Role of Symmetry in analysis of one, two and three dimensional black body radiation

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Abstract :

Nature love symmetry and it always play a significant role in analysis of various physical system. In this research paper I have applied planck theory of black body radiation and concepts of phase space for symmetrical analysis of two and three dimensional black body .Temperature variation and size of black body always affect many physical properties like as no. of photon, energy,specific heat etc. of any thermodynamic system,with the help of mathematical analysis dependency of various physical parameters is completely elaborated in this paper .

Keywords: Planck theory, thermodynamic potentials, black body, physical parameters etc.

Introduction:

In 1900 Max Planck German physicist introduce a revolutionary new idea to explain the distribution of energy among the various wavelengths of the cavity radiation. He assume that atoms of the walls of the cavity radiator behaved as an oscillators and each one have its own characteristics frequency of oscillations. Such oscillators emit and absorb electromagnetic radiant energy with in the cavity and consequently an equilibrium state is setup.

Max Planck made two independent revolutionary assumptions regarding these oscillators.

1. These oscillators can have only certain discrete set of energies given as
   E = nhν ,where ν is the frequency of oscillator and h is planck constant, n is known to be quantum number. this simply means that oscillator can have only the energies hν, 2hν, 3hν and so on.
2. These oscillators do not emit or absorb energy continuously but only in jumps.

In this research paper we are going to discuss effect of temperature and size on various physical properties in various dimension on black body radiation .

One dimensional analysis:

According to planck's distribution law average number of photons in a single mode of frequency ν $\bar{n} = \frac{1}{(e^{h\nu/kT}-1)}$

Since the atoms of cavity walls behaved as an harmonic oscillator therefore average energy of such harmonic oscillator would be

$$< E > = h\nu\,\bar{n} = \frac{h\nu}{(e^{h\nu/kT}-1)}$$

From the concept of phase space total number of harmonic oscillator for one dimensional black body radiation = $\frac{Ldp}{h}$

For the photon which have rest mass zero and travel with velocity of light , we have

E= cp =hν

where symbols have there usual meanings

Now density of states in one dimensional cavity for black body radiation is

$$g_\nu d\nu = \quad 2\frac{L}{c}d\nu \times \bar{n}$$ here 2 is taken because of polarization

$$g_\nu d\nu = \quad 2\frac{L}{c}d\nu \times \frac{1}{(e^{h\nu/kT}-1)}$$

now integrating this from the limit ν=0 to ν=∞ we have

$$\int_0^\infty g_\nu d\nu = \int_0^\infty 2\frac{L}{c} \times \frac{1}{(e^{h\nu/kT}-1)}\, d\nu \quad = \quad \bar{N}$$

Here $\bar{N}$ represents total number of photons consist by the one dimensional black body At any temperature T

Now putting $h\nu/_{kT} = x$ or $\nu = \frac{kT}{h}x$ and dν = $\frac{kT}{h}dx$ in above equation we get

$\bar{N} \propto L$ (Length of the cavity)

And $\bar{N} \propto T$ (Temperature of cavity)

Now energy of radiation U_ν in the frequency range ν to ν+dν

$$U_\nu d\nu = 2\frac{L}{c}d\nu \times < E >$$

$$U_\nu d\nu = 2\frac{L}{c} \times \frac{h\nu}{(e^{h\nu/kT}-1)}d\nu$$

Now to calculate total energy of radiation taking integration of both the side from the limit ν=0 to ν=∞ we have

$$\int_0^\infty U_\nu d\nu = \int_0^\infty 2\frac{L}{c} \times \frac{h\nu}{(e^{h\nu/kT}-1)}\, d\nu$$ = E (total energy)

Now putting $h\nu/_{kT} = x$ or $\nu = \frac{kT}{h}x$ and dν = $\frac{kT}{h}dx$ in above equation we get

E $\propto L$ (length of cavity)

E $\propto T^2$ (T is the Temperature of cavity)

And specific heat at constant volume as well as entropy $\propto T$

Thermodynamic potential like as U (internal energy), F (Helmholtz function), G (Gibb's energy) , H (enthalpy) is directly proportional to second power of temperature.

Two Dimensional Analysis of Black Body Radiation:

From the concept of phase space, number of vibration mode of electromagnetic waves between the frequency range ν to ν+dν is

$$\frac{\iint dxdy \iint dp_x dp_y}{h^2}$$

Since $\iint dxdy$ = A (area of the cavity) and $\iint dp_x dp_y = 2\pi p dp$

Using all these together we have

$$\frac{A\, 2\pi p dp \times 2}{h^2}$$

Factor 2 is multiplied due to polarization (left and right polarization).

