Superior Eccentric Domination in some Quadrilateral Snake Graphs

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ABSTRACT

In 2017 we define superior eccentric domination in graphs. A superior dominating set S of vertices of G is called a superior eccentric dominating set if every vertex of V (G) − S has some superior eccentric vertex in S. A superior eccentric dominating set of G of minimum cardinality is a minimum superior eccentric dominating set and its cardinality is called the superior eccentric domination number and is denoted by $\gamma_{sed}(G)$. In this paper we initiate the study of superior eccentric dominating sets in quadrilateral snake graphs, alternate quadrilateral snake graphs, double quadrilateral snake graphs and double alternate quadrilateral snake graphs.

Keywords:
Superior eccentric vertex, superior dominating set, superior eccentric dominating set, Superior eccentric dominating set of some quadrilateral snake graphs.
***Mathematics subject classification(2010) :* 05C12, 05C69.**

1 *INTRODUCTION*

Let G be a finite, simple, undirected (a, b) graph with vertex set V(G) and edge set E(G), |V(G)| = u, |E(G)| = v . For graph theoretic terminology refer Harary [3], Buckley and Harary [1].
In 2010, Janakiraman, Bhanumathi and Muthammai defined eccentric domination in graphs [5]. K. M Kathiresan and G.Marimuthu introduced the superior domination in graphs and superior distance in graphs [5,6].
A set D ⊆ V is said to be a dominating set in G, if every vertex in V− D is adjacent to some vertex in D. The minimum cardinality of a dominating set is called the domination number and is denoted by $\gamma(G)$. For two vertices u and v in a graph G, the distance from u to v is denoted by d(u,v) and defined as the length of a shortest u–v path in graph G. Let G be a connected graph and v be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u,v) : u\in V\}$. A set D ⊆ V(G) is an eccentric dominating set if D is a dominating set of G and for every v∈ V–D, there exist at least one eccentric vertex of v in D. The minimum cardinality of an eccentric dominating set is called the eccentric domination number and is denoted by $\gamma_{ed}(G)$.

2. Superior Eccentric Dominating Set.

For distinct vertices u and v of a non-trivial connected graph G, let $D_{u,v} = N(u) \cup N(v)$. We define a $D_{u,v}$ – walk as a u–v walk in G that contains every vertex of $D_{u,v}$. The superior distance $d_D(u, v)$ from u to v is the length of a shortest $D_{u,v}$ walk. For each vertex u ∈ V(G), define $d_D(u) = \min\{d_D(u, v) : v\in V(G) - \{u\}\}$. A vertex v (≠u) is called a superior neighbor of u if $d_D(u, v) = d_D(u)$. A vertex u is said to superior dominate a vertex v if v is a superior neighbor of u. A set S of vertices of G is called a superior dominating set if every vertex of V(G) − S is superior dominated by some vertex in S. A superior dominating set of G of minimum cardinality is a minimum superior dominating set and its cardinality is called the superior domination number of G and is denoted by $\gamma_{sd}(G)$. we define the superior eccentricity of v as $e_D(v) = \max\{d_D(u,v) : u \in V(G)\}$. A vertex v of a graph G is said to be a superior eccentric vertex of a vertex u if $d_D(u,v) = e_D(u)$. A vertex u is superior eccentric vertex of G if it is a superior eccentric vertex of some vertex v.

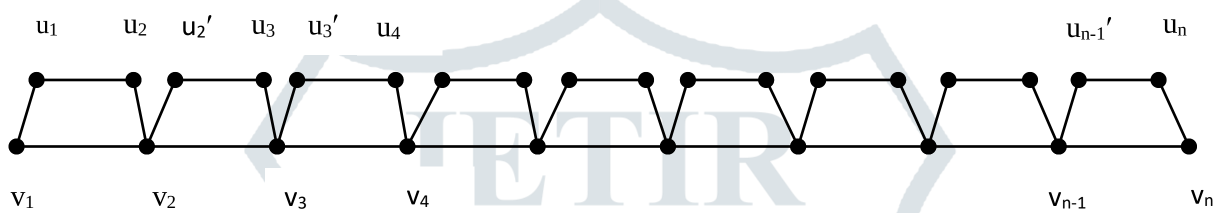
2. Superior Eccentric Domination in Some Graphs:

Definition 2.1

A superior dominating set S of vertices of G is called a superior eccentric dominating set if every vertex of $V(G) - S$ has some superior eccentric vertex in S.

