CONVECTIVE FLOWS DUE TO OSCILLATORY EFFECTS, RESULT AND DISCUSSION

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ABSTRACT

Here the main aim is to study only those cases where either the plate is oscillatory or the suction is oscillatory besides oscillatory temperature distribution. Singh [79] used the oscillatory boundary conditions (BCs) of the flow pattern between two parallel plates at a distance h.

Key word :- Oscillatory, convective, temperature

Mathematical Analysis

(3.1.1) $$u = u_1 e^{i\omega t}, T = T_s \ at\ y = 0,$$

$$u = U_0, T = T_h\ at\ y = h$$

Agarwal & Singh [4] & [5] took the temperature distribution oscillatory about non-zero mean.

(3.1.2) z'=0, u'=0, $$\omega' = 0,$$

$$T' = T'\omega' + t(T'\omega' - T'\alpha)e^{i\omega' t'}$$

$$c' = c'\omega + c(c\omega' c'\alpha)e^{i\omega' t'}$$

(3.1.3) $$y' \to \alpha, u' = u't' = u_0(1 + te^{i\omega' t'})$$

$$\omega' = 0, T' \to T'\alpha, c' \to c'_\alpha$$

Jaiswal & Soundelgaker [12] & [25] the oscillatory parameter were used as follows :

(4.1.4) $$(a)\ \ u=0, \qquad \theta=t, c=1 \quad as \quad y=0,$$

$$(a)\ \ u=ut=1+ee^{i\omega t}, \theta=0, c=0 \quad as \quad y\rightarrow\alpha,$$

$$(c)\ \ u=u(t+\varepsilon e^{i\omega t}u_1,$$

$$(d)\ \ \theta=\theta_0+\varepsilon\, e^{i\omega t}\theta_1,$$

$$(e)\ \ c=c_0+\varepsilon\, e^{i\omega t}c_1$$

soundelgakar used the following equations :-

(3.1.5) $$(a)\, v'_{t'}+v'_{uy'}=u't'+g_{\beta}(T'-T\mu)+\upsilon u'_{y'y'}$$

$$(b)\, \rho' Cp(T'_{t'}+v'U'_{y'})=k(T'_{y'y'})+\mu(u'_{y'})^{2,}$$

$$(c)\, v'_{y'}=0,$$

$$(d)\, v'=v_0'(1+\varepsilon\, e^{i\omega t}).$$

Reducing by non-dimensional variable be obtained.

(3.1.6) $$(a) 1/4U_t+(1+e\,Ae^{i\omega t})=1/4u_t+G_{\theta}+u_{yy}$$

$$(b)\, P/4\theta_t+(1+e\,Ae^{i\omega t})\theta_y=Q_{yy}+P_E q^2{}_y$$

$$(c)\, u=0, \theta=1\ as\ \ y=0$$

$$(d)\, u=u(t)=1+e^{i\omega t}, \theta=0\ as\ \ y\rightarrow\alpha$$

Singh [79] studied M.H.D. problem by using Laplace technique, Birajdar [8] used the following equations & obtained some graphs.

(3.1.7) $$(a)\, ut = G\theta + uyy - M$$

$$(b)\, P_r \theta_t = \theta_{yy}$$

$$u = 0, \theta = 0 \; for \;\; all \;\; y$$

(3.1.8) $$(a)\, t \le 0, u = 1, \theta = Cosnt \; at \; y = 0$$

$$(b)\, t \ge 0, u = 0, \theta = 0 \quad y \to \alpha$$

Lal [41] cosidered Boundary layer equation for the Laminar flow past a porous vertical wall for free convection, when suction velocity is an oscillatory function.

(3.2) OSCILLATORY FLOW PROBLEM

The combined buoyancy of Thermal & mass magnetic flow confined between two parallel plates moving in opposite directions while one plate is oscillatory about a constant mean and the temperature at the plates change accordingly to the law $T = Ts' + (T\varpi' - Ts')\,(1 - \varepsilon e^{i\omega t})$ has been examined. A sort of solution has been developed and analysed in details. We have considered the two dimensional unsteady flow of an incompressible viscous fluid between two parallel plates moving in opposite directions. Let x' axis be chosen along an infinite flat plate moving vertically upwards and y' axis also moving. B_0 is the constant magnetic field in a transverse direction so that the induced magnetic field is negligible. The governing equations are as follows [87].

