A Mathematical Analysis of Some Repairable Redundant Engineering System in Reliability Theory: A Review

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Abstract: High system dependability and availability are essential for industrial expansion since profit is directly proportional to the amount of output, which is in turn proportional to the performance of the system. As a result, appropriate design, optimization at the design stage, and maintenance of a system over its service life may improve the dependability and availability of a system. Because of their widespread usage in power plants, manufacturing systems, and industrial systems, a large number of researchers have investigated the dependability and availability of a wide range of systems and technologies.

Key Word: High System Dependability, Repairable Redundant Engineering, Reliability Theory

I. Introduction

A typical strategy for enhancing a system's dependability and availability is the use of redundancy. Because of the dependability of current electrical and mechanical components, many applications do not need the addition of redundancy in their design to be effective. Redundancy, on the other hand, may be an appealing alternative if the failure cost is high enough.

1.1 Models of Redundancy

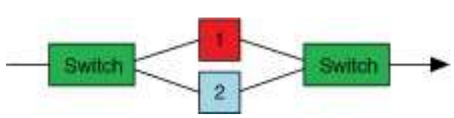
Different approaches to redundancy implementation exist, however the following models reflect the most often used ones in industry. Standby Redundancy, N Modular Redundancy, and 1: N Redundancy are the three primary models discussed in this work.

Standby Redundancy

Back-up redundancy, or "standby redundancy," refers to the use of a second identical unit in case the main fails. In most cases, the secondary unit serves just as a backup and is not used to monitor the system. On takeover of the Device Under Control, the standby unit must reconcile its input and output signals with those of the main unit (DUC). The secondary unit may transmit control signals to the DUC that are out of sync with the previous control signals that came from the main unit because of this "bump" in transmission.

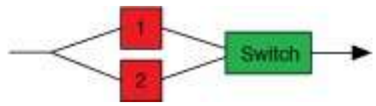
There should also be a third party acting as a watchdog, keeping track of the system to determine when a switchover condition has been reached and to provide commands instructing the system to hand power over to the standby unit and a voter. This form of redundancy often increases system expenses by roughly 2X or less, depending on the software development costs of your organisation. Cold Standby and Hot Standby are the two most common forms of standby redundancy.

Cold Standby



In cold standby mode, the secondary unit is shut down, ensuring the device's durability. As a result, the downtime is larger than in hot standby due to the need to power up the standby unit and bring it online in a predetermined condition. In order to resolve synchronisation difficulties, it is more difficult, but this is exacerbated by the amount of time it takes to bring the standby unit online.

Hot Standby

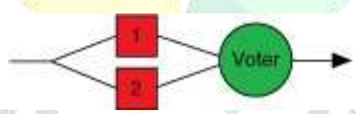


Because it is powered up when in hot standby mode, the secondary unit may be used to keep tabs on the DUC. To avoid the need for a third party, use the secondary unit as the watchdog and/or voter to determine when to switch over. The cold standby design maintains the standby unit's dependability better than this design. The downtime is reduced, which in turn boosts the system's availability. The DMR (Dual Modular Redundancy) and Parallel Redundancy (Parallel Redundancy). It is normal practise to use these naming conventions interchangeably. The key distinction between Hot Standby and DMR is the degree of synchronisation between the primary and secondary units. When using DMR, both the main and secondary units are perfectly in sync. DMR may be used more broadly in certain publications to describe any redundancy model using two units.

N Modular Redundancy

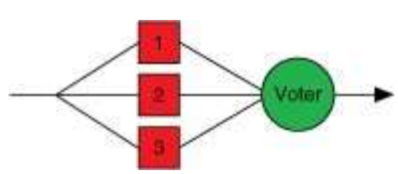
Multiple units operating in parallel, or N Modular Redundancy, is a term for this strategy. Input information is sent to all units simultaneously, ensuring good synchronisation. After comparing their output values, a vote is held to determine which values should be adopted. This model allows for smooth transitions. Hot Standby models often have quicker switchover times. When you just have two options, it might be difficult to determine which one is accurate. It might be difficult to decide on which one to put your faith in most of the time. When there are more than two participants, the issue is simplified, and the winner is generally determined by a vote of the majority or by a tie vote.

