TWO UNIT COLD STANDBY SYSTEM WITH TWOTYPES OF FAILURE AND REPLACEMENT

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ABSTRACT

The Present paper deals with the analysis of two-unit cold standby system with two types of failure. For making the system more effective, it is considered that the unit in the system which is under operationfails with two types say type-Iand type-II. If the unit fails due to type-I then the failed unit goes for repair and if it fails due to type-II then it goes for replacement by new one. Failure time distributions of type-I and type-II areexponential. The repair and replacement time distributions are general. By using regenerative points technique with Markov Renewal Process, the various measures such as MTSF, Availability analysis, Busy period analysis, expected number of visits by the repairman and cost benefit analysis of the system effectiveness are obtained.

Key words: Mean Time to System Failure, Availability, Cost benefit analysis, Regenerative point.

INTRODUCTION

Several authors [2,3,8,10 & 11] working in the felid of reliability have analyzed many engineering systems with the assumption that the failed unit caused due to any type of failure sent for repair without observing the type of failure. But in the real practical situation there exist some systems in which the repair of the failed unit is not possible always and, in such circumstances, to replace the failed unit is the only remedy. Keeping the above view in mind we in this present study consider atwo-unit engineering system in which the unit under operation fails with two types say type-I and type-II. If the unit fails due to type-I then the failed unit goes for repair and if due to type-II then goes for replacement by new one. By using regenerative point technique, the following reliability measures are to be obtained.

(i) Steady state transition probabilities.

(ii) Mean sojourn time.

(iii) Mean time to system failure.

(iv) Point wise and steady state availability of the system.

(v) Expected busy period of the repairman in the time interval (o,t].

(vi) Expected number of the visits by the repairman in (o,t].

(vii) Cost benefit analysis of the system.

MODEL DESCRIPTION AND ASSUMPTIONS

(i) The system consists of two identical units. Initially one unit is operative and other kept as cold standby.

(ii) Upon failure of an operative unit due to type-I, it is sent for repair.

(iii) Upon failure of an operative unit due to type-II, it is sent for replacement.

(iv) The probabilities of type-I and type-II failure are fixed.

(v) Failure rate of the operative unit is constant.

(vi) The repair and replacement time distributions for failed unit are arbitrary.

(vii) Single repair facility is used for repair and replacement.

NOTATIONS AND STATES

α	Constant failure rate of operative unit.
f(.), F(.)	P.d.f. and c.d.f. of the repair time distribution of the failed unit caused by the type-I failure.
G(.), G(.)	P.d. f an c.d.f. of the time to complete replacement caused by the type-II failure.

a_1	Probability of committing first type of failure,
$a_2=(1-a_1)$	Probability of committing second type of failure.
N_0	Normal unit kept as operative
N_S	Normal unit kept as cold standby.
F_{r_1}	Failed unit caused by first type of failure is under repair.
F_{r_2}	Failed unit caused by second type of failure is under replacement.
F_{wr_1}	Failed unit caused by first type of failure is waiting for repair.
F_{wr_2}	Failed unit caused by second type of failure is waiting for replacement.
F_{R_1}	Repair of failed unit caused by first type of failure is continued from earlier state.
F_{R_2}	Replacement of failed unit caused by second type failure is continued from earlier state.
m_1, m_2	Mean time for repair and replacement,

Using the above notations and assumptions, the possible states of the system are:

