Fuzzy Multi-Objective Travelling Salesman Problem

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Abstract

In this paper a method is proposed to solve multi-objective Triangular Fuzzy Travelling Salesman problem. The given FTSP problem is converted to parametric form. Then using appropriate weights, the multi-objective salesman problem is converted to single objective salesman problem. A numerical example of this type TSP is presented in support of the proposed method.

Key words: Travelling Salesman problem, Fuzzy number, Fuzzy arithmetic operations, Fuzzy ranking function.

1. INTRODUCTION:

Travelling Salesman Problems are been of interest in past few decades if we observe new approach for triangular fuzzy traveling salesman problem , Mohanaselvi S & Ganesan K (2012) Fully Fuzzy linear programs with triangular fuzzy number ,Classical numbers was discussed by Abbasbandy [1], Chaudhuri A[4] discussed Fuzzy multi-objective linear programming for Fuzzy TSP discussed Dhansekhar S. [5]. Sobhan Babu,K ,Keshava redid.E & Sundara Murthy.M[12] discussed an exact algorithm for travelling salesman problem, in this paper Fuzzy TSP is solved in different manner without converting it to crisp TSP. Section 2 has Definitions of Fuzzy sets& number ,membership function, and arithmetic operations on Triangular Fuzzy numbers. In Section-3 mathematical formulation of Fuzzy TSP is given and then Algorithm to solve travelling Salesman is presented. In Section-4 Numerical example in support of the algorithm is given, Section-5 contains conclusion.

2. PRELIMINARIES:

Definition-2.1. A Fuzzy set $\tilde{p}$ defined on the set of real number R is said to be a Fuzzy number, if its membership function $\tilde{p}$ $\rightarrow$[0, 1] has the following properties.

(i) $\tilde{p}$ is convex, (ii) $\tilde{p}$ is normal i.e. there exists x ε R such that $\tilde{p}$ (x) =1.

(ii) $\tilde{p}$ is piecewise continuous.

Definition 2.2. A Fuzzy number $\tilde{p}$ on R is a triangular fuzzy number if it's membership function $\tilde{p}$: R$\rightarrow$[0, 1] satisfies following

$$\tilde{p}(x) = \begin{cases} \dfrac{x-p_1}{p_2-p_1}, & p_1 < x \le p_2, \\ \dfrac{p_3-x}{p_3-p_2}, & p_2 \le x \le p_3 \\ 0, & otherwuse \end{cases}$$

If $\tilde{p} = (p_1, p_2, p_3)$ and If t = p_2 is mid value and let δ = (p_2 – p_1) represents left spread, y = (p_3 – p_2) represents right spread of triangular fuzzy number then $\tilde{p} = (p_1, p_2, p_3)$ = (δ, t, y)

Definition-2.3. Arithmetic operations used for parametric form;

(a , b(r),b (r)) + (p , q(r),q (r)) =(a + p, Max{b(r),q(r)}, Max{b(r),q(r)})

(a , b(r),b (r)) - (p , q(r),q (r)) =(a - p, Min{b(r),q(r)}, Min{b(r),q(r)})

Definition-2.4. A triangular fuzzy number $\tilde{p} = (p_1, p_2, p_3)$ $\in$ F(R) can also be represented as an pair $\tilde{p} = (\underline{p}, \overline{p})$ of function $\underline{p}(r),\ \overline{p}(r),\ for, o \le r \le 1,$ which satisfies the following requirements:

(i) $\underline{p}(r)$ is a bounded monotonic increasing right continuous function. (ii) $\overline{p}(r)$ is a bounded monotonic increasing left continuous function. (iii) $\underline{p}(r), \leq \overline{p}(r),\ o \leq r \leq 1,$

It is also represented by $\tilde{p} = (p_O,\ p_L,\ p^U)$ where $p_L = (p_O - \underline{p}),\ p^U = (\overline{p} - p_O)$ are called left fuzziness index function and right fuzziness index function respectively.

$$p_O = \left(\frac{\underline{p}(r) + \overline{p}(r)}{2}\right)_{r=1}$$ is called location index number of $\tilde{p}$.

Definition-2.5. Magnitude of $\tilde{p}$ = $mag\ (\tilde{p}) = \frac{1}{2}\left(\int_0^1 (p^U + 4p_o - p_L)\ f(r)\ dr\right)$

Where f (r) is a non negative and increasing function on [0, 1] with f (0) = 0, f(1)=1 and $\int_0^1 f(r)\ dr = \frac{1}{2}$. The function f (r) can be considered as weighing function and can be chosen according to the situation . In this paper f (r) = r is taken.

