STUDY OF RELATIVE MOTION IN THE SYSTEM OF GRAVITATIONAL FIELD OF FORCE

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Abstract: In this article, it is tried to study the motion of the system in the central gravitational field of force when the centre of mass of the system in moving along elliptical orbit. The equation of the relative motion of the system in the central gravitational field of the force is derived in polar form for the elliptical orbit of the centre of mass. It is obtained in an approximate form of non-linear oscillations. Parametric resonance driven oscillations of a dumb-bell satellite in elliptical orbit in central gravitational field of force under the combined effects of perturbing forces earth magnetic field and oblateness of the earth has been studied. The system comprises of two satellite connected by a light, flexible and inextensible cable, moves like a dumbbell satellite in elliptical orbit, in central gravitational field of force. The gravitational field of the Earth is the main force governing the motion and magnetic field of the Earth and Oblateness of the Earth are considered to be perturbing forces, disturbing in nature.

Keywords: Relative Motion, Gravitational Field, Constraints, Radius vectors, Centre of mass, etc.

I. INTRODUCTION

We have, in this paper, studied the effects of dissipative and disturbing forces on the non-linear oscillation of the system. It is well known that in addition to the gravitational forces, the dissipative and disturbing forces are also present in nature. Though these forces are small, in comparison to the gravitational forces, it is expected that they may exert considerable effect on the oscillation of the system. These forces have been summed up as a dissipative force which arises due to friction of bodies in atmosphere, tidal forces and gravitational radiation, etc, and a periodic force with slowly varying frequency caused by mulfipole moments, the gravitational waves at resonance frequency, etc. (Pyragas et al.,1978). The gravitational forces, small dissipative and disturbing forces are also present in nature. Though these force small in comparison to the gravitational force but exert their vital effects on the motion and stability of the system and produce deformation in the amplitude of the oscillation of the system. These effects are more significant when we study the motion of the system in the linear field of gravitational interaction than that in the linear one. The relative motion of two particles connected by light, flexible and inextensible string in the central gravitational field of Earth was studied first by Beletsky (1969a)and Beletsky and Novikova (1969b). This study was based on the assumptions that the orbit of the centre of mass of the system is circular and the system itself moves in the plane of the orbit of the centre of mass. Later on there appeared a lot of research works in which the effects of different perturbing forces on the system were studied in both linear and non-linear fields. The cable connected satellite system is the mathematical idealization of real space system such as space vehicle and astronaught floating in space, two or multidirectional satellite system, manned space capsule attached booster by cable and spinning to provide artificial gravity for the astronaut and lastly the two satellites at the same time of rendezvous. In order to transport a man successfully to an orbiting space station. Rendezvous in outer space we must be familiar with relative notion of the satellites with respect to orbiting station and more over must predict the stable direction of approach for the rendezvous. Also, in case, the astronaut wants to float in outer space for scientific exploration. The exploration of space science resulted in the formulation of the problem of the relative motion of a system of two cable connected satellites in the central gravitational field of force.

II. MATHEMATICAL DISCUSSIONS

Let us consider radius vectors of the particles $\vec{\rho}_1$ and $\vec{\rho}_2$, is given by,

$$\vec{\rho}_1 = \frac{m_2}{m_1 - m_2} (\vec{r}_1 - \vec{r}_2)$$

$$\vec{\rho}_2 = \frac{m_1}{m_1 + m_2} (\vec{r}_2 - \vec{r}_1) \quad \ldots\ldots \quad (1)$$

It is clear from equation (1) that the motion of the particles can be easily obtain with the help of the motion of the other particles. Hence in order to predict the motion of the system as a whole, it is sufficient to know the motion of the particles of the system. Thus, in our subsequent discussion we shall consider the motion of the particles m_l w.r.t. their centre of mass.

Now we shall obtain the differential equation which determines the motion of one of the particles m_l of the system in the linearlised and normalized form.

In order to linearise the equation of relative motion of the particles w.r.t. the infinitesimal is $\frac{\vec{\rho}_1}{R}$ and $\frac{\vec{\rho}_2}{R}$.

Let us consider on following equations,

$$m_1 \ddot{\vec{r}}_1 + \frac{m_1 \mu \vec{r}_1}{r_1^3} \lambda (\vec{r}_1 - \vec{r}_2) = 0 \quad \ldots\ldots \quad (2)$$

Now, dividing equation (1), by m_1 and m_2 respectively we obtain,

$$\ddot{\vec{r}}_1 + \frac{\mu \vec{r}_1}{r_1^3} = \frac{\lambda}{m_1} (\vec{r}_1 - \vec{r}_2) \quad (3)$$

$$\ddot{\vec{r}}_2 + \frac{\mu \vec{r}_2}{r_2^3} = \frac{\lambda}{m_2} (\vec{r}_2 - \vec{r}_1) \quad (4)$$

Now , subtracting these equations, we get,

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 + \mu \left(\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_2^3} \right) = -\lambda (\vec{r}_1 - \vec{r}_2) \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \quad (5)$$

Putting the values of $\frac{\vec{r}_1}{r_1^3}$ and $\frac{\vec{r}_2}{r_2^3}$ in equation (5) and considering the terms up to the first order Infinitesimals only we get,

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 + \frac{\mu}{R^3} (\vec{r}_1 - \vec{r}_2) - \frac{3\mu}{R^5} \vec{R} \left[\vec{R} (\vec{r}_1 - \vec{r}_2) \right] = -\lambda (\vec{r}_1 - \vec{r}_2) \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \quad \ldots\ldots \quad (6)$$

Now we consider the following relation and obtain,

$$\overrightarrow{r_1} - \overrightarrow{r_2} = \overrightarrow{\rho_1} - \overrightarrow{\rho_2} = \overrightarrow{\rho_1} + \frac{m_1}{m_2}\overrightarrow{\rho_1} = \frac{m_1 + m_2}{m_2}\overrightarrow{\rho_1} = \ell\rho_1 \quad \ldots\ldots \quad (7)$$

The condition for constraints: $|\overrightarrow{r_1} - \overrightarrow{r_2}|^2 \leq \ell^2$ **and it now takes the form,**

$$|\overrightarrow{\rho_1}| \geq \ell \quad (8)$$

Thus, the linearlised and normalized differential equation of motion of the particles m_1 **is given by,**

$$\overrightarrow{\rho_1} = \frac{(m_1 + m_2)\overrightarrow{\rho_1}}{lm_2} \quad \ldots\ldots\ldots \quad (9)$$

The inequality of equation (8), gives the relation for constraints. If the sign holds in equation (8), then the particles moves with loose the string and hence, $\lambda_\alpha = 0$**. The motion takes place with taut string if the equality sign holds good** $\lambda_\alpha \neq 0$**, in equation (5) and this needs to be determined in the process of solving the problem.**

III. CONCLUSION

The present research work deals with polar form of equations of motion of the system the central gravitational field of force for the elliptical orbit of the centre of mass. In this article it is studied the motion of the system in the central gravitational field of force when the centre of mass of the system in moving along elliptical orbit. We found that the equation of dumb- bell satellite in the central gravitational field of force and is in a suitable form of a non-linear oscillator. This describes the non- linear oscillation of the system about its equilibrium position. This research work is in progress. It is hoped that the research may be very useful in the future applications in satellites.

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