Up states

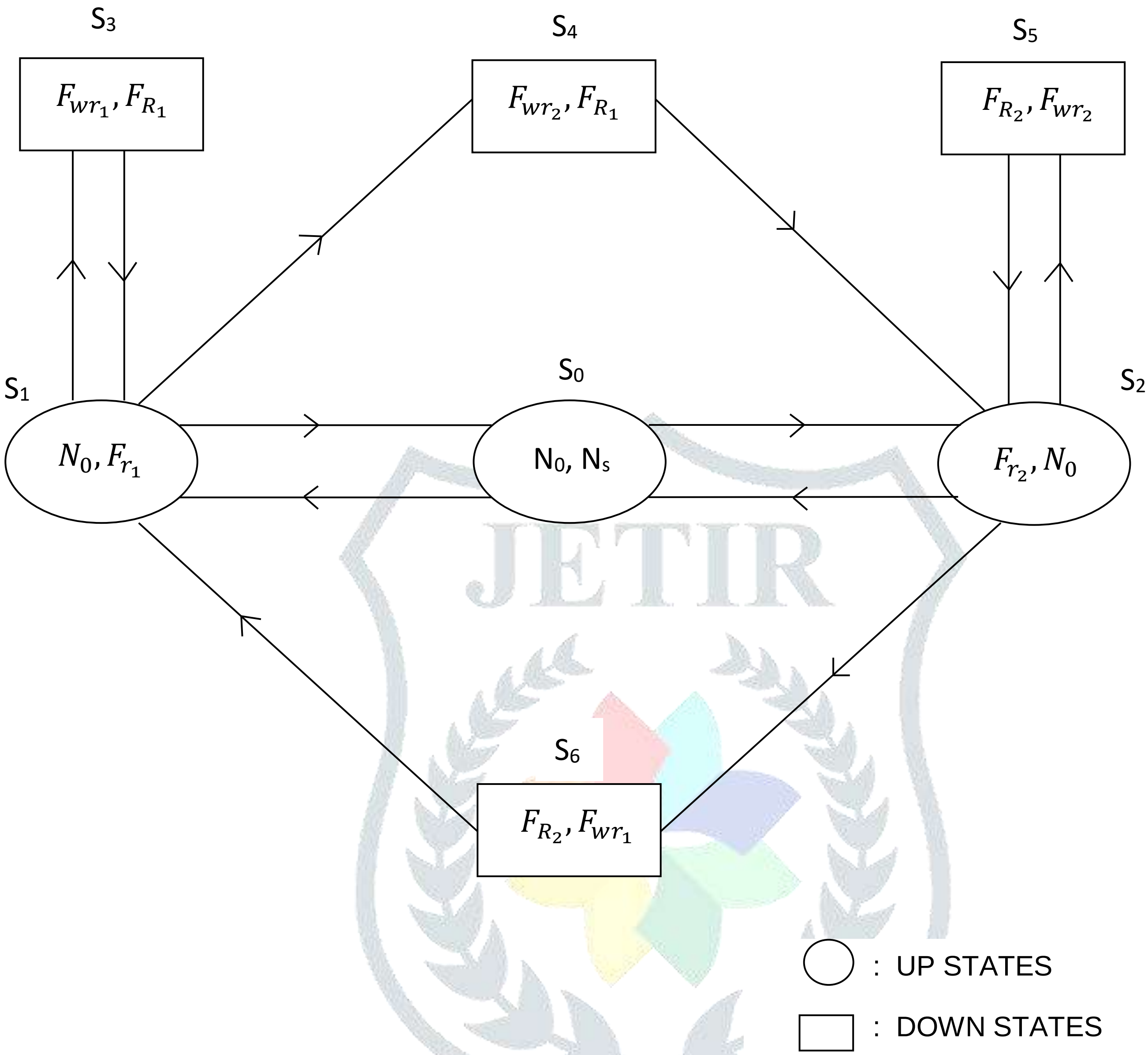
S_o : (N_O, N_S), S_1: (N_O, F_{r_1}), S_2 : (N_{r_2}, N_0)

Down States

S_3: (F_{wr_1}, F_{R_1}), S_4 : (F_{wr_2}, F_{R_1})

S_5: (F_{R_2}, F_{wr_2}), S_6 : (F_{R_2}, F_{wr_1})

The epochs of entrances from $S_1 \rightarrow S_3$, $S_1 \rightarrow S_4$, $S_2 \rightarrow S_5$, and $S_2 \rightarrow S_6$, are non-regenerative and remaining are regenerative. The possible states and transitions between them are shown in the figure.


S3
F_{wr_1}, F_{R_1}
S4
F_{wr_2}, F_{R_1}
S5
F_{R_2}, F_{wr_2}
S1
N_0, F_{r_1}
S0
N_0, N_s
S2
F_{r_2}, N_0
S6
F_{R_2}, F_{wr_1}
: UP STATES
: DOWN STATES

TRANSTION PROBABLITY AND MEAN SOJOURN TIME

The non zero elements of the transition probability,P = (P_{ij}) are as followos.