(3.2.1) $$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y} = g\beta(T'-T'_0) +$$

$$g\beta * (c'-cs') + v\frac{\partial 2u'}{\partial y_1^2} - \sigma\frac{B_0^2 u'}{\rho'}$$

(3.2.2) $$\frac{\partial v'}{\partial t'} = -\frac{\partial p'}{\partial y'}$$

(3.2.3) $$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K'}{\rho' C_p'}\frac{\partial^2 T'}{\partial y'^2} + \frac{v}{C_{\rho'}}\left(\frac{\partial u'}{\partial y'}\right)^2$$

(3.2.4) $$\frac{\partial c'}{\partial t'} + V'\frac{\partial c'}{\partial y'} = D\frac{\partial^2 c'}{\partial y'^2}$$

(3.2.5) $$\frac{\partial v'}{\partial y'} = 0$$

Intergrating (3.2.5) $V' = V'_0(1 + e^{i\omega' t'})$.The BCs are given as follows :

(3.2.6) $$u' = U'_0, v' = v'_0(1 + \varepsilon e^{i\omega' t'}), T' = T'_s,$$

$$C' = C'_s \quad at\ y' = 0$$

(3.2.7) $$u' = -U_{(t)} = +u_0(e^{Mm} + \varepsilon e^{i\omega' t'}),$$

$$T' = T'_s + (T'_\omega - T'_s)(1 + e^{i\omega' t'})$$

$$C' = C'_s + (C_{\omega'} - C'_s)(1 + \varepsilon e^{i\omega' t'}),$$

$$v' = v'_0(1 + \varepsilon e^{i\omega' t'}) \quad at\ y' = d$$

Where ρ' is the density, v' is the kinematic viscosity, σ is the electrical conductivity, β^* the coefficient of volume expansion, β^{**} coefficient of thermal expansion with concentration, c' the specific heat of liquid, B_0 is the magnetic field component.

Substituting non-dimensional variables :-

(3.2.8) $$\eta = y'\frac{v_0}{v}, m = \frac{dv_0}{v}, t = \frac{v_0^2 t'}{4v},$$

$$\omega = \frac{4v\omega'}{v_0^2}, u = \frac{u'}{U_0'} v = \frac{v'}{v_0}, \theta = \frac{T' - T'_s}{T'\omega - T'_s},$$

$$\theta^* = \frac{C' - C'_s}{C'_\omega - C'_s}, P_r = \mu\frac{C_{p'}}{K'}, S_c = \frac{v}{D},$$

$$E = \frac{v_0^2}{C'_p (T'_\omega - T'_s)'} G_r = \frac{vg\beta^* (T'_\omega - T'_0)}{}$$

$$G_c = \frac{vg\beta^{**} (c'_\omega - C'_0)}{C'_p (T'_\omega - T'_s)} \text{and } M = \frac{\sigma\beta_0^2 v}{\rho' v_0^2}$$

Making use of (3.2.1), (3.2.7) and (3.2.8), also assuming that $\frac{\partial p'}{\partial y'}$ is small in Boundary layer, we shall get.

(3.2.9) $$(a)\ \ \eta = 0, \quad u = 1, \quad \theta^* = 1, \quad \theta = 0$$

$$(b)\ \ h = m, \quad u = (e^{Mm} + \varepsilon e^{i\omega t})$$

$$\theta = -(1 + \varepsilon e^{i\omega t}) \text{ and } \theta^* = -(1 + \varepsilon e^{i\omega t})$$

