A MANUFACTURING INVENTORY MODEL WITH SHORTAGE USING PENTAGONAL FUZZY NUMBER

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Abstract

In this paper, a production inventory model with shortage is formulated. The cost parameters are represented as pentagonal fuzzy number. The model is defuzzified by K-preference graded mean integration representation method. The optimum production quantity and shortage level have been obtained. A numerical illustration is given to support the problem

Keywords: Inventory, economic production quantity, Pentagonal fuzzy number, K-Preference.

1. Introduction

The economic order quantity (EOQ) model is first introduced by F. Harris. Inventory problems are common in manufacturing, maintenance service and business operations in general. Often uncertainties may be associated with demand and various relevant costs like those of carrying, shortage and set-up. For several years, classical economic production quantity (EPQ) problems with different variations were solved by many researchers and had been presented in the reference books and survey papers (e.g. Cheng[2],De and Goswami [3] etc,).

In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. However, in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory, originally introduced by Zadeh[8] and Bellman[1], is applicable. Therefore it becomes more convenient to deal such problems with fuzzy set theory rather than probability theory.

In the real world, the fuzzy inventory model with both uncertain parameters and fuzzy variables has been discussed recently. In 1987, Park [7] used fuzzy set concept to treat the inventory problem with fuzzy inventory cost under arithmetic operations of Extension Principle. In 2003, Hsieh [4] proposed a optimization of Fuzzy Inventory Models under K-preference. In [5] C.H. Hsieh discussed Optimization of fuzzy inventory models under fuzzy demand and fuzzy lead time.

Till now there is no fuzzy inventory model using k-preference of the pentagonal fuzzy number. So that in this paper, the manufacturing Inventory model with shortage using k-preference of the pentagonal fuzzy number has been considered in a fuzzy environment. The fuzzy holding cost, setup cost and shortage cost have been represented by the pentagonal fuzzy number. The model is defuzzified by k- preference Graded mean Integration technique. The Numerical example is illustrating the model and the results have been compared with the crisp model.

2. Assumptions and Notations

Assumptions

The following assumptions are made:

(i) A single item is considered

(ii) Lead time is zero

(iii) Production rate is finite

(iv) Shortages are allowed and fully back logged

Notations

t =the interval between production cycle.

t_1=the time in which the production and supply.

t_2=the time in which the supply.

t_3=the time in which the shortage.

t_4=the time in which fulfill the shortage.

Q_1=the inventory level at the end of t_1.

Q_2=the shortage level at the end of t_3.

Q=optimal lot size per cycle.

K=production rate.

R=demand rate.

$\tilde{C}_1$=fuzzy holding cost per unit item per unit time.

$\tilde{C}_2$ =fuzzy shortage cost per unit item per unit time.

$\tilde{C}_3$=fuzzy set up cost per set up.

3. Mathematical Model in Crisp environment

Average total cost is given by,

$$C(t_2,t_3)=\frac{\frac{1}{2}(C_1t_2{}^2+C_2t_3{}^2)RK+C_3(K-R)}{K(t_2+t_3)}$$

By using calculus technique, the analytical expression of t_2 , t_3 have been derived.

$$t_2^{*}=\sqrt{\frac{2C_3C_2(1-R/K)}{R(C_1+C_2)C_1}} \quad ; \quad t_3^{*}=\sqrt{\frac{2C_3C_1(1-R/K)}{R(C_1+C_2)C_2}}$$

by using t_2 and t_3 the analytical expression for optimum lot size and shortage level

$$Q^{*}=\sqrt{\frac{2RC_3(C_1+C_2)}{C_1C_2}}\sqrt{\frac{K}{K-R}} \quad ; \quad Q_2^{*}=\sqrt{\frac{2RC_1C_3}{(C_1+C_2)C_2}}\sqrt{\frac{K-R}{K}}$$

4. Pentagonal Fuzzy Number and its K-Preference Graded Mean Integration Method

Pentagonal fuzzy number: A pentagonal fuzzy number $\tilde{A}$ described as a fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & , \quad a\le x \\ w_A\dfrac{x-a}{b-a} & , \quad a\le x\le b \\ w_A+(1-w_A)\dfrac{x-b}{c-b} & , \quad b\le x\le c \\ w_A+(1-w_A)\dfrac{d-x}{d-c} & , \quad c\le x\le d \\ w_A\dfrac{e-x}{e-d} & , \quad d\le x\le e \\ 0 & , \quad e\ge x \end{cases} \quad \text{where } w_A \ , \ 0.6\le w_A<1 .$$

K-Preference Graded Mean Integration Method of Pentaganol Fuzzy Number

Graded k-preference Integration Representation method, the inverse function of L^{-1} and R^{-1} are L and R respectively, then the graded k-preference h-level value of generalized fuzzy number