$P_{01} = a_1$,$P_{02} = a_2$, $P_{10} = f^*(\alpha)$

$P_{13} = a_1[1 - f^*(\alpha)] = P_{11}^{(3)}$, $P_{14} = a_2[1 - f^*(\alpha)] = P_{12}^{(4)}$

$P_{20} = g^*(\alpha)$, $P_{26} = a_1[1 - g^*(\alpha)] = P_{21}^{(6)}$

$P_{25} = a_2[1 - g^*(\alpha)] = P_{22}^{(5)}$, $P_{31} = P_{42} = 1 = P_{52} = P_{61}$

The above probabilities satisfies the relations

$$P_{01} + P_{02} = 1 = P_{10} + P_{13} + P_{14} = P_{10} + P_{11}^{(3)} + P_{12}^{(4)}$$

$$P_{20} + P_{25} + P_{26} = 1 = P_{20} + P_{21}^{(6)} + P_{22}^{(5)}$$

Mean Sojourn Time

Mean sojourn time μ_i in state Si is defined as the expected time for wichthe systemstays in state S_i before transiting to any other state. Let X_i denote the Sojourn Time in state S_i is given by

$$\mu_i = \int P\,[X_i > t]dt$$

so that we obtain the following relations

$\mu_0 = \frac{1}{\alpha}$, $\mu_i = [1 - f^*(\alpha)]/\alpha$

$\mu_2 = [1 - g^*(\alpha)]/\alpha$,$\mu_3 = \int_o^\infty t\,f(t)dt = \mu_4$

$\mu_5 = \int_o^\infty tg(t)dt = \mu_6$

The conditional mean sojourn time in state S_i, when system transits direct to S_j, is

$m_{ij} = \int td\theta_{ij}(t) = -\int Q_{ji}(t)'dt$

Thus

$m_{o1}= {}^{a_1}/_{\alpha}$, $m_{o2} = {}^{a_1}/_{\alpha}$

$m_{10}= \int t\, e^{-\alpha t}\, f(t)\, dt$

$m_{11}^{(3)}= a_1[\int tdF(t) - \int te^{-\alpha t}\, dF(t)]$

$m_{12}^{(4)}= a_2[\int tdF(t) - \int te^{-\alpha t}\, dF(t)]$

$m_{13}= a_1\left[\frac{1}{\alpha} - \int te^{-\alpha t}dF(t) - \frac{1}{\propto}\int e^{-\alpha t}dF(t)\right]$

$m_{14} \quad = a_2\left[\frac{1}{\alpha} - \int te^{-\alpha t}dF(t) - \frac{1}{\alpha}\int e^{-\alpha t}dF(t)\right]$

$m_{20} \quad =\int te^{-\alpha t}dG(t)$

$m_{21}^{(4)} \quad = a_1[\int tdG(t) - \int te^{-\alpha t}dG(t)]$

$m_{25} \quad = a_2\left[\frac{1}{\alpha} - \int te^{-\alpha t}dG(t) - \frac{1}{\propto}\int e^{-\alpha t}dG(t)\right]$

$m_{26} \quad = a_1\left[\frac{1}{\alpha} - \int te^{-\alpha t}dG(t) - \frac{1}{\propto}\int e^{-\alpha t}dG(t)\right]$

$m_{31} \quad = \int tdF(t) = m_{42}$

$m_{52} \quad = \int tdG(t) = m_{61}$

It can easily be verified that

$$m_{01} + m_{02} = {}^{1}/_{\alpha} = \mu_0$$

$$m_{10} + m_{13} + m_{14} = [1 - f^*(\alpha)]/\alpha = \mu_1$$

$$m_{10} + m_{11}^{(3)} + m_{12}^{(4)} = \int tdF(t) = m_1$$

$$m_{20} + m_{25} + m_{26} = [1 - g^*(\alpha)]/\alpha = \mu_2$$

$$m_{20} + m_{21}^{(6)} + m_{22}^{(5)} = \int tdG(t) = m_2$$

$$m_{31} = m_1, \; m_{42} = m_1 \;, \; m_{52} = m_2 \;, \; m_{61} = m_2$$

MEAN TIME OF SYSTEM FAILURE

To investigate the distribution function $\pi_i(i)$ of the time to system failure with starting state S_i the failed states are regarded as absorbing. Using the

probabilistic arguments the recursive relations among $\pi_i(t)$are

$\pi_0(t) = Q_{01}(t)\$ \pi_1(t) + Q_{02}(t)\$ \pi_2(t)$

$\pi_1(t) = Q_{10}(t)\$ \pi_0(t) + Q_{13}(t) + Q_{14}(t)$

$\pi_2(t) = Q_{20}(t)\$ \pi_0(t) + Q_{25}(t) + Q_{26}(t)$

(1 – 3)

Taking Laplace–Stieltjes Transform of (1 – 3) and solve for $\tilde{\pi}(s)$ and omitting the argument "s" for brevity, we get:

$$\text{MTSF} = E(T) = \frac{d\,\tilde{\pi}_0(s)}{ds}\Big|_{s=0} = \frac{N_1}{D_1} \qquad (4)$$

Where

$$N_1 = \mu_0 + P_{01}\mu_1 + P_{02}\mu_2$$

and

$$D_1 = 1 - P_{01}P_{10} - P_{02}P_{20}$$

AVAILABILITY ANALYSIS

As defined, $M_i(t)$ denotes the probability that the system starting in up stat $S_i \in E$ is up at time t without passing through any regenerating state.

