Generating Negative pedal curve through Inverse function – An Overview

***Ramesha. H.G. Asst Professor of Mathematics. Govt First Grade College, Tiptur.**

Abstract

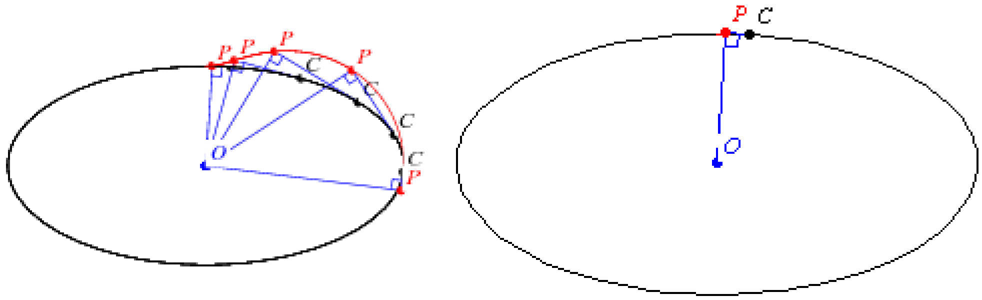
This paper attempts to study the **negative pedal** of a **curve** with fixed point O is therefore the envelope of the lines perpendicular at the point M to the lines. In inversive geometry, an inverse curve of a given curve C is the result of applying an inverse operation to C. Specifically, with respect to a fixed circle with center O and radius k the inverse of a point Q is the point P for which P lies on the ray OQ and OP·OQ = k2. The inverse of the curve C is then the locus of P as Q runs over C. The point O in this construction is called the center of inversion, the circle the circle of inversion, and k the radius of inversion.

An inversion applied twice is the identity transformation, so the inverse of an inverse curve with respect to the same circle is the original curve. Points on the circle of inversion are fixed by the inversion, so its inverse is itself. is a function that "reverses" another function: if the function f applied to an input x gives a result of y, then applying its inverse function g to y gives the result x, and vice versa, i.e., f(x) = y if and only if g(y) = x. The inverse function of f is also denoted. Joseph-Louis Lagrange.The Lagrange inversion theorem (or Lagrange inversion formula, which we abbreviate as LIT), also known as the Lagrange--Bürmann formula, gives the Taylor series expansion of the inverse function of an analytic function. The theorem was proved by Joseph-Louis Lagrange (1736--1813) and generalized by the German mathematician and teacher Hans Heinrich Bürmann (--1817), both in the late 18th century. The Lagrange inversion formula is one of the fundamental formulas of combinatorics. In its simplest form it gives a formula for the power series coefficients of the solution *f(x)* of the function equation f(x)=xG(f(x)) f(x)=xG(f(x)) in terms of coefficients of powers of *G*.Not all functions have inverse functions. Those that do are called invertible. For a function f: X → Y to have an inverse, it must have the property that for every y in Y, there is exactly one x in X such that f(x) = y. This property ensures that a function g: Y → X exists with the necessary relationship with f. Using Riemann-Liouville fractional differential operator, a fractional extension of the Lagrange inversion theorem and related formulas are developed. The required basic definitions, lemmas, and theorems in the fractional calculus are presented.

Key words: differential operator, Lagrange inversion theorem, pedal, curve.

Introduction

A fractional form of Lagrange's expansion for one implicitly defined independent variable is obtained. Then, a fractional version of Lagrange's expansion in more than one unknown function is generalized.



The pedal of a curve C with respect to a point O is the locus of the foot of the perpendicular from O to the tangent to the curve. More precisely, given a curve C, the pedal curve P of C with respect to a fixed point O (called the pedal point) is the locus of the point P of intersection of the perpendicular from O to a tangent to C. The parametric equations for a curve $(f(t), g(t))$ relative to the pedal point (x_0, y_0) are given by

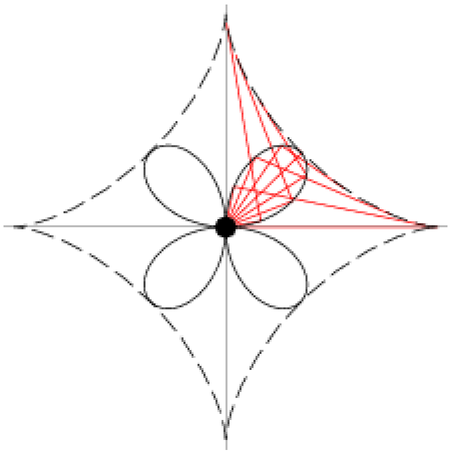
$$x_p = \frac{x_0 f'^2 + f g'^2 + (y_0 - g) f' g'}{f'^2 + g'^2} \quad (1)$$

$$y_p = \frac{y_0 g'^2 + g f'^2 + (x_0 - f) f' g'}{f'^2 + g'^2}. \quad (2)$$

If a curve P is the pedal curve of a curve C, then C is the negative pedal curve of P (Lawrence 1972, pp. 47-48).