Quadrilateral Snake Graph:

The Quadrilateral Snake Graph Q_n is obtained from the path P_n by replacing each edge of the path by a cycle C_4. The minimum cardinality of a superior eccentric domination in quadrilateral snake graph is $\gamma_{sed}(Q_n)$.

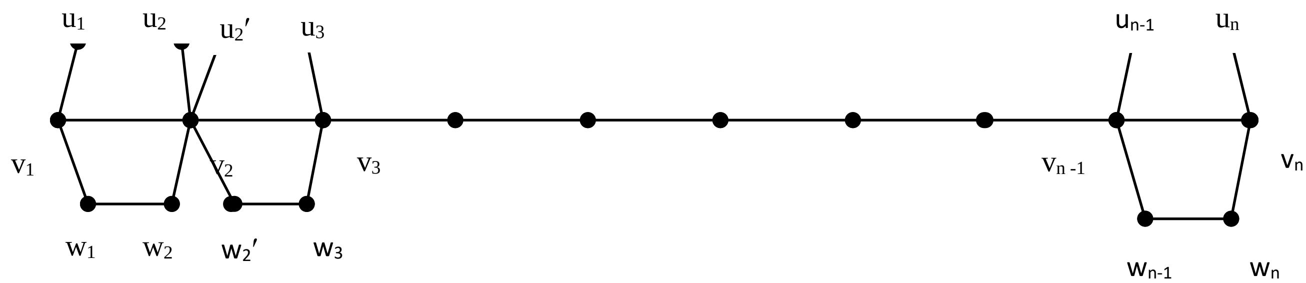


Superior distance $d_D(v_1, v_2) = 7$,
Superior distance $d_D(v_1, u_1) = 3$,
Superior distance $d_D(v_1, v_n) = 3 + d(v_2, v_{n-1}) + 3 = 6 + n - 3 = n + 3$,
Superior distance $d_D(v_i, v_n) = 4 + d(v_i, v_{n-1}) + 3 = 7 + n - i - 1 = n - i + 6$(for $i \geq 2$),
Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_j) + 3 = 4 + (j - 1 - i) + 3 = 7 + (j - 1 - i) = 6 + j - i$(for $i \geq 2, j \leq n - 1$),
Superior distance $d_D(u_1, u_2) = 3$,
Superior distance $d_D(u_1, u_2') = 7$,
Superior distance $d_D(u_1, u_3') = 8$,
Superior distance $d_D(u_1, u_4) = 9$,
Superior distance $d_D(u_1, u_3) = 8$,
Superior distance $d_D(u_1, v_n) = 4 + d(v_2, v_{n-1}) + 3 = 7 + n - 1 - 2 = n + 4$,
Superior distance $d_D(u_1, v_{n-1}) = 4 + d(v_2, v_{n-2}) + 7 = 11 + n - 2 - 2 = n + 7$,
Superior distance $d_D(v_2, u_2) = 7$,
Superior distance $d_D(u_2, u_2') = 6$,
Superior distance $d_D(u_1, u_n) = 3 + d(v_1, v_n) + 3 = 3 + n - 1 + 3 = n + 5$,
Superior distance $d_D(u_i, u_n) = 3 + d(v_i, v_{n-1}) + 3 = 6 + n - i - 1 = 5 + n - i$,
Superior distance $d_D(u_i, u_j) = 3 + d(v_i, v_{j-1}) + 7$

Hence v_2 and v_{n-1} are the superior eccentric vertices of other vertices, u_1 is superior adjacent to v_1, u_1 and u_2 are superior adjacent to each other, v_2 is superior adjacent to u_2'. v_{n-1} is superior adjacent to u_{n-1} $S = \{v_1, v_2, u_1, u_2, u_2', \ldots, v_{n-1}, u_{n-1}\}$ are the superior eccentric dominating set of the quadrilateral snake graph. S is also minimum with this property.