$\tilde{A}=(a_1,a_2,a_3,a_4,a_5;w_A)_{LR}$ is $h[kL^{-1}(h)+(1-k)R^{-1}(h)]$ .Then the graded k-preference integration representation of $\tilde{A}$ is $P_k(\tilde{A})$ with grade w_A, where

$$P^k(\tilde{A})=\frac{\int_0^1 h\left[k\,L^{-1}(h)+(1-k)\,R^{-1}(h)\right]dh}{\int_0^1 h\,dh} \qquad 0< h \le w_A \text{ and } 0\le k \le 1.$$

$$\therefore P^k(\tilde{A})=\frac{1}{3(1-w_A)}\begin{bmatrix} k\left\{b+2c-3c\,w_A+w_A^{\,2}(a-b)-w_A^{\,3}(a-c)\right\} \\ +(1-k)\left\{d+2c-3cw_A+w_A^{\,2}(e-d)-w_A^{\,3}(e-c)\right\}\end{bmatrix}$$

5. Mathematical model in Fuzzy inventory

The above model is fuzzified by Pentagonal fuzzy number

$$\tilde{C}(t_2,t_3)=\frac{\frac{1}{2}(\tilde{C}_1t_2^{\,2}+\tilde{C}_2t_3^{\,2})RK+\tilde{C}_3(K-R)}{K(t_2+t_3)}$$

Defuzzifying the fuzzy average total cost by using K-Preference Graded Mean Integration Representation method is given by

$$P_k(\tilde{C}(t_2,t_3)) = \frac{\frac{1}{2}(P_k(\tilde{C}_1)t_2^{\ 2} + P_k(\tilde{C}_2)t_3^{\ 2})RK + P_k(\tilde{C}_3)(K-R)}{K(t_2+t_3)}$$

By using calculus technique, the Optimal fuzzy lot size and Shortage level have been derived.

$$\tilde{Q}* = \left(\left(\sqrt{\frac{2R\ P_K(C_3)(P_K(C_1)+P_K(C_2))}{P_K(C_1)P_K(C_2)}}\sqrt{\frac{K}{K-R}}\right)\right);\ \tilde{Q}_2* = \left(\left(\sqrt{\frac{2R_1P_K(C_1)P_K(C_3)}{(P_K(C_1)+P_K(C_2))P_K(C_2)}}\sqrt{\frac{K-R_1}{K}}\right)\right)$$

6. Numerical Examples

The demand for an item is 16 units per day. The holding cost is Rs.0.75 per unit time and the cost of shortage is Rs.0.25. The setup cost is Rs.95 per setup and the production rate is 25 units per day, determine the optimum production quantity and shortage level.

K=25, $\tilde{C}_3$ = (80,90,100,110,120) $\tilde{C}_1$ = (0.6,0.7,0.8,0.9,1) $\tilde{C}_2$ = (0.1,0.2,0.3,0.4,0.5).

TABLE 1: OPTIMAL SOLUTION FOR CRISP AND FUZZY MODEL

Case	k	Q*	Q_2*	Avg. TC
Crisp	-	180.678	40.9045	25,987
Crisp	-	205.789	45.789	27,678
Crisp	-	222.246	60.006	28,684
Crisp	-	248.987	75.890	32,678
Crisp	-	255.76	78.90	34,789
Fuzzy	0.1	115.6561	33.1373	21191
Fuzzy	0.5	87.9784	22.9825	17322
Fuzzy	0.9	80.6151	20.8015	16456

Conclusion

- When compare the optimum lot size, shortage level and total average minimum cost in crisp and fuzzy model, fuzzy lot size was higher than crisp lot size.
- And also fuzzy shortage level and average minimum total cost were lower than crisp shortage level and average minimum total cost.
- Finally, we conclude that these models can be executable in the real world.

Reference:

[1]. BellmanR.E., and L.A. Zadeh, "Decision-making in a fuzzy environment", Management Sciences, 17(1970) B141- B164.

[2]. Cheng.T.C.E, "An economic production quantity model with demand-dependent unit cost", European Journal of Operational Research, 40 (1989) 252-256.

[3]. De.S., and Goswami.A, "A replenishment policy for items with finite production rate & fuzzy deterioration rate", OPSEARCH, 38 (2001) 419-430.

[4]. Hsieh. C. H. (2003). Optimization of Fuzzy Inventory models under K-preference Tamsui on ford Journal of Mathematical Sciences 19 (1), (2003) 79-97.

[5]. Hsieh C.H. (2004), Optimization of Fuzzy Inventory Models under Fuzzy Demand and Fuzzy Lead Time, Tamsui on ford Journal of Mathematical Sciences, (20), 21-36.

[6]. Jain.R,"Decision making in the presence of fuzzy variables", IEEE Transaction on Systems, Man and Cybernetics, vol. 6, no.10, pp. 698-703,1976.

[7]. Park.K.S (1987), Fuzzy set theoretic interpretation of economic order quantity, IIIE Transactions on systems, Man and cybernetics, 17, 1082-1084.

[8]. Zadeh.L.A, "Fuzzy Sets", Information and Control, vol. 8, no 3, pp.338-353, 1965

[9]. Zimmermann H. J. (1978). "Fuzzy linear programming with several objective functions", Fuzzy Sets Systems, 1,46-55.