When a closed curve rolls on a straight line, the area between the line and roulette after a complete revolution by any point on the curve is twice the area of the pedal curve (taken with respect to the generating point) of the rolling curve. As an example, consider the real-valued function of a real variable given by f(x) = 5x − 7. Thinking of this as a step-by-step procedure (namely, take a number x, multiply it by 5, then subtract 7 from the result), to reverse this and get x back from some output value, say y, we would undo each step in reverse order. In this case, it means to add 7 to y, and then divide the result by 5. In functional notation, this inverse function would be given by,

{\displaystyle g(y)={\frac {y+7}{5}}.}With y = 5x − 7 we have that f(x) = y and g(y) = x.



The pedal curve of an astroid

$$x = a\cos^3 t \quad (1)$$

$$y = a\sin^3 t \quad (2)$$

with pedal point at the center is the quadrifolium

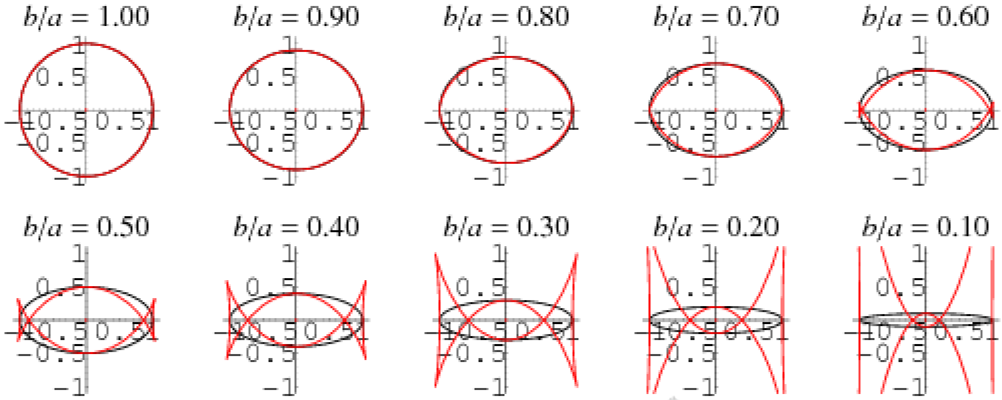
$$x_p = a\cos t\sin^2 t \quad (3)$$

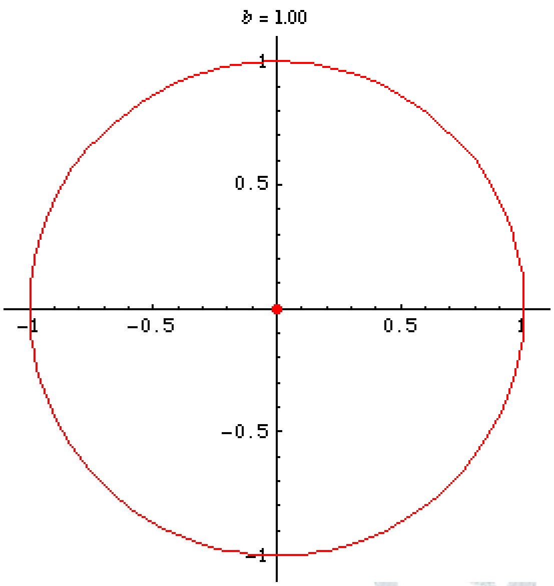
$$y_p = a\cos^2 t\sin t. \quad (4)$$

Objective:

This paper intends to explore **pedal** and **negative pedal** as inverse concepts. Also, to find **Negative pedal** of a **curve** C that can be defined as a **curve** C' such that the **pedal** of C is C'. Stating **negative pedal curve** is a plane curve that can be constructed from another plane curve C and a fixed point P on that curve.

