Discrete Linear Bilevel Programming Problems Using Transformation

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***Abstract*—**In this paper an algorithm is developed to solve the integer linear bilevel programming (ILBP) problem. The ILBP is transferred into a single level programming which can be solved by the algorithm proposed here. This algorithm deals by changing the randomly generated initial population into an initial population satisfying the constraints to improve the efficiency of the algorithm to deal with the constrains.

Keywords- Linear functios; bilevel programming; Integer programming; genetic algorithm; Kuhn-Tucker conditions.

I. INTRODUCTION

The mathematical programming models deal with single decision maker and single objective function which are used for centralized planning systems. Bilevel programming deals with decision processes involving two decision makers with a hierarchical structure. The first level decision maker is termed as the "Leader" and the decision maker at the second level pertains to the objective of "follower". Each decision maker optimizes its own objective function and/or is affected by actions .of planner. Integer Linear bilevel programming (ILBP) problem is a special case of bilevel programming problems in which the objective functions as well as constraints are all linear, and it can be formulated as follows[3]:

(ILBP). $\max f(x,y) = p_1 x + q_1 y.$

Where, y solves

$\max g(x,y)=p_2 x+q_2 .y$

s.t. $A_1 .x+A_2 .y \le b;$

x,y≥0, x, y integers

where f, g : the objective functions of leader-follower; x is an n_1 -dimensional column vector (variables under control of first level) and y is an n_2 -dimensional column vector (variables under control of second level); p_1 , p_2 are n_1 -dimensional row vectors, q_1 and q_2 are n_2 -dimensional row vectors; A_1 is $m \times n_1$-matrix and A_1 is$m \times n_1$ - matrix and b is an m-dimensional column vector. We assume that S defined by common constraints is nonempty and bounded

(ILBP) max f (x,y) = $p_1 x + q_1 y$

where y solves

$$\max g(x, y) = p_2 x + q_2 y \tag{1}$$

s.t. $A_1 x + A_2 y \le b$,

$x, y \ge 0$,

x, y integers

where f, g are the objective functions of the leader and follower, respectively; x is an n_1 dimensional column vector (variables under the control of first level) and y is an n_2 dimensional column vector (variables under the control of second level); p_1 and p_2 are n_1 -dimensional row vectors, q_1 and q_2 are n_2 -dimensional row vectors; A_1 is $m \times n_1$ matrix and A_1 is $m \times n_1$ matrix ;and b is an m-dimensional column vector. We assume that the polyhedron S defined by the common constraints is nonempty and bounded.

Many researchers tackeled the ILBP problem by presenting both theoretical results and application [4],[5]. The algorithmic approaches developed so far for ILBP problem can be classified into four categories[5]

(1) meta-heuristic approach;.

(2) KT conditions .approach;

(3) fuzzy criterion approach;

(4) vertex.enumeration approach

The ILBP is neither continuous everywhere nor convex because the objective function of the first level problem is decided by the solution function of the second level problem which is neither linear nor differential. J. F. Bard [1] proved that ILBP is a NP-Hard problem. The algorithms proposed include genetic algorithms [2], [5], [7], simulated annealing algorithms [9], particle swarm optimization (PSO) [6] etc.

In this study, algorithm is developed/proposed for ILBP problem. DE proposed by Storn and Price[10]optimized real valued multi-modal objective functions. Besides its good convergence properties the algorithm is very simple to implement and finds solution to the problem.

II. ALGORITHMIC DEVELOPMENT

The algorithm proposed in this paper is a parallel direct search method which utilizes NP dimensional parameter vectors

$p_i^G = (p_{i1}^G, p_{i2}^G \ldots\ldots\ldots\ldots p_{iD}^G)$ i=1,2……NP

as a population for each generation G. To be more precise , the basic approach in the algorithm [8],[10] is described in the following steps :

Step1: The initial vector population is chosen randomly from within a user-defined range and should cover the entire parameter space.