Double Quadrilateral Snake Graph:

A Double Quadrilateral Snake Graph DQn consisits of two triangular snakes that have a common path. The minimum cardinality of a superior eccentric domination in Double Quadrilateral Snake Graph is $\gamma_{sed}(DQ_n)$.



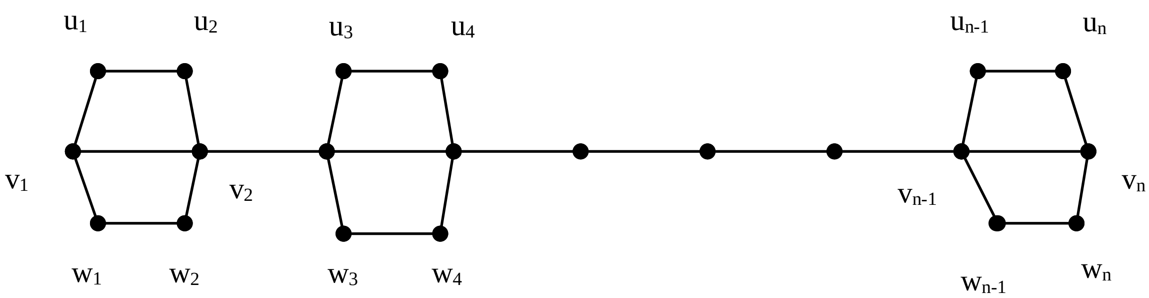
Superior distance $d_D(v_1, v_2) = 13$,
Superior distance $d_D(v_1, u_1) = 5$,
Superior distance $d_D(v_1, u_2) = 6$,
Superior distance $d_D(v_1, w_1) = 5$,
Superior distance $d_D(v_1, w_2) = 6$,
Superior distance $d_D(v_1, v_n) = 6 + n - 3 + 6 = 12 + n - 3 = n + 9$,
Superior distance $d_D(v_i, v_n) = 6 + n - i - 1 + 6 = 12 + n - i - 1 = 11 + n - 1$(for $i > 2$),
Superior distance $d_D(v_i, v_j) = 6 + d(v_i, v_{j-1}) + 6 = 12 + (j - i - 1) + 6 = 11 + j - 1$,
Superior distance $d_D(v_i, v_{i+1}) = 12$,
Superior distance $d_D(u_1, u_2) = 3$,
Superior distance $d_D(u_1, u_2') = 7$,
Superior distance $d_D(u_1, u_3) = 8$,
Superior distance $d_D(u_1, u_n) = 3 + d(v_1, v_n) + 3 = 6 + n - 1 = n + 5$,
Superior distance $d_D(u_i, u_n) = 3 + d(u_i, u_{n-1}) + 3 = 6 + (n - i - 1) = n - i + 5$(for $i > 2$),
Superior distance $d_D(u_i, u_j) = 3 + d(u_i, u_{j-1}) + 5 = 8 + (j - 1 - i) = 7 + j - i$(for $i > 2$, $j < n - 1$),
Superior distance $d_D(w_1, w_2) = 3$,
Superior distance $d_D(w_1, w_2') = 7$,
Superior distance $d_D(w_1, w_3) = 8$,
Superior distance $d_D(w_1, w_n) = 3 + d(v_1, v_n) + 3 = 6 + n - 1 = n + 5$,
Superior distance $d_D(w_i, w_n) = 3 + d(w_i, w_{n-1}) + 3 = 6 + (n - i - 1) = 5 + n - i$(for $i \geq 2$).
Superior distance $d_D(w_i, w_j) = 3 + d(w_i, w_{j-1}) + 5 = 8 + (j - i - 1) = 7 + j - i$.
Superior distance $d_D(v_2, v_n) = 6 + n - 2 + 6 = n + 10$,
Superior distance $d_D(v_2, u_2) = 11$,
Superior distance $d_D(v_2, u_2') = 6$,
Superior distance $d_D(w_2, w_2') = 6$,
Superior distance $d_D(u_1, v_n) = 6 + d(v_2, v_{n-1}) + 5 = 11 + n - 1 - 2 = 11 + n - 3 = n + 8$,
Superior distance $d_D(u_1, v_{n-1}) = 6 + d(v_2, v_{n-2}) + 11 = 17 + n - 2 - 2 = 17 + n - 4 = n + 13$,
Superior distance $d_D(v_2, w_2) = 6 + n - 2 + 6 = n + 10$,
Superior distance $d_D(w_1, v_n) = n + 8$,
Superior distance $d_D(w_1, v_{n-1}) = n + 13$,
Superior distance $d_D(v_2, u_n) = 6 + d(v_2, v_n) + 3 = 9 + n - 2 = n + 7$,
Superior distance $d_D(v_2, u_{n-1}') = 6 + d(v_2, v_{n-1}) + 3$.

