# MHD THREE-DIMENSIONAL FLOW THROUGH A POROUS MEDIUM WITH EXPONENTIAL DECREASING SUCTION

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**ABSTRACT –**

The purpose of this paper is to study the effect of exponential decreasing suction on the MHD 3-dimensional laminar flow of an electrically conducting, viscous and incompressible fluid through a porous medium. The porous medium is bounded by an infinite flat plate and the flow becomes 3-dimensional due to the variation of suction velocity in transverse direction at the plate. A magnetic field is applied perpendicular to the free stream velocity with neglecting induced magnetic field, hall effects, electrical and polarization effects. Analytical expressions for velocity are obtained. The important characteristics of the problem, the skin friction and the components of velocity field are discussed in detail with the help of tables and graphs.

**Key-words:** MHD, Porous Medium, Laminar Flow, Skin Friction, Viscous Fluid.

## 1. INTRODUCTION

Laminar flows with suction are of principal interest because these are quite prevalent in nature. Such flows have many scientific and engineering applications particularly in the fields of agricultural engineering to study the underground water resources, seepage of water in river beds; in aeronautical engineering to reduce drag coefficients and hence to enhance the vehicle power by a substantial amount; in chemical engineering for filtration and purification processes; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. The interest in these types of problem stems from the possibility of reducing the power required to pump oil in a pipe line by suitable addition of water.

The boundary layer suction is one of the most considerable subject and the developments on this subject since world war II, have been reported by **Lachmann (1961)**.

The interaction of the magnetic field on the laminar three dimensional flow of an electrically conducting viscous fluid is of more recent origin and has received considerable interest due to the increasing technical applications of MHD effects. Efforts in this direction have been made by many researchers. **Kishore, Tejpal and Katiyar (1981)** considered unsteady MHD flow through two parallel porous flat plates. **Singh (1991)** analyzed the three dimensional flow and heat transfer along a porous plate in the presence of viscous dissipative heat. **Shiam and Yaghoobinejad (1993)** discussed suction flow along a circular surface. **Gupta & Johari (2001)** analyzed MHD three dimensional flow past a porous plate. **Saxena & Johari (2008)** considered unsteady MHD flow through porous medium and heat transfer past a porous vertical moving plate with heat source. **Sahu & Rajput (2013)** studied thermal diffusion and chemical reaction effects on free convection MHD flow through a porous medium bounded by a vertical surface with constant heat flux.

The object of this paper is to study hydromagnetic effects on the 3-dimensional laminar flow of an electrically conducting viscous incompressible fluid past a porous plate with transverse exponential decreasing suction. The uniform flow is subjected to a transversely applied magnetic field. The mathematical analysis is presented for the hydromagnetic boundary layer flow neglecting the induced magnetic field.

## 2. MATHEMATICAL ANALYSIS

The basic assumptions of the problem are:

1. The flow is three-dimensional, steady and laminar.
2. The fluid is viscous, incompressible and electrically conducting.
3. Hall effects, polarization and electrical effects are neglected.
4. The induced magnetic field is also neglected.
5. The magnetic field is applied perpendicular to the free stream velocity.

Under these assumptions the equations which govern the problem are:

**CONTINUITY EQUATION**

$$\nabla \bar{V} = 0$$

**MOMENTUM EQUATION**

$$(\nabla . \bar{V})\, \bar{V} = -\frac{1}{\rho}\, \nabla \rho + v\, \nabla^2\, \bar{V} + \frac{1}{\rho}\, (\bar{J} \times \bar{B})$$

The last term on the right hand side of above equation is due to electromagnetic field.

Where $\bar{J} \times \bar{B}$ is called Lorentz force and is defined as

$$\bar{J} \times \bar{B} = \partial(\bar{V} \times \bar{B}) \times \bar{B}$$

Consider the steady laminar three-dimensional flow of a viscous incompressible fluid through a porous medium. A transverse decreasing suction is applied in the direction of the flow. A co-ordinate system is introduced with porous plate lying horizontally on x* - z* plane. The x* -axis is taken along the plate i.e. the direction of the flow and y* -axis is taken normal to it directed into the fluid flowing laminarly with free stream velocity U. Since the plate is considered infinite in x*-direction, so all physical quantities will be independent of x*, however, the flow remains three-dimensional due to the variation of the suction velocity distribution of the form:

$$v^* = -v_o \left\{1 + \varepsilon\, e^{-(\lambda U z^*/v)}\right\} \tag{1}$$

Where $V_o > 0$, is the mean suction velocity and $\varepsilon$ ($\varepsilon << 1$) is a very small quantity. The negative sign indicates that the suction is towards the plate.