Thus, we get

$$M_0(t) = e^{-\alpha t} \ , \ M_1(t) = e^{-\alpha t}\bar{F}(t)$$

$$M_2(t) = e^{-\alpha t}\bar{G}(t)$$

Using the arguments of the theory of a regenerative process, the point wise availability $A_i(t)$is satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{11}^{(3)}(t) © A_1(t) + q_{12}^{(4)}(t) © A_2(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{21}^{(6)}(t) © A_1(t) + q_{22}^{(5)}(t) © A_2(t)$$

(5– 7)

Taking the Laplace Transform of the above relation (5 – 7) and solving for $A_0^*(s)$. By omitting the argument 's' for brevity, we get

$$A_o^*(s) = \frac{N_2\,(s)}{D_2(s)} \quad (8)$$

The Steady State Availability, when the system starts from S_i is obtained as follows:

$$A_0 = \lim_{S\to 0} s\, A_0^*(s) = \frac{N_2\,(0)}{D_2'(0)} = \frac{N_2}{D_2} \qquad (9)$$

Where

$$N_2 = \mu_0 \left[P_{10}\left(1 - P_{22}^{(5)}\right) + P_{12}^{(4)} P_{20}\right] + \mu_1 \left[P_{01}\left(1 - P_{22}^{(5)}\right) + P_{02}\, P_{21}^{(6)}\right]$$

$$+ \mu_2 \left[P_{02}\left(1 - P_{11}^{(3)}\right) + P_{01}\, P_{12}^{(4)}\right]$$

And

$$D_2 = \mu_0 \left[\mathrm{P}_{10}\left(1 - \mathrm{P}_{22}^{(5)}\right) + \mathrm{P}_{12}^{(4)}\mathrm{P}_{20}\right] + \mathrm{m}_1\left(\mathrm{P}_{21}^{(6)} + \mathrm{P}_{01}\mathrm{P}_{20}\right) + \mathrm{m}_2\left(\mathrm{P}_{12}^{(4)} + \mathrm{P}_2\mathrm{P}_{10}\right)$$

BUSY PERIOD ANALYSIS

B_i(t) is defined as the probability that the repairman is busy at epoch t starting from $S_i \in$ E. From elementary probalistic arguments, we have

$$B_O(t) = q_{01}(t)© B_1(t) + q_{02}(t)© B_2(t)$$

$$B_1(t) = W_1(t) + q_{10}(t)© B_0(t) + q_{11}^{(3)}(t)©B_1(t) + q_{12}^{(4)}(t)©B_2(t)$$

$$B_2(t) = W_2(t) + q_{20}(t)© B_0(t) + q_{21}^{(6)}(t)©B_1(t) + q_{22}^{(5)}(t)©B_2(t)$$

(10 – 12)

Where

$$W_1(t) = e^{-\alpha t}\bar{F}(t) \quad , \quad W_2(t) = e^{-\alpha t}\bar{G}(t)$$

Taking the Laplace transform of the above aquations and solving for $B_0^*(s)$.And omitting the argument ‘s’ for brevity obtaion

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)} \qquad (13)$$

The Steady State Busy Period, when the system starts from Si, is obtained as follows:

$$B_0 = \lim_{S\to\infty} sB_0^*(s) = \frac{N_3(0)}{\acute{D}_2(0)} = \frac{N_3}{D_2} \qquad (14)$$

Where

$$N_3 = \left[P_{01}\left(1 - P_{22}^{(4)}\right) + P_{02}P_{21}^{(6)}\right]\mu_1 + \left[P_{02}\left(1 - P_{11}^{(3)}\right) + P_{01}P_{12}^{(4)}\right]\mu_2$$

and D_2 is defined as availability analysis.

EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

we defined $V_i(t)$ as the expected number of visits by the repairman in $(0,t]$, geven that the system initially starts from regenerative state S_i. By probabilistic arguments we have the following recursive relations:

$$V_0(t) = Q_{01}(t)\$[1+V_1(t)] + Q_{02}(t)\$[1+V_2(t)]$$

$$V_1(t) = Q_{10}(t)\$\, V_0(t) + Q_{11}^{(3)}(t)\$\, V_1(t) + Q_{12}^{(4)}(t)\$\, V_2(t)$$

$$V_2(t) = Q_{20}(t)\$\, V_0(t) + Q_{21}^{(6)}(t)\$\, V_1(t) + Q_{22}^{(5)}(t)\$\, V_2(t)$$

(15 – 17)

Taking Laplace–Stieltjes Transform of relation (15 – 17) and solving for $\widetilde{V_0}(s)$ by omitting the arguments "s" for brevity, we get

$$\widetilde{V_0}(s) = \frac{N_4(s)}{D_2(s)} \quad (18)$$

In Steady State, the number of visits per unit time in given by

$$V_0 = \underset{t\to\infty}{Lim}\left(\frac{V_0(t)}{t}\right) = \underset{s\to 0}{\text{Lim}}\, s.\widetilde{V_0}(s) = \frac{N_4}{D_2} \quad (19)$$

Where

$$N_4 = \left(1-P_{11}^{(3)}\right)\left(1-P_{12}^{(5)}\right) - P_{12}^{(4)}P_{21}^{(6)} \quad (20)$$

COST BENEFIT ANALYSIS

The expected profit of the system incurred in (0,t] is

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b(t) - K_2\, V_0(t)$$

Where K_0 , K_1 and K_2 be the revenue per unit up time by the system, the cost per unit time for which the repairman is busy and the cost per visit by the repairman respectively.

The expected profit per unit time in steady state is

$$P = \underset{t\to 0}{\text{Lim}}\frac{P(t)}{t} \quad (21)$$

$$P = K_0 A_0 - K_1 B_0 - K_2 V_0 \tag{22}$$

CONCLUSION

In the present system the failure of the operating unit is divided into two types i.e, type-I and type-II. Type-II failure of the unit is serious. In the present study the idea of replacement of the failed unit due to type-II is used to increase the effectiveness of the system. The optimum results are obtained as shown in equation (4), (9), (14) (19) and (22). The behaviour of the MTSF and Profit can also be studied from the equation (4) and (22) with respect to the system failure rate.

REFERENCES

1. Agnihotri, R.K. chand, S. and Mudgal, V.B. Two unit redundant system with adjustable failure rate, crticle human error (C.H.E) and inspection. Microelectronic Reliab.34, 1349-1354 (1994).
2. Agnihotri, R.K, Kumar, V. and Ansari, M.A. Stochastic analysis of a redundant system with two types of failure. Micro election. Raliab.33, 1467-1471 (1993)
3. Agrawal, S.C,Sahani, M., Bansal, S. Reliability characteristic of cold standby redundant system, May 2010. 3(2), IJRRAS.
4. B.S Dhillon and R.B. Mishra. Reliability evaluation of the system with critical human error. MicrolectronReliab. 24, 743-759 (1984)
5. Balagurusamy, E. Reliability Engineering. Tota McGraw Hill Publishing Company Ltd. New Delhi.
6. Barlow, B.E. and Praschan, F. (1965). Mathematical theory of Reliability. John wiley and sons. Inc. New York.
7. Goel, L.R., R.K. Agnihotri and R.Gupta. Profit evaluation of two unit cold standby system with random change in units. Intl.3. system Sci. 23: 367-377 (1992).
8. Gupta R., Varshney.G.A two idendentical unit parallel system with Geometric failure of repair time distributions. J. of Comb. Info. & System Science, Vol. 32, No. 1-4, PP. 127-139 (2007)

9. Hyderi, Pour Darvish and Joorel, J.P.S. Stochastic behaviour of a two unit cold standby redundant system subject to random failure. Microelection. Reliab. 36, 243-246 (1996)

10. Ram, M., Singh, S.B. and Singh, V.V.Stochastic Analysis of a standby system with waiting repairstratigy, IEEE Transaction on System Man and Cybernetics, 43(3), P. 698-707 (2013)

11. Wu, Q., Wu, S. Reliability analysis of a two-unit cold standby reparable systems under Poisson shocks. Appl. Math. Compt. 218(1), 171-182 (2011)