$S = \{ u_1, u_2, \dots, u_{n-1}, u_n\} \cup \{ w_2, w_3, w_4, \dots, w_n\}$ is a superior dominating set. $S = \{ u_1, u_2, \dots, u_{n-1}, u_n\} \cup \{ w_2, w_3, w_4, \dots, w_n\} \cup \{ v_2, v_{n-1}\}$ is a superior eccentric dominating set. $|S| = n + (n - 1) u 2 = 2n + 1$.

Double Alternate Quarilateral Snake Graph:

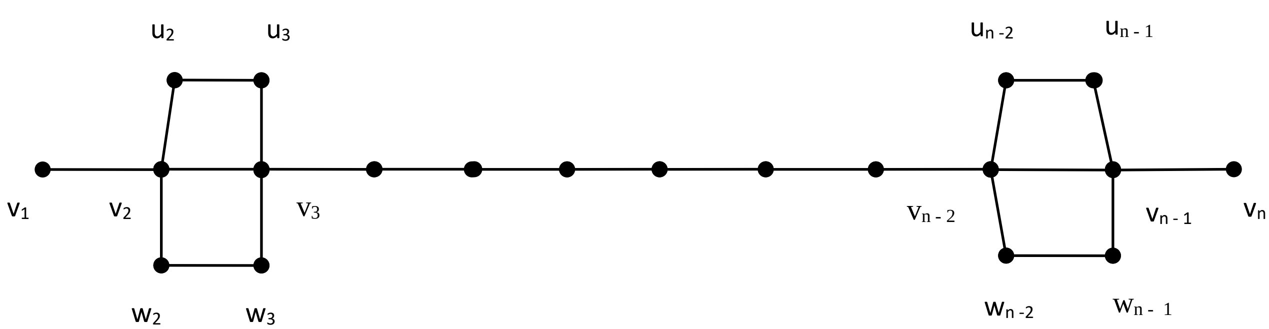
A Double alternate Quadrilateral Snake Graph that have a common path. That is, obtained from a path $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to new vertices v_i, x_i and w_i, y_i respectively and adding the edges v_iw_i and x_iy_i. The minimum cardinality of a superior eccentric domination in double alternate quadrilateral snake graph is $\gamma_{sed}(DAQ_n)$.