An ellipse with parametric equations





For an ellipse with parametric equations

$$x = a\cos t \qquad (1)$$

$$y = b\sin t, \qquad (2)$$

the negative pedal curve with respect to the origin has parametric equations

$$x_n = a\cos^3 t + \frac{(2a^2 - b^2)\cos t\sin^2 t}{a} \qquad (3)$$

$$= \frac{(a^2 + c^2\sin^2 t)\cos t}{a} \qquad (4)$$

$$= a\cos t\left(1 + e^2\sin^2 t\right) \qquad (5)$$

$$y_n = b\sin^3 t + \frac{(2b^2 - a^2)\sin t\cos^2 t}{b} \qquad (6)$$

$$= \frac{\left(a^2 - 2\,c^2 + c^2 \sin^2 t\right) \sin t}{b} \tag{7}$$

$$= \frac{a \sin t \left(1 - 2\,e^2 + e^2 \sin^2 t\right)}{\sqrt{1 - e^2}}, \tag{8}$$

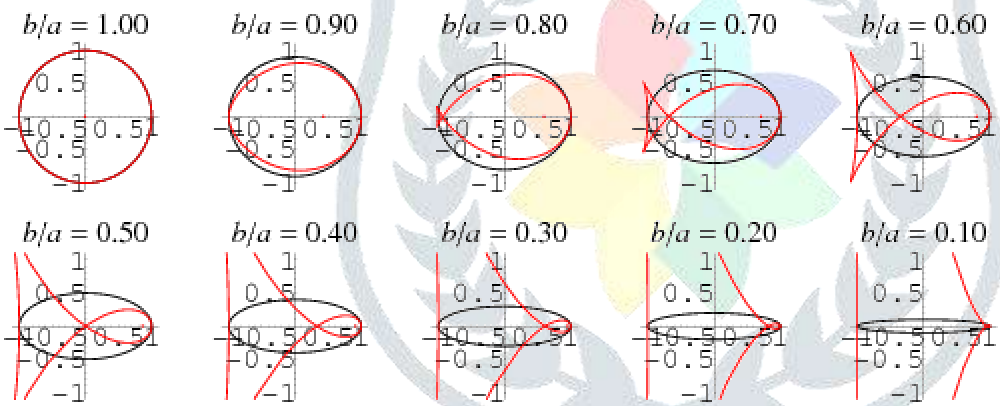
where

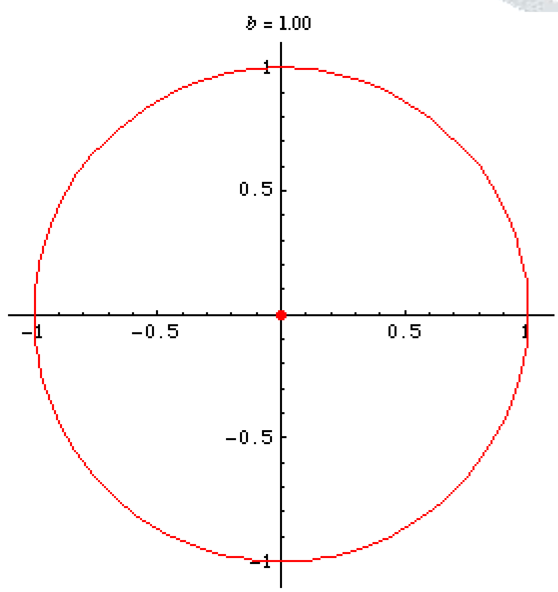
$$c \equiv \sqrt{a^2 - b^2} \tag{9}$$

is the distance between the ellipse center and one of its foci and

$$e \equiv \sqrt{1 - \frac{b^2}{a^2}} = \frac{c}{a} \tag{10}$$

is the eccentricity. For $b/a = 1$, the base curve is a circle, whose negative pedal curve with respect to the origin is also a circle. For $\sqrt{2}/2 < b/a < 1$, the curve becomes a "squashed" ellipse. For $0 < b/a < \sqrt{2}/2$, the curve has four cusps and two ordinary double points and is known as Talbot's curve (Lockwood 1967, p. 157).