A magnetic field of uniform strength $B_O$ is applied along y*-axis and is perpendicular to the free stream. Suppose u*, v* & w* be the velocity components in the directions x*, y* & z* respectively.

Therefore governing equations are

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{2}$$

$$v^*\, \frac{\partial u^*}{\partial y^*} + w^*\, \frac{\partial u^*}{\partial z^*} = v\left\{\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right\} - \frac{\sigma B_O^2 u^*}{\rho} \tag{3}$$

$$v^*\, \frac{\partial v^*}{\partial y^*} + w^*\, \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\, \frac{\partial p^*}{\partial y^*} + v\left\{\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right\} \tag{4}$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \nu \left[\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right] - \frac{\sigma B_O^2 w^*}{\rho} \tag{5}$$

The boundary conditions are

$$\begin{array}{lllll} y^* = o & : u^* = o, & v^* = -v_0 \left[1 + \varepsilon\, e^{-(\lambda U z^*/\nu)}\right], & & w^* = o \\ y^* -> \infty & : u^* = U, & v^* = -v_0 & w^* = o, & \rho^* = \rho^*_\infty \end{array} \tag{6}$$

Equations (2) to (5) reduce to the non-dimensional form:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = o \tag{7}$$

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] - Mu \tag{8}$$

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial \rho}{\partial y} + \left[\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \tag{9}$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial \rho}{\partial z} + \left[\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] - Mw \tag{10}$$

And the boundary conditions (6) to

$$\begin{array}{lllll} y = o & : u = o, & v = -\lambda \left(1 + \varepsilon\, e^{-(\lambda z)}\right), & & w = o \\ y -> \infty & : u = 1, & v = -\lambda & w = o, & \rho = \rho_\infty \end{array} \tag{11}$$

Where the non-dimensional quantities are defined as

$$u = \frac{u^*}{U}, \qquad v = \frac{v^*}{U}, \qquad w = \frac{w^*}{U},$$

$$y = \frac{U y^*}{\nu}, \qquad z = \frac{U z^*}{\nu}, \qquad p = \frac{p^*}{\rho U^2}$$

**SUCTION PARAMETER ($\lambda$):**

$$\lambda = \frac{V_o}{U}$$

**HARTMANN NUMBER (M):**

$$M = \frac{\sigma B_O^2 \nu^2}{\mu U^2} \tag{12}$$

In order to solve differential equations (7) to (10), we assume the solutions s of the following form because the amplitude $\epsilon \ll 1$ is very small:

$$u\,(y,z) = u_o(y) + \epsilon\, u_1\,(y,z) + \dots\dots\dots\dots\dots\dots\dots\dots..$$

$$v\,(y,z) = v_o(y) + \epsilon\, v_1\,(y,z) + \dots\dots\dots\dots\dots\dots\dots\dots..$$

$$w\,(y,z) = w_o(y) + \epsilon\, w_1\,(y,z) + \dots\dots\dots\dots\dots\dots\dots\dots.. \tag{13}$$

$$p\,(y,z) = p_o(y) + \epsilon\, p_1\,(y,z) + \dots\dots\dots\dots\dots\dots\dots\dots..$$

When $\epsilon = 0$, the problem reduces to the two-dimensional flow with constant suction at the plate. In this case equations (7) to (10) reduce to

$$v_0' = 0 \tag{14}$$

$$u_0'' + \lambda\, u_0' - M u_0 = 0 \tag{15}$$

Where primes denote the differentiation with respect to y.