Step2: *Mutation.* In this step, the mutated vectors (i = 1, 2, ⋯ , NP) are generated according to

$$g_i^{G+1} = p_{r1}^G + F \cdot (p_{r2}^G - p_{r3}^G) \qquad (3)$$

with random indexes $r1, r2, r3 \in \{1, 2, \cdots, NP\}$, integer, mutually different and $F > 0$. The randomly chosen integers $r1$, $r2$ and $r3$ are also chosen to be different from the running index i, so that NP must be greater or equal to four to allow for this condition. Weighting factor $F \in [0, 2]$ is a real and constant factor which controls the amplification of the differential variation $(p_{r2}^G - p_{r3}^G)$.

Step3: To increase the diversity of the perturbed parameter vectors, crossover is introduced. To this end, the trial vector

$$h_i^{G+1} = (h_{i1}^{G+1}, h_{i2}^{G+1} \ldots\ldots\ldots\ldots h_{iD}^{G+1}) \qquad (4)$$

is formed, where

$$h_{ij}^{G+1} = \begin{cases} g_{ij}^{G+1}, & \text{if } (randb(j) \le CR) \text{ or } j = rnbr(i) \\ p_{ij}^G, & \text{otherwise,} \end{cases} \qquad (5)$$

In (5), $randb(j) \in [0, 1]$ is the j-th evaluation of a uniform random number generator with outcome. $CR \in [0, 1]$ is the crossover constant which has to be determined by the user. $rnbr(i) \in \{1, 2, \cdots, D\}$ is a randomly chosen index which ensures that h_{ij}^{G+1} gets at least one parameter from g_{ij}^{G+1}

Step4: To decide whether or not it should become a member of generation $G+1$, the trial vector g_{ij}^{G+1} is compared to the target vector p_i^G using the greedy criterion.

Step5: Stop the algorithm if the condition is satisfied or iteration number is larger than the maximal iteration number. Then .best generated solution, which can be obtained by the current best vector kept in all iterations in the earliest time: is reported as the solution for the LBP problem by proposed algorithm.

III. REDUCING ILBP PROBLEM TO A SINGLE LEVEL PROGRAM

At first the ILBP problem can be transferred into a single level programming of the form

(P1) max $f(x,y) = p_1 x + q_1 y$

s.t. $A_1x+A_2y+u=b;$

$zA_2 - v = d_2;$, (6)

$uz = 0,\ vy = 0,;$

$x,y,z,u,v \geq 0.$

x,y integer

Problem (6) is transformed into the following problem

(P2) max p_1x

s.t. $A_1x = b - A_2y_1^G - u_1^G;$

$z'A'_2 - v' = d_2;$

$x, z', v' \geq 0;$ (8)

x integer

In problem (8), z′ and v′ are the variables of z and v that are greater than or equal to zero. Also, A'_2 is the rows of A_2 which is associated with the variables z′. Furthermore, problem (8) can be decomposed into two separate problems as follows:

(P3) $z'A'_2 - v' = d_2$

$z', v' \geq 0$ (9)

(P4) max $p_1 x$

s.t. $A_1 x = b - A_2 y_1^G - u_1^G$ (10)

$x \geq 0$

Such decomposition is allowed for the problem, because problems (9) and (10) do not have common variables. Problem (9) is solved at first. If problem is infeasible, then the target vector is not achievable;otherwise. problem(10) is solved. If problem is infeasible, then p_1^G is unaccessible .

IV NUMERICAL EXAMPLE

Max $-42x+21y+72$

Max $-5x-31y+23$:

s.t. $23.2x-32y \leq 34$:

$2x - 3.1y \leq 24$:

$13x + 54y \leq 96$:

$22.6x + 37y \leq 86$:

$54x + 25y \leq 65.2$:

$34x+54y \geq 78$:

x y ≥0;

The best solution for technique proposed in this paper is given by

(x, y) = (17.4545 ,10.9091) and objective function value is "f = 85.0909" and "g = -50.181".

"IV CONCLUSIONS

The presented procedure designs the algorithm for solving ILBP problems in which the optimal solution of the lower level problem is dependent on the upper level problem.The solution approach makes use of simple transformation method which takes up less computational work.

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