Taking the pedal point at a focus (i.e., $(x, y) = (c, 0)$) gives the negative pedal curve

$$x_n = a\cos t - c\sin^2 t \quad (11)$$

$$y_n = \frac{\left(a^2 - 2c^2 + a\,c\cos t\right)\sin t}{b}. \quad (12)$$

Lockwood (1957) terms this family of curves Burleigh's ovals. As a function of the aspect ratio b/a of an ellipse, the neagtive pedal curve varies in shape from a circle (at $b/a = 1$) to an ovoid (for $\sqrt{2}/2 \le b/a < 1$) to a fish-shaped curve with a node and two cusps to a line plus a loop to a line plus a cusp.

The special case of the negative pedal curve for pedal point $(x, y) = (c, 0)$ and $e^2 = 1/2$ (i.e., $b/a = \sqrt{2}/2$) is here dubbed the fish curve.

Given a curve C and $O = (x_0, y_0)$ a fixed point called the pedal point, then for a point P on C, draw a line perpendicular to OP. The envelope of these lines as P describes the curve C is the negative pedal of C. It can be constructed by considering the perpendicular line segment $((x_1, y_1), (x_2, y_2))$ for a curve C parameterized by (f, g). Since one end of the perpendicular corresponds to the point P, $(x_1, y_1) = (f, g)$.

The equations of the negative pedal curve

Another end point can be found by taking the perpendicular to the OP line, giving

$$(x_2, y_2) = (f, g) + (-(g - y_0), f - x_0), \quad (1)$$

or

$$x_2 = y_0 + f - g \quad (2)$$

$$y_2 = -x_0 + f + g. \quad (3)$$

Plugging into the two-point form of a line then gives

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1), \quad (4)$$

or

$$U(x, y, t) = y - \frac{(x - f)(f - x_0)}{y_0 - g} - g = 0. \quad (5)$$

Solving the simultaneous equations $U(x, y, t) = 0$ and $\partial U/\partial t = 0$ then gives the equations of the negative pedal curve as

$$x_n = -\frac{\left[(g-y_0)^2-(f-x_0)\,f\right]g'+(2\,f-x_0)\,(g-y_0)\,f'}{(f-x_0)\,g'-(g-y_0)\,f'} \quad (6)$$

$$y_n = \frac{\left[(f-x_0)^2-(g-y_0)\,g\right]f'+(2\,g-y_0)\,(f-x_0)\,g'}{(f-x_0)\,g'-(g-y_0)\,f'}. \quad (7)$$

If a curve P is the pedal curve of a curve C, then C is the negative pedal curve of P (Lawrence 1972, pp. 47-48).

The following table summarizes the negative pedal curves for some common curves.

Curve	pedal point	negative pedal curve
cardioid negative pedal curve	origin	circle
cardioid negative pedal curve	point opposite cusp	cissoid of Diocles
circle negative pedal curve	inside the circle	ellipse
circle negative pedal curve	outside the circle	hyperbola
ellipse negative pedal curve with $e > 1/2$	center	Talbot's curve
ellipse negative pedal curve with $e \leq 1/2$	focus	ovoid
ellipse negative pedal curve with $e > 1/2$	focus	two-cusped curve
line	any point	parabola
parabola negative pedal curve	origin	semicubical parabola
parabola negative pedal curve	focus	Tschirnhausen cubic

Conclusion

A surface has **negative curvature** at a point if the surface curves away from the tangent plane in two different directions. The classic example is a saddle, which can be found on your body in the space between your thumb and forefinger, or along the inside of your neck. The concept of a surface of negative curvature can be generalized, for example, with respect to the dimension of the surface itself or the dimension and structure of the ambient space.

Surfaces of negative curvature locally have a saddle-like structure. This means that in a sufficiently small neighbourhood of any of its points, a surface of negative curvature resembles a saddle , not considering the behaviour of the surface outside the part of it that has been drawn). The local saddle-like character of the surface is clearly illustrated in this figure, which shows the principal sections of the surface at an arbitrary point OO. Let 1/R11/R1, 1/R21/R2 be their normal curvatures, i.e. the principal curvatures at the point OO(cf. Principal curvature). According to the classical definition, the Gaussian curvature at OO is the number K=1/R1R2K=1/R1R2. Since K<0K<0, the principal curvatures have different signs, for which reason the principal sections are convex in opposite directions; in Fig.1bthe section 0202 is convex in the direction of the normal nn, while the other is convex in the opposite direction, which fits in with the saddle-like character of the surface. The topological structure in the large ("globally") of a surface of negative curvature can be very different.

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