The corresponding boundary conditions are:

$$\begin{aligned} y = o \quad &: \quad u_0 = o, \quad v_0 = -\lambda \\ y -> \infty \quad &: \quad u_0 = 1,\; v_0 = -\lambda \end{aligned} \tag{16}$$

The solutions of equations (14) & (15) under the boundary conditions (16) are

$$u_0 = 1 - e^{-r_1 y} \tag{17}$$

with

$$\left.\begin{aligned} v_0 &= -\lambda \\ w_0 &= 0 \\ p_0 &= p_\infty \end{aligned}\right\} \tag{18}$$

Where

$$r_1 = \frac{1}{2}\left[\lambda + \sqrt{\lambda^2 + 4M}\,\right]$$

When $\epsilon \neq 0$, substituting (13) in equations (7) to (10) and comparing the coefficients of identical power of $\epsilon$, neglecting the higher powers of $\epsilon$, we get the following equations as the coefficients of $\epsilon$ with the help of equation (18) :

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{19}$$

$$v_1\,\frac{\partial u_0}{\partial y} - \lambda\,\frac{\partial u_1}{\partial y} = \left[\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right] - M u_1 \tag{20}$$

$$-\lambda\,\frac{\partial v_1}{\partial y} = -\,\frac{\partial p_1}{\partial y} + \left[\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right] \tag{21}$$

$$-\lambda\,\frac{\partial w_1}{\partial y} = -\,\frac{\partial p_1}{\partial z} + \left[\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right] - M w_1 \tag{22}$$

These are the linear partial differential equations which describe the three-dimensional flow.

The corresponding boundary conditions become:

$$y = 0 \quad : \quad u_1 = 0, \quad v_1 = -\lambda\, e^{-\lambda z}, \quad w_1 = 0$$

$$y \rightarrow \infty \quad : \quad u_1 = 0, \quad v_1 = 0, \quad w_1 = 0 \quad p_1 = 0 \tag{23}$$

In order to solve these equations we will first consider the equations (19), (21) & (22), being independent of the main flow component $u_1$. We assume $v_1$, $w_1$ & $p_1$ of the form:

$$v_1(y\,,z) = v_{11}(y)e^{-\lambda z} \tag{24}$$

$$w_1(y\,,z) = \tfrac{1}{\lambda}\, v'_{11}(y)\, e^{-\lambda z} \tag{25}$$

$$p_1(y\,,z) = p_{11}(y)\, e^{-\lambda z} \tag{26}$$

Where prime denote the differentiation w, r to y. Equations (24) & 25) have been chosen so that the continuity equation (19) is satisfied. Substituting these equations into equations (21) & (22) and applying the corresponding transformed boundary conditions, we get the solutions of $v_1$, $w_1$, & $p_1$, as:

$$v_1 = \frac{\lambda}{(r_2 - r_{22})}\left\{r_{22}\, e^{-r_2\, y} - r_2\, e^{-r_{22}\, y}\right\} e^{-\lambda z} \tag{27}$$

$$w_1 = \frac{r_2\, r_{22}}{(r_2 - r_{22})}\left\{ e^{-r_2\, y} - e^{-r_{22}\, y}\right\} e^{-\lambda z} \tag{28}$$

$$p_1 = \frac{r_2\, r_{22}}{\lambda(r_2 - r_{22})}\left[\left\{r_{22}(r_{11} - \lambda) - M\right\} e^{-r_{22}\, y} - \left\{r_2(r_1 - \lambda) - M\right\} e^{-r_2\, y}\right] e^{-\lambda z} \tag{29}$$

Where

$$r_2 = \frac{1}{2}\left\{r_1 + \sqrt{r_1^2 + 4\,\lambda^2}\right\}$$

$$r_{11} = \frac{1}{2}\left\{\lambda - \sqrt{\lambda^2 + 4M}\right\}$$

$$r_{22} = \frac{1}{2}\left\{r_{11} + \sqrt{r_{11}^2 + 4\lambda^2}\right\}$$

In order to solve the differential equation (20) for $u_1$, we assume

$$u_1(y,z) = u_{11}(y)\, e^{-\lambda z} \tag{30}$$

Substituting this equation in (20) and solving under the boundary conditions (23), we get –

$$u_1 = \frac{\lambda}{(r_2 - r_{22})}\left\{C_1\, e^{-ny} + C_2\, e^{-(r_1 + r_{22})y} + C_3\, e^{-(r_1 + r_2)y}\right\} e^{-\lambda z} \tag{31}$$

