Study of cable connected satellites system under the influence of small eccentricity in elliptic orbit near the parametric resonance n=1

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ABSTRACT: *In this article, it is tried to study the linear oscillation of the system about the stable position where the system oscillates like a dumb-bell satellite with constant amplitude and phase varying with true anomaly. B.K.M. method has been exploited to get the general solution valid at and near the parametric resonance **n=1.** The equation of the relative motion of the system in the central gravitational field of the force is derived in polar form for the elliptical orbit of the centre of mass. It is obtained in an approximate form of non-linear oscillations.*

KEYWORDS: Rrelative motion, Dumb-bell satellites, Resonance, Oscillations, Perturbative Forces, Eccentricity, etc.

1. INTRODUCTION

The Russian mathematicians Beletsky; V.V. (1965) made significant studies exploring the effect of perturbative forces on cable-connected satellites system. Similar problems have been studied in details by Demine (1968) ; Singh; R.B.;Sinha S.K.; Das; S.K. Sharma; B. and Singh; C.P. Linear oscillation of the system about the stable position of equilibrium for small eccentricity (e) of the orbit at and nearer the parametric resonance n=1. Later on there appeared a lot of research works in which the effects of different perturbing forces on the system were studied in both linear and non-linear fields. We have, in this paper, studied the effects of dissipative and disturbing forces on the non-linear oscillation of the system. It is well known that in addition to the gravitational forces, the dissipative and disturbing forces are also present in nature. Though these forces are small, in comparison to the gravitational forces, it is expected that they may exert considerable effect on the oscillation of the system. These forces have been summed up as a dissipative force which arises due to friction of bodies in atmosphere, tidal forces and gravitational radiation etc. and a periodic force with slowly varying frequency caused by mulfipole moments, the gravitational waves at resonance frequency, etc. (Pyragas et al., 1978).

2. THE MATHEMATICAL DICUSSIONS

The equation of the motion of one of the two satellites system in the central gravitational field of oblate earth under the influence of air resistance, magnetic force and shadow of the earth due to solar pressure in polar form is given by,

$$(1+e\cos v)\psi''-2e\psi'\sin v+3\sin\psi\cos\psi$$
$$=2e\sin v+5A(1+e\cos v)^2\sin\psi\cos\psi+Bo\rho^3(\cos\alpha\sin\psi-\sin\alpha\cos\psi)$$
$$+f\rho^2(e\rho\sin\psi\sin v-\cos\psi)+\frac{c}{\rho}(\sin\psi-e\rho\sin v\cos\psi) \quad ..(1)$$

where, $A=-\frac{3k_2}{p^2}$ **= oblateness force parameter.**

$B_o=\frac{B\sin\theta}{\pi}$ **= Parameter due to shadow of the earth and solar pressure.**

c= magnetic force parameter.

$f = \frac{cup^3}{\sqrt{\mu p}}$ = **Air resistance force parameter**

$\rho = \frac{1}{1+e\cos v}$; **v= True anomaly of the orbit of centre of mass**

p = focal parameter

Here dashes denote differentiation with respect to true anomaly v. The stable equilibrium position is given by-

$$\varphi = \varphi_0 = 0 \text{ and } \psi = \psi_0 = \frac{-f - B_o \sin a}{3-5A-c-B_0\cos a} = d_0 \qquad ...(2)$$

The equation of small oscillation about the stable equilibrium position is obtained by putting $\psi = \psi_0 + \eta = d_o + \eta$ **from (1) assuming e to be small quantity of first order infinitesimal as:**

$$\eta'' + n^2\eta = e\begin{bmatrix} 2\sin v - \eta''\cos v + 2\eta'\sin v + 10A\{d_o + (1+d_0^2)\eta\}\cos v \\ +3B_0\{1-d_0\eta\}\sin a\cos v - 3B_0\{d_0+\eta\}\cos a\cos v \\ +2f\{1-d_0\eta\}\cos v - f\{d_0+\eta\}\sin v \\ +c(d_0+\eta)\cos v - c(1-d_0\eta)\sin v \end{bmatrix} \qquad ...(3)$$

Where, $n^2 = (3-5A)(1-d_0^2) - B_0\cos a - B_0 d_0 \sin a - f d_0 - c$ **...(4)**