(i) n is even



Superior distance $d_D(v_1, v_2) = 9$,
Superior distance $d_D(u_1, u_2) = 3$,
Superior distance $d_D(u_1, v_1) = 3$,
Superior distance $d_D(w_1, v_1) = 3$,
Superior distance $d_D(v_1, v_n) = 7 + n – 3 + 7 = n + 3 + 14 = n + 11$,
Superior distance $d_D(v_i, v_n) = 2 + (n – i – 1) + 6 = 2 + n – i + 6 – 1 = 7 + n - i$ (for $i \geq 2$),
Superior distance $d_D(v_2, v_n) = n + 10$,
Superior distance $d_D(v_2, u_2) = 7$,
Superior distance $d_D(u_1, v_n) = 4 + d(v_2, v_{n-1}) + 6 = 10 + n – 1- 2 = n + 7$,
Superior distance $d_D(u_1, v_{n-1}) = 4 + d(v_2, v_{n-2}) + 6 = 10 + n – 2 -2 = n + 6$,
Superior distance $d_D(v_2, w_2) = 7$,
Superior distance $d_D(w_1, v_n) = n + 7$,
Superior distance $d_D(w_1, v_{n-1}) = n + 6$,
Superior distance $d_D(u_2, v_{n-1}) = 3 + d(v_2, v_{n-2}) + 6 = 9 + n – 4 = n + 5$,
Superior distance $d_D(w_2, v_{n-1}) = n + 5$,
Superior distance $d_D(v_i, v_j) = 2 + d(v_i, v_{j-1}) + 8 = 10 + (j- i -1) = 9 + j – i$(for $i \geq 2$, $j \leq n – 1$),
Superior distance $d_D(u_1, u_2) = 3$,
Superior distance $d_D(u_1, u_3) = 8$,
Superior distance $d_D(u_1, u_n) = 3 + d(v_1, v_n) + 3 = 6 + n – 1 = n + 5$,
Superior distance $d_D(u_i, u_n) = 3 + d(u_i, u_{n-1}) + 3 = 6 + n – i -1= 5 + n – i$(for $i \geq 2$),
Superior distance $d_D(u_i, u_j) = 3 + d(u_i, u_j) + 3 = 6 + j –i -1= 5 + j – i$(for $i \geq 2$, $j \leq n – 1$),
Superior distance $d_D(w_1, w_2) = 3$,
Superior distance $d_D(w_1, w_3) = 8$,
Superior distance $d_D(w_1, w_n) = 3 + d(v_1, v_n) + 3 = n + 5$,
Superior distance $d_D(w_i, w_n) = 3 + d(w_i, w_{n-1}) + 3 = 6 + (n – i -1) = 5 + n – i$(for $i \geq 2$),
Superior distance $d_D(w_i, w_j) = 3 + d(w_i, w_{j-1}) + 3 = 6 + (j- i – 1) = 5 + j – i$.
Hence v_2 and v_{n-1} are superior eccentric vertices of the other vertices. $S = \{u_1, u_3, u_5, . . ., u_{n-1}\} \cup \{w_2, w_4, . . ., w_n\}$ is a superior dominating set . $S_1 = S \cup \{ v_2, v_{n-1}\}$ is a superior eccentric dominating set of the double alternate quadrilateral snake graph. . $|S| = \frac{n}{2} + \frac{n}{2} = n$.

(ii) n is even



Superior distance $d_D(v_1, v_2) = 7$,
Superior distance $d_D(v_1, v_3) = 8$,
Superior distance $d_D(v_1, v_n) = 2 + d(v_2, v_{n-1}) + 6 = 8 + n - 1 - 2 = n + 5$,
Superior distance $d_D(u_2, u_3) = 3$,
Superior distance $d_D(w_1, v_1) = 4$,
Superior distance $d_D(v_i, v_n) = 8 + n - 1 - i + 2 = 10 - 1 + n - i = 9 + n - i$,
Superior distance $d_D(v_2, u_2) = 5$,
Superior distance $d_D(u_2, v_n) = 3 + d(v_2, v_{n-1}) + 2 = 5 + n - 1 + 2 = n + 6$,
Superior distance $d_D(u_2, v_{n-1}) = 1 + d(v_2, v_{n-1}) + 6 = 7 + n - 1- 2 = n + 4$,
Superior distance $d_D(v_i, v_j) = 8 + d(v_i, v_{j-1}) + 8 = 16 + j - 1 - i = 15 + j - i$(for $i \geq 2, j \leq n - 1$)
Superior distance $d_D(u_2, u_{n-1}) = 1 + d(v_2, v_{n-1}) + 1 = 2 + n - 3 = n - 1$,
Superior distance $d_D(v_2, v_n) = 6 + d(v_2, v_{n-1}) + 1 = 7 + n - 1 -2 = 7 + n - 3 = n +4$,
Superior distance $d_D(v_2, v_{n-1}) = 6 + d(v_2, v_{n-1}) + 6 = 12 + n - 1 - 2 = 12 + n - 3 = n + 9$.
Hence v_2 and v_{n-1} are superior eccentric vertices of the other vertices. $S = \{u_2, u_3, \ldots, u_{n-2}, u_{n-1}\} \cup \{w_2, w_3, \ldots, w_{n-2}, w_{n-1}\}$ are the superior dominating set $S_1 = S \cup \{v_2, v_{n-1}\}$ are the superior eccentric dominating set the double alternate quadrilateral snake graph.