Where $n = \frac{1}{2}\left\{\lambda + \sqrt{\lambda^2 + 4(\lambda^2 + M)}\right\}$

$$C_1 = C_2 - C_3, \qquad C_2 = \frac{r_2}{r_{22}}, \qquad C_3 = \frac{r_1 r_{22}}{r_2^{\,(3r_1 - \lambda)}}$$

## 3. RESULT & DISCUSSION

We now discuss the important flow characteristics of the problem. Knowing the velocity field we can obtain the expressions for the shear stress components in the x* & z*-directions in the non-dimensional form as:

$$\tau_x = \frac{\tau_x^*}{\rho U^2} = \left[\frac{\partial u}{\partial y}\right]_{y=0}$$

$$= u_0'(o) + \varepsilon\, u_1'(o)$$

$$= r_1 + \frac{\varepsilon\lambda}{(r_2 - r_{22})}\left\{C_2(r_1 + r_{22} - n) - C_3(r_1 + r_2 - n)\right\} e^{-\lambda z} \tag{32}$$

$$\tau_z = \frac{\tau_z^*}{\rho U^2} = \left[\frac{\partial w}{\partial y}\right]_{y=0}$$

$$= w_0'(o) + \varepsilon\, w_1'(o)$$

$$= \varepsilon\, r_2\, r_{22}\, e^{-\lambda z} \tag{33}$$

The effect of the hartmann number (M) and Suction parameter ($\lambda$) with $\varepsilon = 0.2$, z = 0 on the main flow velocity profiles u are shown in Fig.1. From this Figure it may be seen that velocity increases when the Hartmann number and Suction parameter increase and the differences of the velocity decrease with the increase of M & $\lambda$.

Figs. 2(a) & 2(b) give the variations of the transverse velocity component $w_1$ for different values of Hartmann number (M) and Suction parameter ($\lambda$) at z = -0.5 & z = 0.5 respectively. A study of these Figures show that the transverse velocity component $w_1$ increases with the increase of Hartmann number (M) & Suction parameter ($\lambda$) both. It can also seen from these Figures that the transverse velocity component $w_1$ increases with the decreases of M at a constant value of $\lambda$ but $w_1$ decreases with the decreases of $\lambda$ at a constant Hartmann number.

The numerical values of skin-frictions have been listed in Table 1(a) & 1(b) for various values of M and $\lambda$ at $\varepsilon = 0.2$, z=0. A study of Table 1 (a) shows that main flow skin-friction ($\tau_x$) increases when the Hartmann number (M) and Suction parameter ($\lambda$) increase. And Table 1(b) shows that skin-friction in the direction perpendicular to the main flow ($\tau_z$) decreases with the increases of Hartmann number at a constant $\lambda$ but it is increases with the increases of Suction parameter at a constant M.

**Table 1(a) : Main flow skin-friction ($\tau_x$) at $\varepsilon = 0.2$, z=0**

| $\lambda$ | M | | |
|---|---|---|---|
| | 0 | 2 | 4 |
| 0.5 | 0.5500 | 1.6407 | 2.3203 |
| 1.0 | 1.1000 | 2.1130 | 2.6746 |

**Table 1(b) : Skin-friction in the direction perpendicular to the main flow ($\tau_z$) at $\varepsilon = 0.2$, z=0**

| $\lambda$ | M | | |
|---|---|---|---|
| | 0 | 2 | 4 |
| 0.5 | 0.0809 | 0.0667 | 0.0625 |
| 1.0 | 0.3236 | 0.2984 | 0.2835 |