(i) $\tilde{p} \succeq \overline{\delta}$ *if and onlyif* $mag\ (\tilde{p}) \geq mag\ (\overline{\delta})$.

(ii) $\tilde{p} \preceq \overline{\delta}$ *if and onlyif* $mag\ (\tilde{p}) \leq mag\ (\overline{\delta})$.

(iii) $\tilde{p} \approx \overline{\delta}$ *if and onlyif* $mag\ (\tilde{p}) = mag\ (\overline{\delta})$.

However in this paper as the problem is not converted into crisp TSP so magnitude of fuzzy number is not used.

3. MATHEMATICAL FORMULATION OF FUZZY MULTI-OBJECTIVE TRAVELLING SALESMAN PROBLEM:

$$\tilde{z}_1 = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij}\ x_{ij}$$

$$\tilde{z}_2 = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{d}_{ij}\ x_{ij}$$

$$\tilde{z}_3 = \min \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{t}_{ij}\ x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ \sum_{j=1}^{n} x_{ij} = 1,\quad x_{ij} + x_{ji} \leq 1,\ x_{ij} \geq 0, i \neq j \leq n$$

$$x_{ij} = \begin{cases} 1, if\ \ city\ 'j'\ is\ \ visited\ from\ city\ 'i' \\ 0,\ \ otherwise \end{cases}$$

$\tilde{C}_{i\,j}$ is the cost of travelling from city ‘i’ to city ‘j’ in fuzzy form.

$\tilde{d}_{i\,j}$ is the distance of travelling from city ‘i’ to city ‘j’ in fuzzy form.

$\tilde{t}_{i\,j}$ is the time of travelling from city ‘i’ to city ‘j’ in fuzzy form.

$\tilde{C}_{i\,i}, \tilde{d}_{i\,i}, \tilde{t}_{i\,i}$ is taken infinite to insure no visit of salesman from city ‘i’ to’ i’

3.1 Algorithm to solve the travelling Salesman problem :-

Step 1: We express all the cost, distance, time in parametric form.(core, left fuzziness function, right fuzziness function)

Step 2: Then the multi-objective TSP is converted to single objective TSP by giving suitable weights to the cost, distance and time objects.

Step 3: Then without converting to Crisp TSP, we solve directly the given Salesman problem by subtracting row minima, followed by column minima.(Using Def.2.3)

Step 4: The optimal solution of single objective TSP is also optimal solution of given multi-objective TSP.

4. NUMERICAL ILLUSTRATION :

Following is a five city, cost, distance, time fuzzy travelling salesman problem.

Table-1 Cost matrix of TSP with triangular fuzzy number

	C_1	C_2	C_3	C_4	C_5
C_1	∞	(22, 24, 26) (6, 7, 8) (6, 8, 10)	(18, 22, 26) (3, 5, 7) (8, 10, 12)	(9, 11, 13) (2, 3, 4) (1, 2, 3)	(19, 20, 21) (9, 10, 11) (1, 2, 3)
C_2	(12, 10, 14) (2, 4, 6) (7, 8, 9)	∞	(14, 16, 18) (1, 2, 3) (2, 3, 4)	(15, 18, 21) (5, 6, 7) (2, 3, 4)	(28, 30, 32) (4, 5, 6) (2, 3, 4)
C_3	(21, 24, 27) (4, 5, 6) (1, 2, 3)	(26, 28, 30) (6, 7, 8) (2, 4, 6)	∞	(13, 15, 17) (4, 5, 6) (3, 4, 5)	(7, 10, 13) (8, 10, 12) (4, 5, 6)
C_4	(7, 10, 13) (8, 10, 12) (4, 5, 6)	(6, 10, 14) (1, 2, 3) (4, 5, 6)	(8, 10, 12) (3, 5, 7) (1, 3, 5)	∞	(6, 8, 10) (3, 5, 7) (2, 4, 6)
C_5	(14, 16, 18) (1, 2, 3) (2, 3, 4)	(9, 12, 15) (7, 9, 11) (3, 5, 7)	(18, 20, 22) (1, 3, 5) (1, 2, 3)	(14, 16, 18) (1, 2, 3) (2, 3, 4)	∞