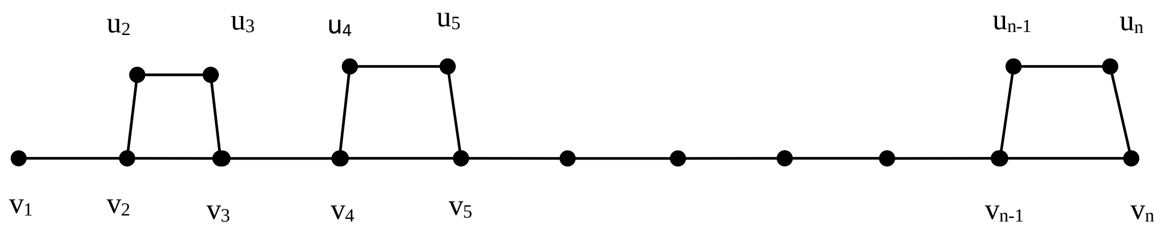
(iii) n is odd

Superior distance $d_D(v_1, v_2) = 7$,
Superior distance $d_D(v_1, v_3) = 8$,
Superior distance $d_D(v_1, v_n) = 2 + d(v_2, v_{n-1}) + 6 = 8 + n - 1 - 2 = n + 5$,
Superior distance $d_D(v_i, v_n) = 8 + d(v_i, v_{n-1}) + 5 = 13 + n -1 - i = 11 + n - i$(for $i \geq 2$),
Superior distance $d_D(v_i, v_j) = 2 + d(v_i, v_{j-1}) + 2 = 4 + (j - i - 1) = 3 + j - i$,
Superior distance $d_D(u_2, u_3) = 3$,
Superior distance $d_D(u_2, u_n) = 3 + d(v_2, v_n) + 5 = 8 + n - 2 = n +6$,
Superior distance $d_D(u_i, u_n) = 4 + d(u_i, u_{n-1}) + 3 = 7 + n - i - 1= 6 + n - i$(for $i \geq 2$),
Superior distance $d_D(v_2, v_n) = 6 + n - 2 + 6 = 10 + n$,
Superior distance $d_D(v_2, u_2) = 5$,
Superior distance $d_D(u_2, v_n) = 3 + d(v_2, v_{n-1}) + 3 = 6 + n - 1 - 2 = n +3$,
Superior distance $d_D(v_2, v_{n-1}) = 7 + d(v_3, v_{n-2}) + 7 = 14 + n - 2 - 3 = n + 9$,
Superior distance $d_D(u_2, v_{n-1}) = 3 + d(v_2, v_{n-2}) + 2 = 5 + n - 2 - 2 = n + 1$.
Hence v_2 and v_{n-1} are superior eccentric vertices of the other vertices. $S =\{v_2, v_4, \ldots, v_{n-3}, v_{n-1}\} \cup \{u_3, u_5, \ldots, u_n\} \cup \{w_3, w_5, \ldots, w_n\}$ are the superior dominating set $S_1 = S \cup \{v_2, v_{n-1}\}$ are the superior eccentric dominating set the double alternate quadrilateral snake graph.

Alternate quadrilateral snake graphs:

An alternate quadrilateral snake graph $A(Q_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i u_{i+1}(alternatively) to new vertices v_iw_i respectively and then joining v_i and w_i. That is every alternate edge of a path is replaced by a cycle C_4. The minimum cardinality of a superior eccentric domination in quadrilateral snake graph is $\gamma_{sed}(AQ_n)$.