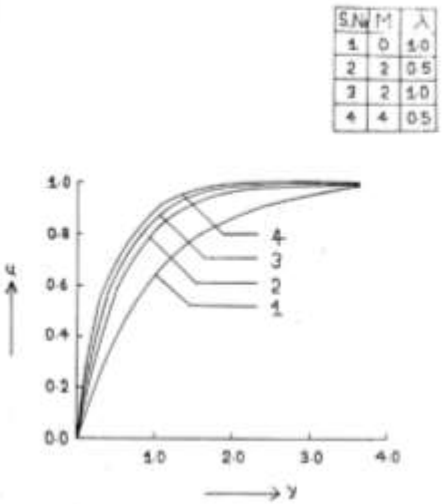


FIG. 1 VELOCITY PROFILES FOR $\epsilon = 0.2, Z = 0$

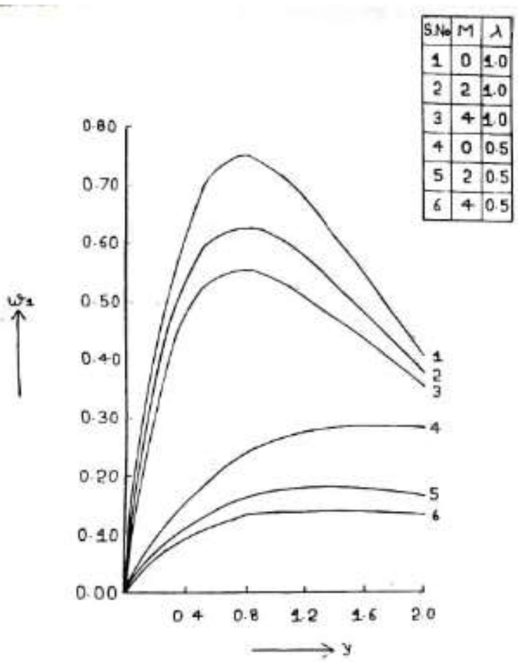


FIG. 2(a) TRANSVERSE VELOCITY PROFILES FOR $Z = -0.5$

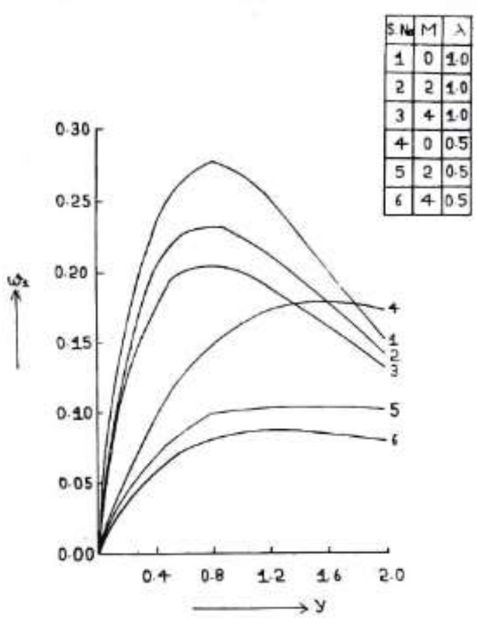


FIG. 2(b). TRANSVERSE VELOCITY PROFILES FOR $Z = 0.5$

# NOMENCLATURE

| | |
|---|---|
| $\bar{B}$ | Magnetic field |
| $B_0$ | Magnetic field component along $y^*$ - axis |
| M | Hartmann number |
| $p^*$ | Pressure |
| P | Dimensionless pressure |
| $P_\infty^*$ | Pressure at $y^* = \infty$ |
| $\bar{J}$ | Current density vector |
| $u^*, v^*, w^*$ | Velocity Components in the directions x*, y* & z* respectively |
| u, v, w | Dimensionless velocity components in the directions x, y & z respectively |
| U | Free stream velocity |
| $\bar{V}$ | Velocity vector |
| $V_0$ | Mean suction velocity |
| $x^*, y^*, z^*$ | Co-ordinate system |
| x, y, z | Dimensionless co-ordinate system |
| $\rho$ | Density |
| $\sigma$ | Electrical conductivity |
| $\upsilon$ | Kinematic viscosity |
| $\mu$ | Coefficient of viscosity |
| $\lambda$ | Suction parameter |
| $\tau_x$ | Main flow skin-friction |
| $\tau_z$ | Skin-friction in the direction perpendicular to the main flow |

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