Table-2 Cost matrix of TSP with triangular fuzzy number in parametric form

	C_1	C_2	C_3	C_4	C_5
C_1	∞	(24, 2-2r, 2-2r) (7, 1-r, 1-r) (8, 2-2r, 2-2r)	(22, 4-4r, 4-4r) (5, 2-2r, 2-2r) (10, 2-2r, 2-2r)	(11, 2-2r, 2-2r) (3, 1-r, 1-r) (2, 1-r, 1-r)	(20, 1-r, 1-r) (10, 1-r, 1-r) (2, 1-r, 1-r)
C_2	(12, 2-2r, 2-2r) (4, 2-2r, 2-2r) (8, 1- r, 1- r)	∞	(16, 2-2r, 2-2r) (2, 1- r, 1- r) (3, 1- r, 1- r)	(18, 3-3r, 3-3r) (6, 1-r, 1-r) (3, 1- r, 1- r)	(30, 2-2r, 2-2r) (5, 1- r, 1- r) (3, 1- r, 1- r)
C_3	(24, 3-3r, 3-3r) (5, 1-r, 1-r) (2, 1- r, 1- r)	(28, 2-2r, 2-2r) (7, 1- r, 1- r) (4, 2-2r, 2-2r)	∞	(15, 2-2r, 2-2r) (5, 1- r, 1- r) (4, 1-1r, 1-1r)	(10, 3-3r, 3-3r) (10, 2-2r, 2-2r) (5, 1- r, 1- r)
C_4	(10, 3-3r, 3-3r) (10, 2-2r, 2-2r) (5, 1- r, 1- r)	(10, 4-4r, 4-4r) (2, 1- r, 1- r) (5, 1- r, 1- r)	(10, 2-2r, 2-2r) (5, 2-2r, 2-2r) (3, 2- 2r, 2- 2r)	∞	(8, 2-2r, 2-2r) (5, 2-2r, 2-2r) (4, 2- 2r, 2- 2r)
C_5	(16, 2-2r, 2-2r) (2, 1- r, 1- r) (3, 1-1r, 1-1r)	(12, 3-3r, 3-3r) (9, 2-2r, 2-2r) (5, 2-2r, 2-2r)	(20, 2-2r, 2-2r) (3, 2-2r, 2-2r) (2, 1-r, 1-r)	(16, 2-2r, 2-2r) (2, 1- r, 1- r) (3, 1-1r, 1-1r)	∞

Table-3 Cost matrix of TSP after multiplication of weights 0.5, 0.3, 0.2

	C_1	C_2	C_3	C_4	C_5
C_1	∞	(12, 1-r, 1-r) (2.1, 0.3-0.32, 0.3r-0.3r) (1.6, 0.4-0.4r,0 .4-0.4r)	(11, 2-2r, 2-2r) (1.5, 0.6-.6r, 0.6r-.6r) (2, 0.4-.4r, 0.4-.4r)	(5.5, 1-r, 1-r) (0.9, 0.3-0.3r, 0.3-0.3r) (0.4, 0.2-0.2r, 0.2-0.2r)	(10, 0.5-.5r, 0.5-.5r) (3, 0.3-.3r, 0.3-.3r) (0.4, 0.2 - 0.2r, 0.2 -0.2r)
C_2	(6, 1-r, 1-r) (1.2, 0.6 - 0.6, 0.6- 0.6r) (1.6, 0.2-.2r, 0.2-.2r)	∞	(8, 1-r, 1-r) (0.6, 0.3-0.3r, 0.3-0.3r) (0.6, 0.2-0.2r, 0.2-.02r)	(9, 1.5-1.5r, 1.5-1.5r) (1.8, 0.3 -0.3r, 0.3-0.3r) (0.6, 0.2 -0.2r, 0.2-0.2r)	(15, 1-1r, 1-1r) (1.5, 0.3 -0.3r, 0.3-0.3r) (0.6, 0.2 -0.2r, 0.2-0.2r)
C_3	(12, 1.5-1.5r, 1.5-1.5r) (1.5, 0.3 -0.3r, 0.3-0.3r) (0.4, 0.2 -0.2r, 0.2-0.2r)	(14, 1-r, 1-r) (2.1, - 0.3 -0.3r, 0.3-0.3r) (0.8, 0.4 -0.4r, 0.4-0.4r)	∞	(7.5, 1-r, 1-r) (1.5, 0.3 -0.3r, 0.3-0.3r) (0.8, 0.2 -0.2r, 0.2-0.2r)	(5, 1.5- 1-1.5r, 1.5-1.5r) (3, 0.6 -0.6r, 0.6-0.6r) (1, 0.2 -0.2r, 0.2-0.2r)
C_4	(5, 1.5 - 1.5r, 1.5-1.5r) (3, 0.6 -0.6r, 0.6-0.6r) (1, 0.2 -0.2r, 0.2-0.2r)	(5, 2-2r, 2-2r) (0.6, 0.3 -0.3r, 0.3-0.3r) (1, 0.2-0.2r, 0.2-.02r)	(5, 1-r, 1-r) (1.5, 0.6 -0.6r, 0.6-0.6r) (0.6, 0.4 -0.4r, 0.4-0.4r)	∞	(4, 1-r, 1-r) (1.5, 0.6 -0.6r, 0.6-0.6r) (0.8, 0.4 -0.4r, 0.4-0.4r)
C_5	(8, 1-r, 1-r) (0.6, 0.3-0.3r, 0.3-0.3r) (0.6, 0.2-0.2r, 0.2-0.2r)	(6, 1.5- 1-1.5r, 1.5-1.5r) (2.7, 0.6 -0.6r, 0.6-0.6r) (1, 0.4 -0.4r, 0.4-0.4r)	(10, 1-r, 1-r) (.9, 0.6 -0.6r, 0.6-0.6r) (0.4, 0.2 -0.2r, 0.2-0.2r)	(8, 1-r, 1-r) (0.6, 0.3-0.3r, 0.3-0.3r) (0.6, 0.2-0.2r, 0.2-0.2r)	∞

