Gauss's Law for Magnetism: The Concept of Magnetism - An Overview

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Abstract

This paper studies the concept of magnetism in physics, Gausss law for magnetism is one of the four Maxwell's equations that underlie classical electrodynamics. It states that the magnetic field B has divergence equal to zero, in other words, that it is a solenoidal vector field. It is equivalent to the statement that magnetic monopoles do not exist. Magnetism is one aspect of the combined electromagnetic force. It refers to physical phenomena arising from the force caused by magnets, objects that produce fields that attract or repel other objects. A magnetic field exerts a force on particles in the field due to the Lorentz force, according to Georgia State University's HyperPhysics website. The motion of electrically charged particles gives rise to magnetism. The force acting on an electrically charged particle in a magnetic field depends on the magnitude of the charge, the velocity of the particle, and the strength of the magnetic field. All materials experience magnetism, some more strongly than others. Permanent magnets, made from materials such as iron, experience the strongest effects, known as ferromagnetism. With rare exception, this is the only form of magnetism strong enough to be felt by people. Magnetic fields are generated by rotating electric charges, according to HyperPhysics. Electrons all have a property of angular momentum, or spin. Most electrons tend to form pairs in which one of them is "spin up" and the other is "spin down," in accordance with the Pauli Exclusion Principle, which states that two electrons cannot occupy the same energy state at the same time. In this case, their magnetic fields are in opposite directions, so they cancel each other. However, some atoms contain one or more unpaired electrons whose spin can produce a directional magnetic field. The direction of their spin determines the direction of the magnetic field, according to the Non-Destructive Testing (NDT) Resource Center. When a significant majority of unpaired electrons are aligned with their spins in the same direction, they combine to produce a magnetic field that is strong enough to be felt on a macroscopic scale. Magnetic field sources are dipolar, having a north and south magnetic pole. Opposite poles (N and S) attract, and like poles (N and N, or S and S) repel, according to Joseph Becker of San Jose State University. This creates a toroidal, or doughnut-shaped field, as the direction of the field propagates outward from the north pole and enters through the south pole.

The Gauss Law in magnetism says that "Magnetic monopoles do not exist in nature". Gauss Law says that divergence of magnetic field B is zero. From our above understanding of divergence, this means there is no source or sink of the magnetic field anywhere in the universe. But we know that the magnetic field has a source, the magnet.

Key words: Non-Destructive Testing, Magnetic monopoles , Gauss law, magnetic field.

Introduction

The Earth itself is a giant magnet. The planet gets its magnetic field from circulating electric currents within the molten metallic core, according to HyperPhysics. A compass points north because the small magnetic needle in it is suspended so that it can spin freely inside its casing to align itself with the planet's magnetic field. Paradoxically, what we call the Magnetic North Pole is actually a south magnetic pole because it attracts the north magnetic poles of compass needles.

People soon learned that they could magnetize an iron needle by stroking it with a lodestone, causing a majority of the unpaired electrons in the needle to line up in one direction. According to NASA, around A.D. 1000, the Chinese discovered that a magnet floating in a bowl of water always lined up in the north-south direction. The magnetic compass thus became a tremendous aid to navigation, particularly during the day and at night when the stars were hidden by clouds.

Other metals besides iron have been found to have ferromagnetic properties. These include nickel, cobalt, and some rare earth metals such as samarium or neodymium which are used to make super-strong permanent magnets.

Magnetism takes many other forms, but except for ferromagnetism, they are usually too weak to be observed except by sensitive laboratory instruments or at very low temperatures. Diamagnetism was first discovered in 1778 by Anton Brugnams, who was using permanent magnets in his search for materials containing iron. According to Gerald Küstler, a widely published independent German researcher and inventor, in his paper, "Diamagnetic Levitation — Historical Milestones," published in the Romanian Journal of Technical Sciences, Brugnams observed, "Only the dark and almost violet-colored bismuth displayed a particular phenomenon in the study; for when I laid a piece of it upon a round sheet of paper floating atop water, it was repelled by both poles of the magnet."

Bismuth has been determined to have the strongest diamagnetism of all elements, but as Michael Faraday discovered in 1845, it is a property of all matter to be repelled by a magnetic field.

Diamagnetism is caused by the orbital motion of electrons creating tiny current loops, which produce weak magnetic fields, according to HyperPhysics. When an external magnetic field is applied to a material, these current loops tend to align in such a way as to oppose the applied field. This causes all materials to be repelled by a permanent magnet; however, the resulting force is usually too weak to be noticeable. There are, however, some notable exceptions.

Pyrolytic carbon, a substance similar to graphite, shows even stronger diamagnetism than bismuth, albeit only along one axis, and can actually be levitated above a super-strong rare earth magnet. Certain superconducting materials show even stronger diamagnetism below their critical temperature and so rare-earth magnets can be levitated above them. (In theory, because of their mutual repulsion, one can be levitated above the other.)