Dual Modular Redundancy



Using two functionally comparable units, Dual Modular Redundancy (DMR) can control the DUC from either side. In DMR, the most difficult part is figuring out when to switch to the backup unit. You have to determine what to do if the two units monitoring the application disagree. Assuming the secondary unit is more trustworthy than the main unit, you must either generate a tiebreaker vote or designate the secondary unit as the default winner. As long as the main unit is in control, and frequent diagnostics are conducted on the secondary unit to ensure its dependability, it may be more trustworthy. Due to the additional hardware and software development time, a DMR system often has a higher overall cost than a non-redundant system.

Triple Modular Redundancy



TMR (Triple Modular Redundancy) employs three functionally identical modules to offer redundant backups. In aerospace applications, where failure costs are exceedingly high, this strategy is highly prevalent. There are two primary reasons why TMR is more dependable than DMR. Having two "standby" units instead of only one is the most apparent rationale for the switch. The second reason is that diversity platforms, also known as diversity programming, are often used in TMR. It is possible to avoid common mode failure by using various software or hardware platforms on your redundant systems. For example, one hardware platform may be used for units 1 and 2, whereas PXI or Compact FieldPoint would be used for unit 3. LabVIEW and Lab-Windows/CVI can be used for two of the systems, while LabVIEW can be used for the third. You may even have various development teams working on different systems at the same time, if necessary. To tackle the same issue, all systems have the same basic criteria.

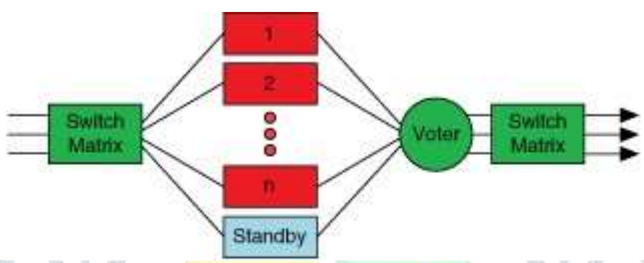
The voter chooses the unit that will be responsible for running the programme. Choosing which system to believe is done democratically and by majority rule using TMR. It's up to the elector to determine which method to believe if he or she gets three distinct replies. As a result, making the transfer is simple and quick. At least three times as expensive, TMR is a downside to this strategy.

Quadruple Modular Redundancy

In order to improve dependability, quadruple modular redundancy (QMR) differs from triple modular redundancy (TMR) by using four instead of three modules. Consequence number one is the 4X rise in system costs.

1:N Redundancy

Using the 1:N approach, a single backup may take the place of any of the active systems, allowing for a more flexible system architecture. A single standby unit serves as a backup for a number of main units, making this method far less expensive than the alternatives. Only if all the main units have identical functions can the standby unit back up any of the primary units in case one of them fails.



Choosing when to switch and having a switch matrix that can appropriately and effectively redirect the signals are other disadvantages of this strategy.

II. Literature Review

Ke & Chu (2007), One working unit and a spare are considered in the steady-state availability (A) of a repairable system. In this study, they compare four computationally viable and efficient ways to computing A's confidence intervals. The asymptotically normal and highly consistent natural estimator A of A is defined as the ratio of the operating unit sample mean lifespan to the sum of the operating unit sample mean lifetime and the system sample mean downtime. The confidence intervals for derived using four different bootstrap methods as explained in this article.

Zoulfaghari et al. (2014), In the redundancy allocation problem (RAP), reliability optimization and availability optimization are two categories of generic optimization issues that are traditionally tackled. Availability optimization, on the other hand, has been investigated by a smaller number of researchers than reliability optimization. If a component is repairable, it is presumed that the complete component is repairable. Furthermore, a numerical example is provided to demonstrate the effectiveness of the suggested GA. One of the most often suggested algorithms in the literature is shown to be outperformed by the proposed GA, according to experimental data.

Huang & Ke (2009), Four designs for a repairable system with two major components/units and one standby are examined in this study. As a result, the four configurations have been programmed to monitor standby failure and potential unit restarts. Each of the main and backup systems is considered to have an exponential failure time distribution. A k-stage Erlang distribution is used to model the repair and reboot times. In this article, the mean time to failure (or MTTF) and steady-state availability (or A) are explicitly expressed for four different designs and compared. The C/B criteria is used to compare certain distribution parameter values and unit cost values. Repairable systems are classified according to MTTF, A, C/B, and B, where B is either A or MTTF. Many managers and manufacturers will benefit from this document, even if they are unsure which configuration is best among the four. Based on the characteristics and weights selected, managers may use these findings to pick the optimal configuration. This study compares four possible configurations of a two-unit online repairable system with a single standby. Reliability and availability models for redundant repairable systems defined by detection, switching failure, and reboot have not before been discussed in this context.