Table-4 Single objective TSP converted by adding Cost, Distance and Time in Table 3

	C_1	C_2	C_3	C_4	C_5
C_1	∞	(15.7, 1 - r, 1 - r)	(14.5, 2 - 2r, 2 - 2r)	(6.8, 1 - r, 1 - r)	(10.7, 0.5-0.5r, 0.5 -0.5r)
C_2	(8.8, 1 - r, 1 - r)	∞	(9.2, 1 - r, 1 - r)	(11.4, 1.5-1.5r, 1.5-1.5r)	(17.1, 1 - r, 1 - r)
C_3	(13.9, 1.5-1.5r, 1.5 - 1.5r)	(16.9, 1 - r, 1 - r)	∞	(9.8, 1 - r, 1 - r)	(9, 1.5 - 1.5r, 1.5 - 1.5r)
C_4	(9,, 1.5 - 1.5r, 1.5 - 1.5r)	(6.6, 2 - 2r, 2 - 2r)	(7.1, 1 - r, 1 - r)	∞	(6.3, 1 - r, 1 - r)
C_5	(9.2, 1 - r, 1 - r)	(9.7, 1.5 - 1.5r, 1.5 - 1.5r)	(11.3, 1 - r, 1 - r)	(9.2, 1 - r, 1 - r)	∞

Table-5 Row Reduced Cost Matrix

	C_1	C_2	C_3	C_4	C_5
C_1	∞	(8.9, 1 - r, 1 - r)	(7.7, 1 - r, 1 - r)	0	(3.9, 0.5-0.5r, 0.5 -0.5r)
C_2	0	∞	(0.4, 1 - r, 1 - r)	(2.6, 1 - r, 1 - r)	(8.3, 1 - r, 1 - r)
C_3	(4.9, 0.5-0.5r, 0.5 -0.5r)	(7.9, 1 - r, 1 - r)	∞	(0.8, 1 - r, 1 - r)	0
C_4	(2.7, 1 - r, 1 - r)	(0.3, 1 - r, 1 - r)	(0.8, 1 - r, 1 - r)	∞	0
C_5	0	(0.5, 1 - r, 1 - r)	(2.1, 1 - r, 1 - r)	0	∞

Table-6 Column Reduced cost matrix

	C_1	C_2	C_3	C_4	C_5
C_1	∞	(8.6, 1 - r, 1 - r)	(7.3, 1 - r, 1 - r)	0	(3.9, 0.5-0.5r, 0.5 -0.5r)
C_2	0	∞	0	(2.6, 1 - r, 1 - r)	(8.3, 1 - r, 1 - r)
C_3	(4.9, 0.5-0.5r, 0.5 -0.5r)	(7.6, 1 - r, 1 - r)	∞	(0.8, 1 - r, 1 - r)	0
C_4	(2.7, 1 - r, 1 - r)	0	(0.4, 1 - r, 1 - r)	∞	0
C_5	0	(0.2, 1 - r, 1 - r)	(1.7, 1 - r, 1 - r)	0	∞

The optimum Solution is $C_1 \rightarrow C_4 \rightarrow C_2 \rightarrow C_3 \rightarrow C_5 \rightarrow C_1$

Fuzzy optimal distance: (3, 1-r, 1-r) + (2, 1-r, 1-r) + (2, 1-r, 1-r) + (10, 2 - 2r, 2 - 2r) + (2, 1 - r, 1 - r) = (19, 2 – 2r, 2-2r).