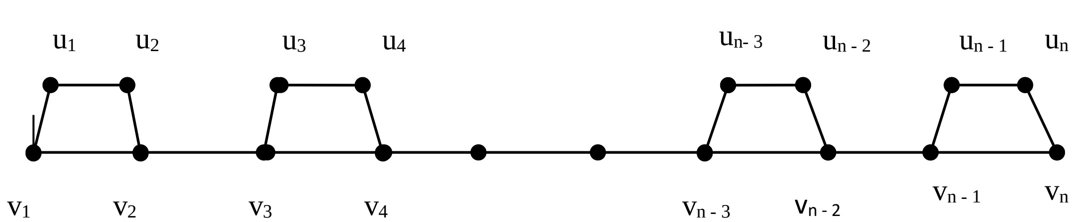
(i) n is odd



Superior distance $d_D(v_1, v_2) = 5$,
Superior distance $d_D(v_1, v_3) = 6$,
Superior distance $d_D(v_1, v_n) = 2 + d(v_2, v_{n-1}) + 4 = 6 + n - 3 = n + 3$,
Superior distance $d_D(v_i, v_n) = 4 + d(v_i, v_{n-1}) + 3 = 7 + n - i - 1 = 6 + n - i$(for $i \geq 2$),
Superior distance $d_D(v_i, v_j) = 2 + d(v_i, v_{j-1}) + 2 = 4 + (j - i - 1) + 2 = 4 +(j - i -1) = 4 + j - i - 1 = 3 + j - i$(for $i \geq 2$, $j \leq n - 1$),
Superior distance $d_D(u_2, u_3) = 3$,
Superior distance $d_D(u_2, u_n) = 3 + d(v_1, v_n) + 3 = 3 + n - 1 + 3 = n + 5$,
Superior distance $d_D(u_i, u_n) = 4 + d(u_i, u_{n-1}) + 3 = 7 + n - i - 1 = 6 + n - i$(for $i \geq 2$),
Superior distance $d_D(v_2, v_n) = 4 + n - 2 + 3 = n + 5$,
Superior distance $d_D(v_2, u_2) = 5$,
Superior distance $d_D(u_2, v_n) = 3 + d(v_2, v_{n-1}) + 3 = 6 + n - 1- 2 = n + 3$,
Superior distance $d_D(v_2, v_{n-1}) = 5 + d(v_3, v_{n-2}) + 5 = n + 5$,
Superior distance $d_D(u_2, v_{n-1}) = 3 + d(v_2, v_{n-2}) + 2 = 5 + (n - 2 - 2) = n - 4 + 5 = n + 1$.

Hence v_2 and v_{n-1} are the superior eccentric vertices of other vertices.
$S^* = \{ v_4, v_6, v_8, \ldots, v_{n-3} \} \cup \{ u_3, u_5, u_7, \ldots, u_{n-2}, u_n\}$ is a superior dominating set.
$S = \{ v_1, v_2, v_{n-1}\} \cup \{ v_4, v_6, v_8, \ldots, v_{n-3} \} \cup \{ u_3, u_5, u_7, \ldots, u_{n-2}, u_n\}$ is a superior eccentric dominating set of the double alternate quadrilateral snake graph.
Therefore $|S| = n - 3 - 4 + n - 3 + 3 = 2n - 7$.

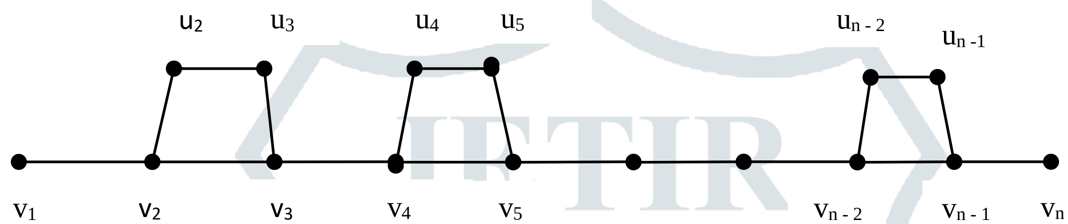
(ii)n is even



Superior distance $d_D(v_1, v_2) = 5$,
Superior distance $d_D(v_1, v_3) = 8$,
Superior distance $d_D(v_1, v_n) = 4 + d(v_2, v_{n-1}) + 3 = 6 + n - 1 - 2 = 6 + n - 3 = n + 3$,
Superior distance $d_D(v_i, v_n) = 2 + d(v_i, v_{n-1}) + 3 = 5 + n - 1 - i = 4 + n - i$(for $i \geq 2$),
Superior distance $d_D(v_i, v_j) = 2 + d(v_i, v_{j-1}) + 2 = 4 + (j - i - 1) = 3 + j - i$(for $i \geq 2$, $j \leq n -1$),
Superior distance $d_D(v_2, v_n) = 4 + n - 2 + 4 = n + 6$,
Superior distance $d_D(v_2, u_2) = 5$,