Huang et al. (2016), Using more efficient and effective techniques to speed up convergence and increase optimization model solution correctness has been a primary emphasis in recent research on reliability-redundancy allocation challenges. However, no more research has been done on the reliability-redundancy allocation model. In this study, the authors use the notion of survival signature to simplify the optimization model for allocating resources based on dependability and redundancy. The survival signature summarises the structure of a system's information. As an optimization problem, the reliability-redundancy allocation issue is stated to maximise system dependability while meeting certain constraints. When a constraint-free optimization problem is needed, an adaptive penalty function is given. The unconstraint optimization is then solved using a heuristic approach known as stochastic fractal search. A component's redundancy level may be assigned using the relative significance of its components, which can be quantified using the (joint) structure importance. The suggested technique minimises the size of the optimization problem while still providing insight into the allocation of system dependability and redundancy.

Gavrilov & Gavrilova (2001), There is a broad theory of system failure called reliability theory. Predicting age-related failure rates for a given system design (reliability structure) and the predetermined reliability of its constituent parts is now possible. Systems with a constant failure rate but no ageing components are predicted by reliability theory to degrade (fail more often) as they age if they are redundant in irreplaceable components. As a result, systems redundancy is directly linked to the onset of old age. Reliability theory also predicts a slowing of mortality in old age, as well as a plateauing of mortality in old age, as a result of redundancy depletion. Taking into consideration the early imperfections (defects) in freshly created systems, the theory explains why death rates in many species grow exponentially with age (the Gompertz rule). Furthermore, it explains why organisms "prefer" to die in accordance with the Gompertz rule, but technological equipment often fail in accordance with the Weibull (power) law When organisms perish according to the Weibull rule, theoretical requirements are specified: organisms should be devoid of basic faults and imperfections. We may use this theory to derive an all-encompassing, age-independent failure law, with the Gompertz and Weibull laws serving as specific examples of the broader failure law. Using this hypothesis, it is possible to explain why mortality disparities across populations (within a particular species) decrease over time and why mortality convergence is seen. With a few, extremely generic and reasonable assumptions, dependability theory provides remarkable predictive and explanatory potential. A viable approach to creating a mathematically grounded, biologically specified theory of how to extend human life expectancy seems to be dependability theory.

Barlow (2003), With the 1961 publication of "Multi-component systems, structures, and their dependability," the mathematical theory of reliability has been claimed to constitute a distinct subject. Since then, mathematicians have used advanced approaches like queueing theory, statistical analysis and probability to address engineering dependability issues. How the 1965 book5 "Mathematical Theory of Reliability" got to be written is what they are going to discuss here. On the basis of probabilistic principles of ageing, certain personal historical viewpoints will be presented. Schur functions and their Bayesian implications for reliability studies will also be discussed.

Shekhar et al. (2016), When it comes to production in the digital age, Industry 4.0 represents the fourth industrial revolution. When it comes to the production process, the most important consideration is the continual relationship between decision-makers, machinery and products. Fault-tolerant machining systems with different forms of machining impediment are the primary focus of this study. An essential aspect of computer and communication systems, manufacturing and production systems, security systems, etc., is a fault-tolerant redundant repairable machining system. There is a major issue in the current research about random failures of operating and standby units, as well as probabilistic common-cause failure of the machining system, about an automated changeover of the available standby unit in place of the failed operational unit. M operating units share the burden generally, but short-term overload persists until the system has at least MK units with a degradable failure rate. For system designers and decision-makers, numerical simulations, comparative analysis of queue characteristics, and optimum analysis of queue characteristics are also done substantially.

III. Reliability Design

The dependability theory as well as the related methodology have been developed over a period of time in many stages. During the course of the growing process, three major technological areas were developed:

- ✓ Engineering tasks linked to reliability include system reliability study, design evaluation, and other related tasks.
- ✓ Reliability engineering Operational analysis, which involves examination of failures and remedial measures.

- ✓ It is also known as reliability mathematics, and it is comprised of statistics and associated mathematical knowledge.