Paramagnetism occurs when a material becomes magnetic temporarily when placed in a magnetic field and reverts to its nonmagnetic state as soon as the external field is removed. When a magnetic field is applied, some of the unpaired electron spins align themselves with the field and overwhelm the opposite force produced by diamagnetism. However, the effect is only noticeable at very low temperatures, according to Daniel Marsh, a professor of physics at Missouri Southern State University.

Other, more complex, forms include antiferromagnetism, in which the magnetic fields of atoms or molecules align next to each other; and spin glass behavior, which involve both ferromagnetic and antiferromagnetic interactions. Additionally, ferrimagnetism can be thought of as a combination of ferromagnetism and antiferromagnetism due to many similarities shared among them, but it still has its own uniqueness, according to the University of California, Davis.

Objective:

This paper intends to explores Gauss' Law for magnetism which applies to the magnetic flux through a closed surface. Also, the area vector which points out from the surface.

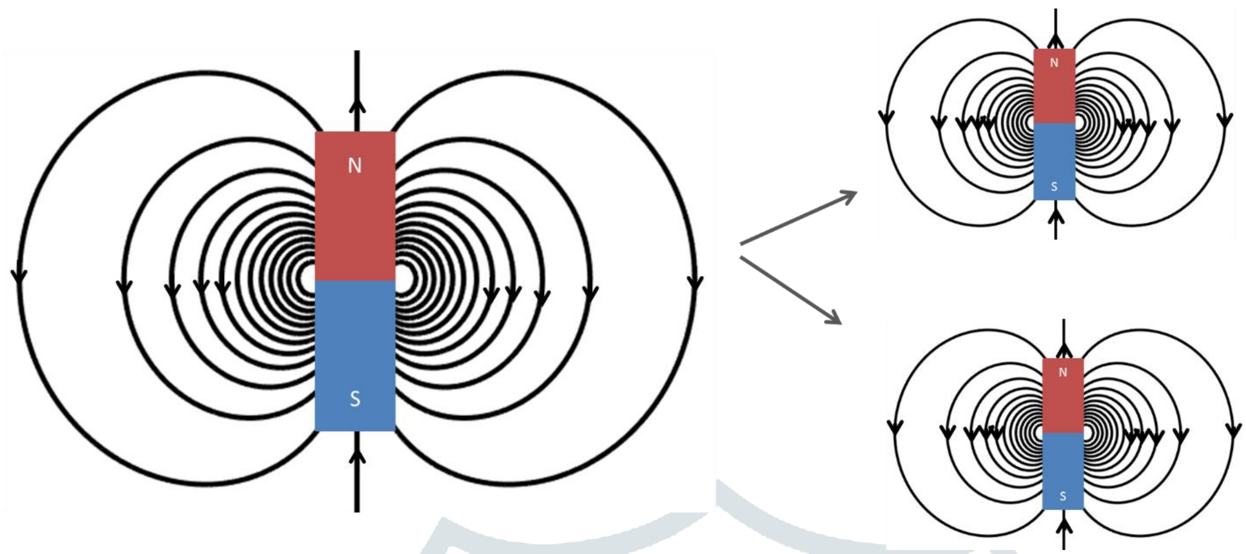
Gauss's law for magnetism the core components

When a wire is moved in a magnetic field, the field induces a current in the wire. Conversely, a magnetic field is produced by an electric charge in motion. This is in accordance with Faraday's Law of Induction, which is the basis for electromagnets, electric motors and generators. A charge moving in a straight line, as through a straight wire, generates a magnetic field that spirals around the wire. When that wire is formed into a loop, the field becomes a doughnut shape, or a torus. According to the Magnetic Recording Handbook (Springer, 1998) by Marvin Cameras, this magnetic field can be greatly enhanced by placing a ferromagnetic metal core inside the coil.

In some applications, direct current is used to produce a constant field in one direction that can be switched on and off with the current. This field can then deflect a movable iron lever causing an audible click. This is the basis for the telegraph, invented in the 1830s by Samuel F. B. Morse, which allowed for long-distance communication over wires using a binary code based on long- and short-duration pulses. The pulses were sent by skilled operators who would quickly turn the current on and off using a spring-loaded momentary-contact switch, or key. Another operator on the receiving end would then translate the audible clicks back into letters and words.

A coil around a magnet can also be made to move in a pattern of varying frequency and amplitude to induce a current in a coil. This is the basis for a number of devices, most notably, the microphone. Sound causes a diaphragm to move in an out with the varying pressure waves. If the diaphragm is connected to a movable magnetic coil around a magnetic core, it will produce a varying current that is analogous to the incident sound waves. This electrical signal can then be amplified, recorded or transmitted as desired. Tiny super-strong rare-earth magnets are now being used to make miniaturized microphones for cell phones, Marsh told Live Science.