Fuzzy optimal cost :(11, 2 – 2r, 2-2r)+ (10, 4 - 4r, 4 - 4r) + (16, 2 - 2r, 2 - 2r) + (10, 3 - 3r, 3 - 3r) + (16, 2 – 2r, 2-2r) = (63, 4 - 4r, 4 - 4r)

Fuzzy optimal tine :(2, 1-r, 1-r) + (5, 1-r, 1-r) + (3, 1-r, 1-r) + (5, 1 - r, 1 - r) + (3, 1 - r, 1 - r) = (18, 1 – r, 1- r).

The value of r	**Optimal Distance (19, 2-2r, 2-2r)**	**Optimal Cost (63, 4-4r, 4-4r)**	**Optimal Time (18, 1 - r, 1 - r)**
r = 0	(17, 19, 21)	(59, 63, 67)	(17, 18, 19)
r = 0.25	(17.5, 19, 20.5)	(60, 63, 66)	(17.25 18, 18.75)
r = 0.5	(18, 19, 20)	(61, 63, 65)	(17.5 18, 18.5)
r = 0.75	(18.5, 19, 19.5)	(62, 63, 64)	(17.75 18, 18.25)
r = 1	(19, 19, 19)	(63, 63, 63)	(18, 18, 18)

5. CONCLUSION :

Thus in this paper Triangular Fuzzy Multi-objective TSP is solved without converting it into crisp problem. In literature FTSP are solved generally by converting them into crisp problem. Here at the end the optimal solution is in terms of parametric form i.e in terms of 'r'. so it depends on decision maker to choose values of 'r' ranging between 0 and 1. In the given numerical example the optimal solution has been given for r = 0, r =0.25 , r = 0.5, r = 0.75, r = 1.hence for future scope other fuzzy number based transportation ,Assignment problem can be solved by the above method without converting into crisp problems. So an alternative way of fuzzy TSP is proposed.

REFERENCES :

[1] Abbasbandy S and Hajjari T (2009) new approach for ranking of trapezoidal fuzzy numbers Computers and Mathematics with Applications 57pp 413–419

[2] Angel E, Bampis E, Gourvès, L (2004). Approximating the Pareto curve with Local Search for Bi-Criteria TSP (1, 2) Problem, Theor. Comp.Sci., 310(1-3): 135-146.

[3] Bellman R E and Zadeh L A (1970) Decision-making in fuzzy environment Management sciences 17(4) 141-164.

[4] Chaudhuri A, De K (2011). Fuzzy multi-objective linear programming for traveling salesman problem. Afr. J. Math. Comp. Sci. Res., 4(2): 64-70

[5] Dhanasekar S Hariharan S & Sekar P(2013) Classical Travelling Salesman Problem (TSP) based Approach to Solve Fuzzy TSP using Yager's Ranking. International Journal of Computer Applications 975 -987.

[6] Jaszkiewicz A (2002). Genetic Local Search for Multiple Objectives Combinatorial Optimization. Eur. J. Oper. Res., 137(1): 50-71.

[7] Javadia B, Saidi-Mehrabad M, Haji A, Mahdavi I, Jolai F, Mahdavi-AmiriN (2008). No-wait flow shop scheduling using fuzzy multi-objective linear programming. J. Franklin Inst., 345: 452-467.

[8]Kumar A,,Gupta A,"Methods for solving fuzzy Assignment problems and fuzzy Travelling salesman problems with different membership functions"Fuzzy inf.&Engg.(2011) Vol(1),pp03-21.

[9] Ming Ma Menahem, Friedman Abraham kande(1999) "A new fuzzy arithmetic" Fuzzy sets and systems 10(8) 83-90.

[10] Mohanaselvi S & Ganesan K (2012) Fully Fuzzy linear programs with triangular fuzzy number International journal of Mathematical Archieve 3(5) 1838-1845

[11] Sobhan Babu,K & Sundara Murthy.M,(2010) "An efficient algorithm for Variant Bulk Transportation Problem", International Journal of Engineering Science and Technology, Vol.2(7), pp.2595-2600.

[12] Sobhan Babu,K ,Keshava redid.E & Sundara Murthy.M (2012)"an exact algorithm for travelling salesman problem" Vol.3(1) .pp.317-321.