CLASSICAL MECHANICS AND THOMAS PRECESSION TO SPIN – ORBIT INTERACTION ENERGY OF HYDROGN ATOM

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ABSTRACT:

We contemplate a semi-classical mechanics to obtain the spin-orbit interaction energy in an easy method for example the hydrogen atom. The energy of the interaction ΔE can be evaluated in the rest frame of the electron about which the nucleus moves in a circular orbit and the electron is affected situated within a current loop. The interface energy ΔE is produced by the coupling of the screwed up electric dipole due to the electric field E_n of the core. We point out the presence of an additional force exerted on a spinning electron due to the appearance of its electric dipole moment in the rest frame of the nucleus. After studying the motions of the electron spin and orbital angular moments, it is obtained that the total angular momentum averaged over the orbit is not a constant of the motion in the Thomas Precession. Considering during the spin flip transition the radius of the electron's orbit is constant. Our model predicts that the energy of the system changes by the factor ½ budding in nature as a consequence of equilibrium and the variation of the kinetic energy of the electron. The orbit radius for immobile, total angular momentum for the circular orbit system including nonzero orbit angular momentum is the radius of the Bohr model in the ground states.

INTRODUCTION:

The classical analysis of spin-orbit coupling in hydrogen atoms by Thomas took an important part in sustaining the hypothesis of electron spin as highly developed by Uhlenbek and Goudsmit. The later effectively explained the spectra of the multiples structure of atom and anomalous Zeeman Effect based on the hypothesis about the appropriate magnetic moment of the electron and is correlated with the own rotational motion of the electron (spin). However, the calculation of a spin orbit coupling was found double of the measured value of the hydrogen atom. This divergence was eliminated by Thomas in 1926 by using a classical relativistic approach. He pointed out those successive Lorentz transformations with none collinear relative velocities lead to an extra spatial revolution of the coordinate axes of the reference frames involved. In this way he showed that the doublet separation due to spin- orbit coupling is half the value derived by Uhlenbek and Goudsmit. Later the problem of spin-orbit interface was solved by the relativistic quantum electron theory as given by Dirac. Nonetheless, it is comforting that this effect has a classical explanation especially because on account of Thomas precession in the spin-orbit coupling of the hydrogen atom is often considered a confirmation of special relativity, which is still of current interest.

THEORY:

The starting point of the inner magnetic field experienced by an electron revolving in a one-electron atom is uncomplicated to realize if we think about the motion of the nucleus from the point of view of the electron. In a frame of reference fixed on the electron, the charged nucleus revolves around the electron and the electron is affected, sited inside a current loop, which develops the magnetic field on the electron (from Ampere's law) is given by

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0 \, ze}{4\pi} \quad \frac{\vec{v} \times \vec{r}}{r^3} \qquad (1)$$

Where $\vec{J}$ is the current element and is given by $\vec{J} = -\, z\, e\, \vec{v}$. Z and e are the charges of the nuclear and electron respectively and $\vec{v}$ represents velocity of the electron

From the Coulomb's law, the electric field acting on the electron is given by

$$\vec{E} = \frac{ze}{4\pi\varepsilon_0} \; \frac{\vec{r}}{r^3}$$

$$\text{or,} \quad z\, e = 4\pi\varepsilon_0 \; \frac{\vec{r}}{r^3} \; \vec{E} \qquad (2)$$

Using equation (2) in equation (1), we get

$$\vec{B} = -\, \mu_0 \varepsilon_0 \, \vec{v} \times \vec{E}$$

Where μ_0 is the permeability and ε_0 is the permittivity of the medium

But the velocity of the plane electromagnetic wave in vacuum is $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, then we have,

$$\vec{B} = -\frac{1}{C^2} \, \vec{v} \times \vec{E} \qquad (3)$$

This magnetic field $\vec{B}$ affects the electron due to the charged nucleus.

It can be assumed that he electron and its spin magnetic dipole moment have different orientations in the inner magnetic field of the atoms and these orientations have their different potential energies. The oriental potential energy(ΔE) of the magnetic dipole moment ($\overrightarrow{\mu_s}$) due to spin of an electron in this magnetic field is given by

$$\Delta E = -\, \vec{\mu}_s \cdot \vec{B} \qquad (4)$$

But as, $$\overrightarrow{\mu_s} = -\frac{g_s \, \mu_B}{\hbar} \cdot \vec{S}$$ Where

$\mu_B =$ the Bohr magneton.

Thus due to the magnetic dipole moment, the oriental potential energy may also be expressed in terms of the electron's spin angular momentum $\vec{S}$, is given by

$$\Delta E = \frac{g_s \, \mu_B}{\hbar} \, \vec{S} \cdot \vec{B} \qquad (5)$$

Where g_s denotes spin g-factor.