(3.2.10) $$(a)\frac{1}{4}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial \eta}(1+\varepsilon A e^{i\omega t})=$$

$$Gr\theta+Gc\theta^{*}+\frac{\partial^{2}u}{\partial \eta^{2}}-M^{2}u,$$

$$(b)\ \frac{P_r}{u}\frac{\partial \theta}{\partial t}+P_r\frac{\partial \theta}{\partial \eta}(1+\varepsilon A e^{i\omega t})=$$

$$\frac{\partial^{2}\theta}{\partial \eta^{2}}+P_rE\left(\frac{\partial u}{\partial \eta}\right)^{2}$$

$$(c)\ \frac{S_c}{u}\frac{\partial \theta^{*}}{\partial t}+S_c(1+\varepsilon A e^{i\omega t}).$$

$$\frac{\partial \theta^{*}}{\partial \eta}=\frac{\partial^{2}\theta^{*}}{\partial \eta^{2}}$$

In order to solve above equations for velocity, temperature, concentration distribution in the neighbourhood of plate, we assume

(3.2.11) $$(a)u(\eta,t)=u_0(\eta)+\varepsilon\, e^{i\omega t}u_1(\eta)($$

$$(b)\theta(\eta,t)=\theta_0(\eta)+\varepsilon\theta_1^{*}(\eta)e^{i\omega t}$$

$$(c)\theta^{*}(\eta,t)=\theta_0^{*}(\eta)+\varepsilon\theta_1^{*}(\eta)e^{i\omega t}$$

Substituting (3.2.11) in (3.2.9) and (3.2.10) and neglecting terms of e^2 and higher powers, we obtain

(3.2.12) $$(a)\frac{d^{2}u_0}{d\eta^{2}}-\frac{du_0}{d\eta}-M^{2}U_0=$$

$$-G_r\theta_0 - G_c\theta_0^*$$

$$(b)\frac{d^2u_1}{d\eta^2} - A\frac{du_1}{d\eta} - \frac{1\omega}{4}u_1 - M^2u_1 =$$

$$-G_r\theta_1 - G_c\theta_1^* + A\frac{du_0}{d\eta}$$

$$(c)\frac{d^2\theta_0}{d\eta^2} - P_r\frac{d\theta_0}{d\eta} = -P_rE\left(\frac{du_0}{d\eta}\right)^2$$

$$(d)\frac{d^2\theta_1}{d\eta^2} - P_r\frac{d\theta_1}{d\eta} - \frac{i\omega}{4}P_r\theta - P_r\frac{d\theta_0}{d\eta}$$

$$-2P_rE\frac{du_0\,du_1}{d\eta\,d\eta}$$

$$(e)\frac{d^2\theta_0^*}{d\eta^2} - S_c\frac{d\theta_0^*}{d\eta} = 0$$

$$(f)\frac{d^2\theta_1^*}{d\eta^2} - S_c\frac{d\theta_1^*}{d\eta} - \frac{S_c i\omega\theta_1^*}{4} = S_c\frac{d\theta_0^*}{d\eta}$$

(3.2.13) $$(a)\ \ \eta = 0,\ u_0 = 1\ u_1 = 0,$$

$$\theta_0 = 0,\quad \theta_1 = 0,\quad \theta^*{}_0 = 1,\quad \theta^*{}_1 = 0,$$

$$(b)\eta = m,\qquad u_0 = +e^{Mm}, u_1 = -1,.$$

$$\theta_0 = -1,\ \theta_1 = -1,$$

$$\theta^{*}{}_{0} = -1, \theta^{*}{}_{1} = -1,$$

(3.2.12) on account of (3.2.13) yields

(3.2.14) $$\theta_0^* = \frac{1+e^{Scm} - 2e^{Sc\eta}}{e^{Scm} - 1}$$

(3.2.15) $$\theta_0^* = A_1 e^{\alpha 1\eta} + A_2 e^{\alpha 2\eta} + A_3 e^{\alpha 3\eta}$$