For the photon, energy momentum relations and using planck expression

$$E = pc = h\nu$$

$$P = \frac{h\nu}{c} \text{ and } dp = \frac{h d\nu}{c}$$

Therefore the number of vibrational modes of electromagnetic waves between frequency range ν to ν+dν is

$$\frac{A}{h^2} \times 2\pi \left(\frac{h\nu}{c}\right) \times \frac{h d\nu}{c} \times 2$$

$$= \frac{4\pi A}{c^2} \nu d\nu$$

Now the number of quantum states of photons between the frequency range ν to ν+dν is

$$N(\nu)d\nu = \frac{4\pi A}{c^2} \nu d\nu \times \text{occupation index}$$

$$= \frac{4\pi A}{c^2} \nu d\nu \times \frac{1}{(e^{h\nu/kT} - 1)}$$

As the energy of each photon in given frequency ranges is hν, therefore radiant energy in this segment of spectrum is

$$U(\nu)d\nu = \frac{4\pi A}{c^2} \nu d\nu \times \frac{h\nu}{(e^{h\nu/kT} - 1)}$$

And therefore the radiant energy per unit area is

$$u(\nu)d\nu = \frac{4\pi}{c^2} \nu d\nu \times \frac{h\nu}{(e^{h\nu/kT} - 1)}$$

now to calculate the total number of photons at any temperature T in two dimensional black body radiation, limit of frequency is taken from 0 to ∞, in above equation

$$N(T) = \int_0^\infty N(\nu)d\nu = \int_0^\infty \frac{4\pi A}{c^2} \nu d\nu \times \frac{1}{(e^{h\nu/kT} - 1)}$$

Now putting $h\nu/_{kT} = x$ or $\nu = \frac{kT}{h} x$ and $d\nu = \frac{kT}{h} dx$ in above equation

we get

$$N(T) = \frac{4\pi A}{c^2} \left(\frac{kT}{h}\right)^2 \int_0^\infty \frac{x\, dx}{e^x - 1}$$

Therefore we have

$N(T) \propto T^2$ and $N \propto A$ (Area of the black body radiation)

This expression clearly shows that number of photons in two dimensional cavity is proportional to second power of temperature (T) and area (A) of the cavity (size).

At equilibrium the total energy of two dimensional black body radiation inside an enclosure of area (A) at any Temperature (T)

Integrating above equation taking the limit of frequency from 0 to ∞

$$\int_0^\infty U(\nu)d\nu = \int_0^\infty \frac{4\pi A}{c^2}\nu d\nu \times \frac{h\nu}{(e^{h\nu/kT}-1)}$$

Now putting $h\nu/_{kT} = x$ or $\nu = \frac{kT}{h}x$ and $d\nu = \frac{kT}{h}dx$ in above equation we get

$$U(T) = \frac{4\pi Ah}{c^2}\left(\frac{kT}{h}\right)^3 \int_0^\infty \frac{x^2\,dx}{e^x-1} \quad \text{or}$$

$$U(T) \propto A \text{ (Area of black body radiation)}$$

$$U(T) \propto T^3$$

These expressions clearly shows that total energy of black body in two dimension is directly proportional to third power of temperature (T) and Area (A) of the black body radiation.

Thermodynamic potential U (internal energy), F(Helmholtz function), G(Gibb's energy), H (enthalpy) is directly proportional to third power of temperature because all these physical parameters have the same temperature dependency.

Entropy S and Specific heat at constant volume is directly proportional to second power of temperature because these two physical parameters have the same temperature dependency here.

Three Dimensional Analysis of Black Body Radiation:

From the concept of phase space, number of vibration mode of electromagnetic waves between the frequency range ν to ν+dν is

$$\frac{\iiint dxdydz \iiint dp_x dp_y dp_z}{h^3}$$

Since $\iiint dxdydz = V$ (Volume of the cavity) and $\iiint dp_x\,dp_y dp_z = 4\pi p^2 dp$