Gauss's Law for Magnetic Fields



When a bar magnet is cut in two, you get two bar magnets.

Gauss's law for magnetism states that no magnetic monopoles exists and that the total flux through a closed surface must be zero. This page describes the time-domain integral and differential forms of Gauss's law for magnetism and how the law can be derived. The frequency-domain equation is also given. At the end of the page, a brief history of the Gauss's law for magnetism is provided.

Integral equation

The Gauss's law for magnetic fields in integral form is given by:

∮Sb·da=0,∮Sb·da=0,

where:

- bb is the magnetic flux

The equation states that there is no net magnetic flux bb (which can be thought of as the number of magnetic field lines through an area) that passes through an arbitrary closed surface SS. This means the number of magnetic field lines that enter and exit through this closed surface SS is the same. This is explained by the concept of a magnet that has a north and a south pole, where the strength of the north pole is equal to the strength of the south pole (Fig. 35). This is equivalent to saying that a magnetic monopole, meaning a solitary north or south pole, does not exist because for every positive magnetic pole, there must be an equal amount of negative magnetic poles.

Differential equation

Gauss's law for magnetic fields in the differential form can be derived using the divergence theorem. The divergence theorem states:

$\int V(\nabla\cdot f)dv=\oint Sf\cdot da,\int V(\nabla\cdot f)dv=\oint Sf\cdot da,$

where ff is a vector. The right-hand side looks very similar to Using the divergence theorem, Equation is rewritten as follows:

$0=\oint Sb\cdot da=\int V(\nabla\cdot b)dv.0=\oint Sb\cdot da=\int V(\nabla\cdot b)dv.$

Because the expression is set to zero, the integrand $(\nabla\cdot b)(\nabla\cdot b)$ must be zero also. Thus the differential form of Gauss's law becomes:

$\nabla\cdot b=0.\nabla\cdot b=0.$

Derivation using Biot-Savart law

Gauss's law can be derived using the Biot-Savart law, which is defined as:

$b(r)=\mu 04\pi\int V(j(r')dv)\times\ r\text{–}\hat{}|r-r'|2,b(r)=\mu 04\pi\int V(j(r')dv)\times\ r\_\hat{}|r-r'|2,$

where:

- $b(r)b(r)$ is the magnetic flux at the point rr
- $j(r')j(r')$ is the current density at the point $r'r'$
- $\mu 0\mu 0$ is the magnetic permeability of free space.

Taking the divergence of both sides of Equation (51) yields:

$\nabla\cdot b(r)=\mu 04\pi\int V\nabla\cdot(j(r')dv)\times\ r\text{–}\hat{}|r-r'|2.\nabla\cdot b(r)=\mu 04\pi\int V\nabla\cdot(j(r')dv)\times\ r\_\hat{}|r-r'|2.$

To carry through the divergence of the integrand in Equation (52), the following vector identity is used:

$\nabla\cdot(A\times B)=B\cdot(\nabla\times A)-A\cdot(\nabla\times B).\nabla\cdot(A\times B)=B\cdot(\nabla\times A)-A\cdot(\nabla\times B).$

Thus, the integrand becomes:

$[j(r')\cdot(\nabla\times\ r\text{–}\hat{}|r-r'|2)]-[\ r\text{–}\hat{}|r-r'|2\cdot(\nabla\times j(r'))][j(r')\cdot(\nabla\times\ r\_\hat{}|r-r'|2)]-[\ r\_\hat{}|r-r'|2\cdot(\nabla\times j(r'))]$

The first part of Equation is zero as the curl of r–^|r−r′|2 r_^|r−r′|2 is zero. The second part of Equation becomes zero because jj depends on r′r′ and ∇∇ depends only on rr. Plugging this back into the right-hand side of the expression becomes zero. Thus, we see that:

$\nabla \cdot b(r)=0, \nabla \cdot b(r)=0,$

which is Gauss's law for magnetism in differential form.

Differential equation in the frequency-domain

The equation can also be written in the frequency-domain as:

$\nabla \cdot B=0. \nabla \cdot B=0.$

Units

Magnetic flux	bb	T	tesla
Electric current density	jj	Am2Am2	ampere per square meter

Constants

Magnetic constant	μ0=4π×10−7NA2≈1.2566370614...×10−6T·mAμ0=4π×10−7NA2≈1.2566370614...×10−6T·mA

Discoverers of the law

Gauss's law for magnetism is a physical application of Gauss's theorem (also known as the divergence theorem) in calculus, which was independently discovered by Lagrange in 1762, Gauss in 1813, Ostrogradsky in 1826, and Green in 1828. Gauss's law for magnetism simply describes one physical phenomena that a magnetic monopole does not exist in reality. So this law is also called "absence of free magnetic poles".