Where $\alpha_1 = \dfrac{S_c + \sqrt{S_c^2 + i\omega S_c}}{2}$

$$\alpha_2 = \frac{S_c - \sqrt{S_c^2 + i\omega S_c}}{2}$$

$$A_2 = \frac{e^{Scm}(8AS_c + i\omega) - i\omega - 8S_c A e^{\alpha 2m}}{i\omega(e^{Scm} - 1)(e^{\alpha 2m} - e^{\alpha 1m})}$$

$$A_2 = \frac{8ASc^{\alpha 1m} - e^{Scm}(8AS_c + iu) + i\omega}{i\omega(e^{Scm} - 1)(e^{\alpha 2m} - e^{\alpha 1m})}$$

$$A_3 = \frac{8S_c A}{i\omega(e^{Scm} - 1)}$$

Now from (3.2.12) (a), (f) and (3.2.14), we get :

(3.2.16) $$\frac{d^2 u_0}{d\eta^2} - \frac{du_0}{d\eta} - M^2 u_0 =$$

$$-G_r\theta_0 - G_c \frac{1+e^{Scm} - 2e^{2Sc\eta}}{e^{Scm} - 1}$$

(3.2.16) and (3.2.12) (a) is reduced to

(3.2.17) $$\theta_0 = -\frac{1}{G_r}(G_c\theta_0^* + \frac{d^2u_0}{d\eta^2} - \frac{du_0}{d\eta} - M^2u_0)$$

Thus finally, we get from (3.2.17) and (3.2.12) (c) :

(3.2.18) $$\frac{d^4u_0}{d\eta^4} - (1+P_r)\frac{d^3u_0}{d\eta^3} + P_r(-M^2)\frac{d^2u_0}{d\eta^2} + P_rM^2\frac{du_0}{d\eta} - P_rEG_r\left(\frac{du_0}{d\eta}\right)^2 = \frac{(Gc2S_c^2 + 2G_cP_rS_c)e^{Sc\eta}}{e^Sc^m - 1}$$

We shall deal a particular case when S_c has numberical value equal to M and also we take :

(3.2.19) $$\frac{(-M^3 + P_rM^2 - P_rEG_rM^2)(e^{Mm} - 1)}{(G_r2M^2 + 2P_rG_c} = 1$$

Therefore, we obtain

$$\theta_0 = \frac{e^{2M\eta}P_rE}{P_r2M - 4M^2} + \frac{e^{\Pr\eta}(P_rM - M^2) - P_rE + P_rEe^{2Mm}}{(P_rM - M^2)(1 - e^{\Pr M})} +$$

where

$$A_4 = \frac{P_r E}{P_r 2M - 4M^2}$$

$$A_5 = \frac{(P_r M - M^2 - P_r E + P_r E e^{2Mm})}{(P_r M - M^2)(1 - e^{\Pr M})}$$

$$A_6 = \frac{P_r E e^{\Pr M} - P_r M + M^2 + P_r e^{2Mm} E}{(P_r M - M^2)(1 - e^{\Pr M})}$$

(3.2.22) $u_0 = e^{M\eta}$

Substituting (3.2.15), (3.2.20) and (3.2.21), we obtain :

(3.2.22) $$\frac{d^2 u_1}{d\eta^2} - A\frac{du_1}{d\eta} - \frac{i\omega u_1}{4} - M^2 u_1 = G_r \theta_1 -$$

$$G_c (A_1 e^{\alpha 1\eta} + A_2 e^{\alpha 2\eta} +$$

$$A_3 e^{Sc\eta}) + AMe^{\eta M}$$

and

(3.2.23) $$\frac{d^2\theta_1}{d\eta^2} - P_r \frac{d\theta_1}{d\eta} - i\omega P_r \theta_1$$

$$\Pr(A_4 M e^{2M\eta} + A_5 P_r e^{\Pr \eta})$$

$$-2P_r E M e^{M\eta} \frac{du_1}{d\eta}$$

Let us take

(3.2.24) $$u_1 = \frac{1-e^{M\eta}}{e^{Mm}-1}$$

From (3.2.19) and (3.2.23) after using boundary conditions, we get

(3.2.25) $$\theta_1 = A_6 e^{2M\eta} - A_7 e^{\Pr\eta} + A_8 e^{\alpha 3\eta} + A_9 e^{\alpha 4\eta}$$