This energy has been evaluated in a frame of reference fixed on the electron in which the charged nucleus revolves around the electron but we are also paying attention to find this energy in a frame of reference fixed on the nucleus in which the electron revolves around the nucleus. In Thomas Precession the relativistic transformation of velocities affect the result due to this the transformation of velocities comes back to the nucleus rest frame, so the oriental potential energy is reduced by a factor of 2. Thus, the spin- orbit interface energy is given by

$$\Delta E_{S.O} = \frac{1}{2} \frac{g_s \ \mu_B}{\hbar} \vec{S} \cdot \vec{B} \qquad (6)$$

The factor of 2 can not be ignored because the result will become complicated. It is feeling convenient to express equation (6) in terms of the scalar product of the spin and orbital angular momentum vectors.

The force experienced by electron due to the electric field is given by

$$\vec{F} = -\, e\vec{E}$$

(7) Where $\vec{E}$ is an electric field and $-\,e$ be the charge on the electron.

The relation between the force and the potential energy is given by

$$\vec{F} = -\frac{dv(r)}{dr} \frac{\vec{r}}{r} \qquad (8)$$

Where $\frac{\vec{r}}{r}$ is a unit vector which gives the direction of $\vec{F}$ in the radial direction. With this substitution, equation (7) becomes,

$$-\, e\vec{E} = -\frac{dv(r)}{dr} \frac{\vec{r}}{r}$$

Or, $$\vec{E} = \frac{1}{e} \frac{dv(r)}{dr} \frac{\vec{r}}{r} \qquad (9)$$

With the substitution of equation (9) in equation (3), we get,

$$\vec{B} = \frac{1}{ec^2} \frac{1}{r} \frac{dv(r)}{dr} \vec{v} \times \vec{r} \qquad (10)$$

The orbital angular momentum ($\vec{L}$)is given by

$$\vec{L} = \vec{r} \times \mathrm{m}\vec{v} = -\, \mathrm{m}\vec{v} \times \vec{r}$$

With this substitution equation (10) becomes

$$\vec{B} = \frac{1}{mec^2} \frac{1}{r} \frac{dv(r)}{dr} \vec{L} \qquad (11)$$

Using equation (11) in equation (6) we get,

$$\Delta E_{\ S.O} = \frac{1}{2} \frac{1}{mec^2} \ \frac{1}{r} \frac{g_s \ \mu_B}{\hbar} \frac{dv(r)}{dr} \vec{S} \cdot \vec{L} \qquad (12)$$

Evaluating g_s and μ_B , we obtain

$$\Delta E_{S.O} = \frac{1}{2}\frac{1}{m^2c^2}\ \frac{1}{r}\ \frac{dv(r)}{dr}\ \vec{S}\cdot\vec{L} \qquad (13)$$

Thomas first derived this equation in 1926 with the help of the Bohr model, Schrodinger wave equation and relativistic kinematics. However, this result is entirely in agreement with the results of the Dirac's relativistic quantum mechanics.

C0NCLUSION:

We have summarized the approaches to the conventional derivation of the spin – orbit coupling within H-atom suggested by Thomas and Frenkel and showed that both approaches symmetrically ignore the additional force acting between the electrons and nucleus due to the relativistic polarization of the orbiting electron. This result agrees qualitatively with the solution of the Dirac-Coulomb equation, where the position of the maximum of the radial part of electron's wave function differ from each other for different values of the quantum number j and the relative variation in position of the maximum has the same order of magnitude $(Z\alpha)^2$, in the semi classical expression.

In this paper, I have tried to show that the correct ½ factor may be derived without remedy to Thomas's precession. Because the kinetic energy K, the potential energy V, and the interface energy E between the electron's magnetic moment $\boldsymbol{\mu}$ and the nuclear E-field may be evaluated in the rest-frame of the nucleus if the orbit of the electron must continue a constant radius.

The spin-orbit energy evaluated in the electron's rest-frame is endorsed by the corresponding evaluation in the rest-frame of the nucleus. The results of the present paper indicate that the conclusion of Thomas could be aligned with the spin-orbit energy as found in the rest-frame of the nucleus. The results of the prior sections show that the remaining energy goes into the kinetic energy K of the orbiting electron as seen in the rest-frame of the nucleus.

References:

1. Schiff, L.I.(1964): *Quantum Mechanics; Mc Graw Hill Internatinal, Hong Kong*, p 64-65,

2. Verma, H.C. (2009*): Quantum Physics; Surya Publication, Noida, pm 89-*

3. Jain, K.M. (2003) *: Introductory Quantum Mechanics and Spectroscopy*; South Asia Publications, New Delhi. 142

4. Resonance, July2015,

5. Current Science, July 1-15, 2015

6. *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles .Robert Eisberg and Robert Resnick; Wiley India (P) Ltd. New Delhi.*

7. Thomas, L.H. (1926):" *The motion of the spinning electron*" Nature(London) 117, 514.

8. Dirac, P. A. M. (1928): "*The quantum theory of the electron", Proc. R. Soc. London*

9. Sommerfield, A. (1926): *Wave – Mechanics: Supplementary Volume to* Atomic *Structure and Spectral Lines* (Dutton, New York).

!0. Moller, C.(1972): *Theory of relativity ,* (Clarendon, Oxford,)