Superior distance $d_D(u_1, u_2) = 3$,
Superior distance $d_D(u_1, u_n) = 3 + d(v_1, v_n) + 3 = 6 + n – 1 = 5 + n$,
Superior distance $d_D(u_2, v_n) = 3 + d(v_2, v_{n-1}) + 3 = 3 + n – 1 -2 -3 +3 = n + 3$,
Superior distance $d_D(v_2, v_{n-1}) =4 + d(v_3, v_{n-2}) + 5 = 9 + n – 2 - 2 = n + 5$,
Superior distance $d_D(u_2, v_{n-1}) = 3 + d(v_2, v_{n-2}) + 5 = 8 + n – 4 = n + 4$,
Superior distance $d_D(u_1, v_n) = 4 + d(v_2, v_{n-1}) + 3 = 7 + n – 3 = n + 4$,
Superior distance $d_D(u_1, v_{n-1}) = 4 + d(v_2, v_{n-2}) + 5 = 9 + n – 2 – 2 = n + 5$,
Superior distance $d_D(u_i, u_n) = 3 + d(u_i, u_{n-1}) + 3= 6 + n – i – 1 = 5 + n – i$(for $i \geq 2$),
Superior distance $d_D(u_i,u_j) = 3 + d(u_i, u_{j-1}) + 3 = 6 + j – 1 – i = 5 + j - i$(for $i \leq 2, j \geq n – 1$)
Hence v_2 and v_{n-1} are the superior eccentric vertices of other vertices.
$S = \{ u_1, u_3, u_5, . . ., u_{n-1}\} \cup \{ v_4, v_6, . . ., v_{n-2}\} \cup \{v_2, v_{n-1}\}$
are the superior eccentric dominating set of the alternate quadrilateral snake graph.
Therefore $|S| = n + 2$.

(iii) n is even



Superior distance $d_D(v_1, v_2) = 5$,
Superior distance $d_D(v_1, v_3) = 6$,
Superior distance $d_D(v_1, v_n) = 2 + n – 1 + 2 = n + 3$, $N_D(v_i, v_n) = 6 + d(v_i, v_{n-1}) + 2 = 8 + n – 1 – i = 7 + n – i$(for $i \geq 2$),
Superior distance $d_D(v_i, v_j) = 6 + d(v_i, v_{j-1}) + 6 = 12 + j –i -1 = 11 + j – i$(for $i \geq 2, j \leq n -1$),
Superior distance $d_D(u_2, u_3) = 3$,
Superior distance $d_D(u_2, u_{n-1}) = 1 + d(v_2, v_{n-1}) + 1 = 2 + n – 3 = n – 1$,
Superior distance $d_D(v_2, v_n) = 4 + n – 3 = n + 1$,
Superior distance $d_D(v_2, u_2) = 5$,
Superior distance $d_D(u_2, v_n) = 3 + d(v_2, v_{n-1}) + 3 = 6 + n – 1- 2 = 6 + n – 3 = n + 3$,
Superior distance $d_D(v_2, v_{n-1}) = 5 + d(v_3, v_{n-2}) + 5 = 10 + n – 2 – 3 = n + 5$,
Superior distance $d_D(u_2, v_{n-1}) = 3 + d(v_2, v_{n-2} + 5 = 8 + n – 2 -2 = n + 4$,
Hence v_2 and v_{n-1} are the superior eccentric vertices of the other vertices.
$S = \{ u_1, u_2, u_5, …, u_{n-1}\} \cup \{ v_4, v_6, …, v_{n-2}\} \cup \{ v_2, v_{n-1}\}$ are the superior eccentric dominating set of the alternate quadrilateral snake graph.
terefore $|S| = n + 2$.

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