People had long been noticing that when a bar magnet is divided into two pieces, two small magnets are created with their own south and north poles. This can be explained by Ampere's circuital law: the bar magnet is made of many circular currents rings, each of which is essentially a magnetic dipole; the macroscopic magnetism is from the alignment of the microscopic magnetic dipoles. Because a small current ring always generates an equivalent magnetic dipole, there is no way of generating a free magnetic charge. So far, no magnetic monopole has been found in experiments, despite that many theorists believe a magnetic monopole exists and are still searching for it.However, as pointed out by Pierre Curie in 1894, magnetic monopoles can exist conceivably. Introducing fictitious magnetic charges to the Maxwell's equations can give Gauss's law for magnetism the same appearance as Gauss's law for electricity, and the mathematics can become symmetric. When this modulated electrical signal is applied to a coil, it produces an oscillating magnetic field, which causes the coil to move in and out over a magnetic core in that same pattern. The coil is then attached to a movable speaker cone so it can reproduce audible sound waves in the air. The first practical application for the microphone and speaker was the telephone, patented by Alexander Graham Bell in 1876. Although this technology has been improved and refined, it is still the basis for recording and reproducing sound.

The applications of electromagnets are nearly countless. Faraday's Law of Induction forms the basis for many aspects of our modern society including not only electric motors and generators, but electromagnets of all sizes. The same principle used by a giant crane to lift junk cars at a scrap yard is also used to align microscopic magnetic particles on a computer hard disk drive to store binary data, and new applications are being developed every day.

What does gauss mean?

Surprisingly it is still an often misunderstood term, even among companies that rely on magnetic components.Essentially, gauss refers to the intensity of the magnetic field or, put another way, how much magnetic field is in a given area. One unit of gauss is one line of flux in a 1 cm square surface area. Another way to think of it is in terms of flux density. So if you can imagine a sugar cube and one line of flux coming from the North Pole to the South Pole that's what one gauss is. It doesn't necessarily pertain to how far the magnetic field reaches, however the distance will be proportional to flux density and magnet geometry.

In more technical terms, gauss is still a measurement of field strength; it is a location variable and also a vector (with direction), which means a different location in a space has different gauss reading and direction associated with it.

Magnet pull strength vs gauss

Gauss is different than pull strength of a magnet in general and a higher gauss does not necessarily lead to higher pull strength. A gauss reading is used in applications that field strength functions as the primary parameter, such as a sensor application. In most cases gauss level cannot be used to compare the field strength between magnets unless the magnets have the same geometry, and the gauss readings are measured at the same location.

What does gauss measure?

In measurement terms, gauss, abbreviated as G or Gs, is the cgs unit of measurement of a magnetic field B, which is also known as the "magnetic flux density" or the "magnetic induction". One gauss is defined as one maxwell per square centimeter. The cgs system has been augmented by the SI system, which uses the tesla (T) as the unit for B. One Tesla = 10,000 gauss!

Okay, back to Earth. Since the earth's magnetic field is about 0.5 gauss, and the pizza-shaped refrigerator magnet you got from your local pizzeria is 10 gauss, one could mistakenly conclude that the magnet from Al's Pizza is more powerful than the one around the planet. That would be a frightening thought. The magnetic field can reach everywhere in a space, but its strength is reduced as distance increases. That is why air shipment regulates a certain gauss value at a distance of 15 feet. Usually the gauss level is very small at this distance and can only be measured with a very accurate and sensitive gauss meter.

Magnetic field calculator

A calculation may be helpful in estimating field strength at a distance, but it wouldn't take shielding / packaging into consideration. Working out the calculation by hand can be a long and difficult process. Fortunately, Adams offers a shortcut – a free Magnetic Field Calculator that measures both gauss level and pull force. Our calculations are based on the size and material type for an individual magnet.

Conclusion

The property of magnet to attract or repel other substance is known as **Magnetism**. We know that, there exist an imaginary magnetic field lines around a magnet which is the main source, responsible for the behaviour of the magnets. When these magnetic field lines penetrates through an area perpendicularly, then (The average of the magnetic field lines) X (the area through which it penetrates) is known as **Magnetic Flux**. Gauss law in magnetism states that the magnetic flux through any closed surface is zero.

Three types of Application of Gauss's Law

- Electric Field intensity due to Infinitely long uniformly Charged Wire
- Electric Field due to Plane Sheet
- Electric Field due to Spherical shell

Thus the net electric flux through any closed surface is equal to 1/ times the net electric charge within that closed surface (or imaginary Gaussian surface

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