An accurate evaluation of the failure rate of a system is required to develop a better manner of balancing the expense of failure reduction against the benefit of improvement. One fundamental approach is the quantified reliability evaluation. In the past, equipment and hardware dependability was measured primarily by mechanical means. Learning from failure and experimentation is the key to developing a solid technology. It was utilised prior to the formalisation of data collecting and analysis procedures. Formal data collecting procedures were used to exercise the feedback principle throughout the design phase in order to improve inherent dependability. Reliability study is based on failure data. The failure rate was estimated by manipulating failure data. Quality control had the greatest statistical impact on dependability throughout the 1940s. The previous methodologies are no longer practicable in the face of more complex objects as equipment and systems get larger, more sophisticated, and more costly. Because of the exponential rise in the complexity and expense of sophisticated products like jet aeroplanes and nuclear power plants, learning from past failures is almost impossible.

The statistical and probability fields of mathematics are significantly reliant on each other when it comes to estimating a complicated system's dependability. Probability may be used to investigate complicated systems and events, even at a basic level. In order to answer questions like "What is the likelihood of that happening?" or "How much do we anticipate to earn if we make the decision?" the language of probability has been modified. After the Korean War, quantitative dependability began to be extensively employed, and statistical techniques were used to quantify it. The Weibull distribution was initially suggested by Weibull in 1951 and is now commonly known as the Weibull distribution. Aeronautical Radio, Inc. (ARINC) was established up by the airlines to collect and examine damaged tubes and return them to the tube maker to solve the issue of dependability. When it came to enhancing the dependability of various tube types, the ARINC team had great success. Since its inception in 1950, the ARINC programme has focused on military dependability issues.

Early failure, wear-out failure, and "chance failure" are three types of common failures. In order to deal with these failures, new mathematical procedures are required, and different methodologies must be employed for their estimations. The likelihood of a component wear-out failure occurring over any given time of operation may be predicted analytically based on its failure distribution, for example, where failures frequently cluster around the mean wear-out life of components. Conversely, the likelihood of an early or chance failure occurring throughout the course of an operating period may be determined analytically; these failures have a different distribution than wear-out failures and occur at random intervals.

In the era of mass manufacturing, the expense of ensuring product dependability is significant for producers. The goal is to strike a balance between dependability and advantages. This has resulted in an increased need for quantitative reliability-assessment methods. Modeling strategies for predicting the dependability of equipment and systems are developed by employing a standard component's recurring failure rate to compute and estimate its reliability. Data can now be sorted and analysed more quickly thanks to advances in computer technology.

Fault Tree Analysis (FTA), Fault Mode Effect and Catastrophic Analysis (FMECA), and redundancy system design and environmental screening/stress test methodologies are frequently used in the electric engineering domain. FORM and SORM have been developed in structural engineering during the last several decades, respectively. A major problem with traditional approaches is that they fail to characterise the nature of a micro-process breakdown. In addition, FMEA is a good technique for analysing system dependability since it is an intelligent response surface approach based on a simplified model. It is possible to examine and estimate a complex system's dependability and maintainability via the use of Monte Carlo simulation (MCS). System risk assessment utilises a variety of tools, including FTA. In a fault tree, the failure and error probabilities of fundamental events do not need to be determined with precision when utilising the FTA with fuzzy failure probability. When used in conjunction with probability theory, fuzzy theory may be an invaluable tool. The topic of reliability prediction, based on the idea of repeating component failure rates, has been contentious in recent years. The failure rates of complex goods or systems are not necessarily the consequence of component failures, which are often discovered under similar operating and environmental circumstances. Complex systems' dependability may be affected by a broad range of issues, from software components to human factors to operational manuals to constantly changing environmental conditions. Additionally, the models for system reliability and their interrelationships have become increasingly complex. Conventional reliability theory's assumptions are also its limitations. So, in order to enhance the standard dependability theory, fuzzy set theory and MCS techniques have been established.

IV. Reliability-Cantered Maintenance

With RCM, or reliability-cantered maintenance as it is more often known, companies may better control equipment failure risk by defining and implementing operational rules, routine maintenance procedures, and capital improvements.

4.1 Reliability theory

It is true that reliability theory focuses on the impact of mean time to repair on overall system failure rates in critical systems, but that approach fails to address an important performance criterion: operational failures, which are fundamentally different from unsafe failures in that they are the result of the system-level response to avoid unsafe failures, are what is required.

V. Conclusion

System dependability and availability are critical for industrial development since profit is directly related to the quantity of production, and the amount of output is in turn proportional to how reliable and available the system is. Therefore, good system design, optimization at the design stage, and system maintenance during a system's service life may all contribute to improving the dependability and availability of a system. Many researchers have explored the reliability and availability of a broad variety of systems and technologies, owing to their extensive use in power plants, manufacturing systems, and other industrial environments.

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