3. THE SOLUTIONS OF OSCILLATORY SYSTEM

Now, let us construct the general solution of the oscillatory system based on B.K.M. method which will be valid at and near the parametric resonance n=1. Assuming e to be small parameter, the solution in the first approximation of equation (3) at the parametric resonance n=½ can be sought in the form :

$$\eta = a\cos k$$

$$\text{Where, } k = \frac{v}{2} + \theta \qquad ...(5)$$

The amplitude a and phase θ **present in (5) must satisfy the system of ordinary differential equations :**

$$\frac{da}{dv} = eA_1(a,\theta)$$

$$2\frac{d\theta}{dv} = (2n-1) + 2eB_1(a,\theta) \qquad ...(6)$$

where, $A_1(a,\theta)$ and $B_1(a,\theta)$ are periodic solutions periodic with respect to θ of the system of partial differential equations:

$$(2n-1)\frac{\partial A_1}{\partial \theta} - 4anB_1 = \frac{1}{\pi}\int_0^{2\pi} f_0(v,\eta,\eta',\eta'')\cos kdk$$

and

$$a(2n-1)\frac{\partial B_1}{\partial \theta} + 4nA_1 = -\frac{2}{\pi}\int_0^{2\pi} f_0(v,\eta,\eta',\eta'')\sin kdk \qquad \text{...(7)}$$

Where, $f_0(v,\eta,\eta',\eta'')$ is given by :

$$\begin{aligned} f_0(v,\eta,\eta',\eta'') = {} & (2-c-fd_0)\sin v - 2an\sin k\sin v + cd_0 a\cos k\sin v + an^2\cos v\cos k \\ & - fa\cos k\sin v + (10Ad_0 + 3B_0\sin\alpha - 3B_0 d_0\cos\alpha + f + cd_0)\cos v \\ & + \{10A(1+d_0^{\,2}) + c - 3B_0 d_0\sin\alpha - 3B_0\cos\alpha - 2fd_0\} a\cos k\cos v \end{aligned} \qquad \text{...(8}$$

Now substituting the value of $f_0(v,\eta,\eta',\eta'')$ from (8) on the R.H.S of (7) and then we get on integrating

$$(2n-1)\frac{\partial A_1}{\partial \theta} - 4anB_1 = \mu\cos 2\theta - v\sin 2\theta$$

$$and\; a(2n-1)\frac{\partial B_1}{\partial \theta} + 4nB_1 = -\mu\sin 2\theta - v\cos 2\theta \qquad \text{...(9)}$$

where,

$$\mu = [an^2 - 2an + 10A(1+d_0^{\,2})a + ca - 3B_o a d_o\sin\alpha - 3B_0 a\cos\alpha - 2fad_0]$$
$$\text{and } v = [cd_0 a - fa] \qquad \text{...(10)}$$

Now, the periodic solutions with respect to θ of the system of equation (9) can be easily obtained as –

$$A_1 = \frac{2}{2n+1}(-v\cos 2\theta - \mu\sin 2\theta)$$

$$B_1 = \frac{1}{a(2n+1)}(v\sin 2\theta - \mu\cos 2\theta) \qquad \text{...(11)}$$

Putting the values of A_1 and B_1 from (11) in (6), we obtain –

$$\frac{da}{dv} = \frac{-2e}{2n+1}(\mu\sin 2\theta + v\cos 2\theta)$$

$$and\; \frac{d\theta}{dv} = \left(n - \frac{1}{2}\right) - \frac{e}{a(2n+1)}(\mu\cos 2\theta - \mu\sin 2\theta) \qquad \text{... (12)}$$

The system of equation given by (12) can be written as –

$$\frac{da}{dv} = \frac{1}{a}\frac{\partial \varphi}{\partial \theta}$$

$$\frac{d\theta}{dv} = -\frac{1}{a}\frac{\partial \varphi}{\partial a} \qquad \text{...(13)}$$

where,

$$\varphi = \frac{ae}{(2n+1)}(\mu\cos 2\theta - v\sin 2\theta) - \frac{\left(n - \frac{1}{2}\right)a^2}{2} \qquad \text{...(14)}$$

Clearly, the system of equations given in (13) is in canonical form must have a first integral of the form:

$$\varphi = c_0' = \text{constan}$$

Which reduces the problem to quadrature? Here, Co' is the constant of integration.

