \in X\} \cup \{[x, \min(M_A(x), M_B(x)), \max(N_A(x), N_B(x))] \mid x \in X\}$$
$$= \{[x, [\mathrm{Max}(M_A(x), \min(M_A(x), M_B(x)))], [\mathrm{Min}(N_A(x), \max(N_A(x), N_B(x)))]] \mid x \in X\}$$
$$= \{[x, [\mathrm{Max}(M_A(x), M_B(x))], [\mathrm{Min}(N_A(x), N_A(x))]] \mid x \in X\}$$
$$= \{[x, M_A(x), N_A(x)] \mid x \in X\}$$
$$= A$$

Hence (i) is proved. Similarly (ii) can be proved by analogously.

Proposition 3.5 Let $A, B \in$ intuitionistic fuzzy set(X) then,

$(i). (A \cap B) \cup (A \cap \neg B) = A, \quad (ii). (A \cup B) \cap (A \cup \neg B) = A$

$(iii). (A + B) @ (A \cdot B) = (A @ B), \quad (iv). (A + B) \cup (A \cdot B) = (A \cdot B)$

$(v). (A + B) \cap (A \cdot B) = (A + B), \quad (vi). \neg(A + B) \rightarrow (A \cdot B) = (A \cdot B)$

$(vii). [(A \cdot B) \cup (A @ B)] @ [(A + B) \cap (A @ B)] = (A @ B)$

$(viii). [(A \cdot B) \cup (A \neq B)] @ [(A + B) \cap (A \neq B)] = (A @ B)$

$(ix). [(A \cdot B) \cup (A \$ B)] @ [(A + B) \cap (A \$ B)] = (A @ B).$

Proof(vii).

Prove that $[(A \cdot B) \cup (A @ B)] @ [(A + B) \cap (A @ B)] = (A @ B)$

$$[(A \cdot B) \cup (A @ B)] = \{[x, M_A(x)M_B(x), N_A(x) + N_B(x) - N_A(x)N_B(x)] \mid x \in X\} \cup \left\{\left[x, \left(\frac{M_A(x) + M_B(x)}{2}, \frac{N_A(x) + N_B(x)}{2}\right)\right] \mid x \in X\right\}$$

$$= \left\{\left[\begin{matrix} x, \max\left(M_A(x)M_B(x), \frac{M_A(x) + M_B(x)}{2}\right), \\ \min\left(N_A(x) + N_B(x) - N_A(x)N_B(x), \frac{N_A(x) + N_B(x)}{2}\right) \end{matrix}\right] \mid x \in X\right\}$$

$$= \{[x, M_A(x)M_B(x), N_A(x) + N_B(x) - N_A(x)N_B(x)] \mid x \in X\} \quad \ldots\ldots\ldots\ldots(3)$$

$$[(A + B) \cap (A @ B)] = \left[\begin{matrix} \{[x, M_A(x) + M_B(x) - M_A(x)M_B(x), N_A(x)N_B(x)] \mid x \in X\} \\ \cap \\ \left\{\left[x, \left(\frac{M_A(x) + M_B(x)}{2}, \frac{N_A(x) + N_B(x)}{2}\right)\right] \mid x \in X\right\} \end{matrix}\right]$$

$$= \left\{\left[x, \min\left(M_A(x) + M_B(x) - M_A(x)M_B(x), \frac{M_A(x)+M_B(x)}{2}\right), \max\left(N_A(x)N_B(x), \frac{N_A(x)+N_B(x)}{2}\right)\right] \mid x \in X\right\}$$

$$= \{[x, M_A(x) + M_B(x) - M_A(x)M_B(x), N_A(x)N_B(x)] \mid x \in X\} \quad \ldots\ldots\ldots\ldots(4)$$

From (3) and (4), we get

$$[(A \cdot B) \cup (A @ B)] @ [(A + B) \cap (A @ B)] = \left[\begin{matrix} \{[x, M_A(x)M_B(x), N_A(x) + N_B(x) - N_A(x)N_B(x)] \mid x \in X\} \\ @ \\ \{[x, M_A(x) + M_B(x) - M_A(x)M_B(x), N_A(x)N_B(x)] \mid x \in X\} \end{matrix}\right]$$

$$= \left\{\left[x, \left(\frac{M_A(x)M_B(x) + M_A(x) + M_B(x) - M_A(x)M_B(x)}{2}, \frac{N_A(x) + N_B(x) - N_A(x)N_B(x) + N_A(x)N_B(x)}{2}\right)\right] \mid x \in X\right\}$$

$$= \left\{\left[x, \left(\frac{M_A(x)+M_B(x)}{2}, \frac{N_A(x)+N_B(x)}{2}\right)\right] \mid x \in X\right\}$$

$$= A @ B$$

$$[(A \cdot B) \cup (A @ B)] @ [(A + B) \cap (A @ B)] = (A @ B)$$

Hence (vii) is proved.

Similarly (i), (ii), (iii), (iv), (v), (vi), (viii) and (ix) can be proved by analogously.

4. CONCLUSION

In this paper, few identities connected with intuitionistic fuzzy sets were introduced and proved. In future, the application of these identities will be proposed.

REFERENCES

1. Atanassov K. T. "Intuitionistic fuzzy sets." *Fuzzy sets and Systems* 20, no. 1 (1986): 87-96.
2. Atanassov K T. "More on intuitionistic fuzzy sets." *Fuzzy sets and systems* 33, no. 1 (1989): 37-45.
3. Atanassov K T. "New operations defined over the intuitionistic fuzzy sets." *Fuzzy sets and Systems* 61, no. 2 (1994): 137-142.

4. De, Supriya Kumar, Ranjit Biswas, and Akhil Ranjan Roy. "Some operations on intuitionistic fuzzy sets." *Fuzzy sets and Systems* 114, no. 3 (2000): 477-484.

5. De, Supriya Kumar, Ranjit Biswas, and Akhil Ranjan Roy. "An application of intuitionistic fuzzy sets in medical diagnosis." *Fuzzy Sets and Systems* 117, no. 2 (2001): 209-213.

6. Ezhilmaran. D and Sudharsan. S. "Two New Operator Defined Over Intuitionistic Fuzzy Sets." International Journal of Mathematical Archive 5, no. 8 (2014): 1-7.

7. Zadeh, L. A. "Fuzzy sets." *Information and control* 8, no. 3 (1965): 338-353.

8. Parvathi, Rangasamy, Beloslav Riecan, and Atanassov K T. "Properties of some operations defined over intuitionistic fuzzy sets." *Notes on Intuitionistic Fuzzy Set* 18, no. 1(2012): 1–4.

9. Riecan, Beloslav, and Atanassov K T. "n-extraction operation over intuitionistic fuzzy sets."*Notes on Intuitionistic Fuzzy Set* 12, no. 4 (2006): 9-11.

10. Riecan, Beloslav, and Atanassov K T. "operation division by „n" over intuitionistic fuzzy sets." *Notes on Intuitionistic Fuzzy Set* 16, no. 4 (2010): 1-4.

11. Vasilev, T, "Four equalities connected with intuitionistic fuzzy sets." *Notes on Intuitionistic Fuzzy Sets*14, no. 3 (2008): 1-4.

12. Verma, Rajkumar, and Bhu Dev Sharma. "Intuitionistic fuzzy sets: some new results." *Notes on Intuitionistic Fuzzy Sets* 17, no. 3 (2011): 1-10.