Where $$\alpha_3 = \frac{P_r + \sqrt{P_r^2 + i\varpi P_r}}{2}$$

and $$\alpha_4 = \frac{P_r - \sqrt{P_r^2 + i\omega P_r}}{2}$$

(3.2.26) $$A_6 = \frac{2P_r M A_4 (e^{Mm}-1) + 2P_r E M^2}{(e^{Mm}-1)(4M^2 - 2P_r M - i\frac{\omega}{4}}$$

$$A_7 = \frac{4}{i\omega} A_5 P_r^2$$

$$A_8 = \frac{A_6 e^{2Mm} - A_7 e^{\rho rm} + A_7 e^{\alpha 4m} + A_6 e^{\alpha 2m}}{e^{\alpha 4m} - e^{\alpha 3m}}$$

$$A_9 = \frac{e^{\alpha 3m}(A_6 - A_7) - A_6 e^{2Mm} + A_7 e^{\Pr m} - 1}{e^{\alpha 4m} - e^{\alpha 3m}}$$

Provided

$$A=\frac{\frac{iw}{4}(1-e^{M\eta})+M^2+(e^{Mm}-1)}{(1-Me^{M\eta})}\times\begin{bmatrix}-G_rA_6e^{2M\eta}+G_rA_7e^{\Pr\eta}\\ -A_8G_re^{\alpha 3\eta}\\ -G_rA_9e^{\alpha 4\eta}\\ -G_cA_1e^{\alpha 1\eta}\\ -G_cA_2e^{\alpha 2\eta}\\ -A_3e^{M\eta}\end{bmatrix}$$

Substituting (3.2.14), (3.2.15), (3.2.20), (3.2.21), (3.2.24), (3.2.25) in (3.2.11) yields.

(3.2.27) $(a)\quad u=e^{M\eta}+\dfrac{\varepsilon e^{i\varpi t}(1-e^{M\eta})}{e^{Mm}-1}$

$$(b)\quad \theta=A_4e^{2M\eta}+A_5e^{\Pr\eta}+A_6+\varepsilon e^{i\omega t}$$

$$X(A_6e^{2M\eta}-A_7e^{\rho r\eta}+$$

$$A_8e^{\alpha 3\eta}+A_9e^{\alpha 4\eta})$$

$$(c)\quad \theta^*=\frac{1+e^{Scm}-2e^{Sc\eta}}{e^{Scm}-1}+\varepsilon e^{i\omega t}.$$

$$(A_1e^{\alpha 1\eta}+A_2e^{\alpha 2\eta}+A_3e^{Sc.\eta})$$

Sking friction in non dimensional from is given by -

(3.2.28) $Q=-\left(\dfrac{\partial u}{\partial\eta}\right)_{\eta=0}=M\left(1+\dfrac{\varepsilon e^{i\omega t})}{e^{Mm}-1}\right)$

or

$$(b)\, Q = M\left(1+\frac{\varepsilon Cos\omega t)}{e^{Mm}-1}\right)+\frac{Mi\varepsilon Sin\omega t}{e^{Mm}-1},$$

or

$$(c) \quad Q = M + \frac{M\varepsilon\sqrt{2}}{e^{Mm}-1}(Cos(\omega t - 45))$$

or

$$(d) \quad Q = \tau_m + BCos(\omega t + \omega_1)$$

where τ is the mean skin friction equal to M.

and

$$B = \frac{M\varepsilon\sqrt{2}}{e^{mM}-1}$$

is amplitude of skin friction.

The phase angle ω_1 of sking friction = 0

The rate of heat transfer of the plate is

$$Q = -\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} = -(2MA_4 +$$

$$A_5P_r + \varepsilon e^{i\omega t}A_{64}2M - A_7P_r + A_8\alpha_3 + \alpha_4A_9)$$

$$Q = [P_r(A_7 - P_r) -$$

$$2M(A_6\varepsilon e^{i\omega t} +$$

$$A_4 - A_8\alpha_4A_9)]$$

3.3 CONCLUSION

Thus we have seen that the solution of the equation of motion originated is soluable. We have obtained value of rate of heat transfer of the plate as well as skin friction parameters. If we neglect the oscillatory terms then the above problem gives results similar to Soundelgaker [87].

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