But we are interested in qualitative study of the problem and hence we shall analyze the integral curves in the phase plane (a, θ). In order to plot the integral curves, let us put the equation (13) in the form:

$$\left(4n^2 - 1\right)a^2 - 4ae\left(\mu \cos 2\theta - v \sin 2\theta -\right) + co = 0 \quad \text{...(15)}$$

where, $co = 4(2n+1)c'o = \text{constan}$

The integral curves have been plotted in fig.1 and fig.2 for n=0.495, n=0505 respectively for different values of a, e, v, μ and c_0. Since curves drawn are closed, so we get the stability.

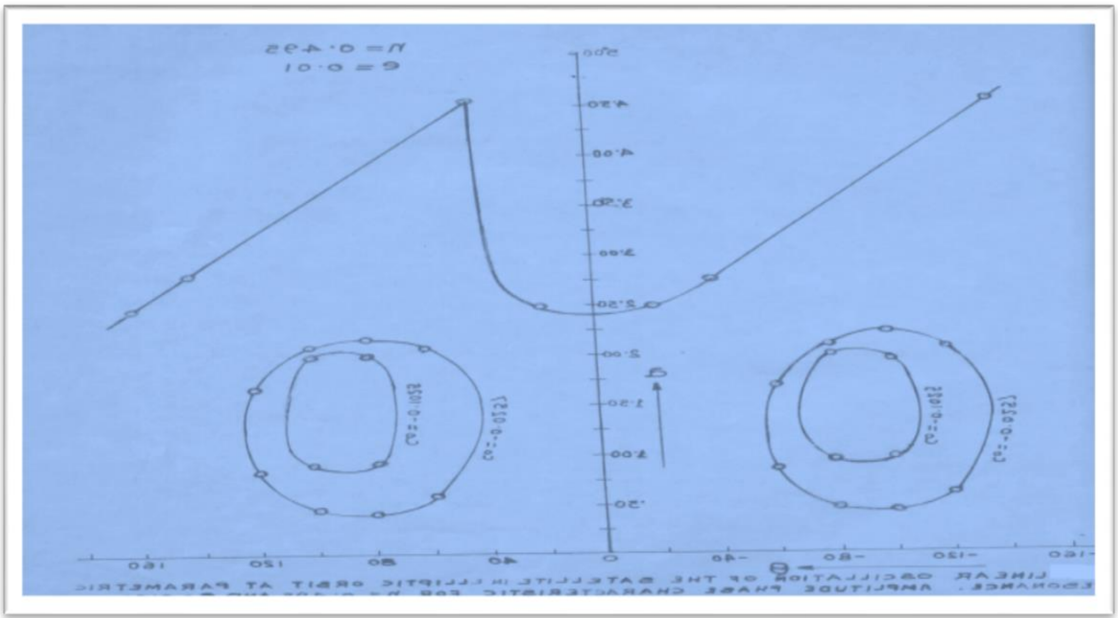

Figure 1: Resonance curve in case of parametric resonance at, $n = 1$

4. CONCLUSIONS

The present research work deals with polar form of equations of motion of the system the central gravitational field of force for the elliptical orbit of the centre of mass. In this article it is studied the motion of the system in the central gravitational field of force when the centre of mass of the system in moving along elliptical orbit. We found that the equation of dumb- bell satellite in the central gravitational field of force and is in a suitable form of a non-linear oscillator. This describes the non- linear oscillation of the system about its equilibrium position. This research work is in progress. It is hoped that the research may be very useful in the future applications in satellites.

ACKNOWLEDGMENT

One of the authors Dr. Uday Narayan Jha takes this opportunity to thank Prof. (Dr.) Byas Sharma (M.Sc., Ph.D), Retired Professor, University Department of Mathematics, B.R.A. Bihar University, Muzaffarpur, Bihar (INDIA), for numerous discussions, valuable suggestion and help in preparing the present research paper for publication. I also feel indebted to some scientific spiritualists who provided us the enthusiasm to undertake scientific studies with a wholesome attitude.

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