Using all these together we have

$$\frac{V 4\pi p^2 dp \times 2}{h^3}$$

Factor 2 is multiplied due to polarization (left and right polarization). For the photon, energy momentum relations and using planck expression

$$E = pc = h\nu$$

$$P = \frac{h\nu}{c} \text{ and } dp = \frac{hd\nu}{c}$$

Therefore the number of vibrational modes of electromagnetic waves between frequency range ν to ν+dν is

$$= \frac{V}{h^3} \times 4\pi\left(\frac{h\nu}{c}\right)^2 \times \frac{hd\nu}{c} \times 2$$

$$= \frac{8\pi V}{c^3}\nu^2 d\nu$$

Now the number of quantum states of photons between the frequency range ν to ν+dν is

$$N(\nu)d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu \times \text{occupation index}$$

$$= \frac{8\pi V}{c^3}\nu^2 d\nu \times \frac{1}{(e^{h\nu/kT}-1)}$$

As the energy of each photon in given frequency ranges is hν, therefore radiant energy in this segment of spectrum is

$$U(\nu)d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu \times \frac{h\nu}{(e^{h\nu/kT}-1)}$$

And therefore the radiant energy per unit volume is

$$u(\nu)d\nu = \frac{8\pi}{c^3}\nu^2 d\nu \times \frac{h\nu}{(e^{h\nu/kT}-1)}$$

now to calculate the total number of photons at any temperature T in three dimensional black body radiation, limit of frequency is taken from 0 to ∞, in above equation

$$N(T) = \int_0^\infty N(\nu)d\nu = \int_0^\infty \frac{8\pi V}{c^3}\nu^2 d\nu \times \frac{1}{(e^{h\nu/kT}-1)}$$

Now putting $h\nu/_{kT} = x$ or $\nu = \frac{kT}{h}x$ and $d\nu = \frac{kT}{h}dx$ in above equation we get

$$N(T) = \frac{8\pi V}{c^3}\left(\frac{kT}{h}\right)^3 \int_0^\infty \frac{x^2\,dx}{e^x-1}$$

Therefore we have

$N(T) \propto T^3$ and $N \propto V$ (Volume of the black body radiation)

This expression clearly shows that number of photons in three dimensional cavity is proportional to third power of temperature (T) and volume (V) of the cavity (size)

At equilibrium the total energy of three dimensional black body radiation inside an enclosure of volume V at any Temperature (T)

Integrating above equation taking the limit of frequency from 0 to ∞

$$\int_0^\infty U(\nu)d\nu = \int_0^\infty \frac{8\pi V}{c^3}\nu^2 d\nu \times \frac{h\nu}{(e^{h\nu/kT}-1)}$$

Now putting $h\nu/_{kT} = x$ or $\nu = \frac{kT}{h}x$ and $d\nu = \frac{kT}{h}dx$ in above equation we get

$$U(T) = \frac{8\pi Vh}{c^3}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3\,dx}{e^x-1} \quad \text{or}$$

$U(T) \propto V$ (Volume of black body radiation)

$U(T) \propto T^4$

These expressions clearly shows that total energy of black body in three dimension is directly proportional to fourth power of temperature (T) and volume (V) of the black body radiation.

Thermodynamic potential U (internal energy), F(Helmholtz function), G(Gibb's energy) , H (enthalpy) is directly proportional to fourth power of temperature because all these physical parameters have the same temperature dependency.

Entropy S and Specific heat at constant volume is directly proportional to third power of temperature because these two physical parameters have the same temperature dependency here.

Conclusions:

Table 1.1: Comparision of physical properties of Three Dimensional Black Body Radiation and Two Dimensional Black Body Radiation

Physical properties	One Dimensional Black Body	Two Dimensional Black Body	Three Dimensional Black Body
No. of photons in the cavity (size)	Directly proportional to length	Directly proportional to Area	Directly proportional to Volume
No. of photons in the cavity (temperature T)	Directly proportional to T	Directly proportional to T^2	Directly proportional T^3
Energy (dependency on size)	Directly proportional to length	Directly proportional to Area	Directly proportional to Volume
Energy (dependency on temperature T)	Directly proportional to T^2	Directly proportional to T^3	Directly proportional to T^4
Thermodynamic potentials	Directly proportional to T^2	Directly proportional to T^3	Directly proportional to T^4
Entropy	Directly proportional to T	Directly proportional to T^2	Directly proportional to T^3
Specific heat at constant volume	Directly proportional to T	Directly proportional to T^2	Directly proportional to T^3

With the help of symmetrical analysis, various physical properties in different dimensions can be easily evaluated and when dimension of black body is change then physical properties of black body is also change and role of symmetry is very vital. Number of photons in the cavity depends upon the size and it is proportional to T^D. Here D is the dimension. Energy also depends upon the size of the cavity and it is also proportional to T^{D+1}. Thermodynamic potentials also depends upon temperature and proportional to T^{D+1}.Entropy of the cavity in black body radiation is also depends upon temperature and proportional to T^D. Specific heat of the cavity is also depends upon the temperature and proportional to T^D.

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