

Kirchhoff's Circuit Laws of Equalities on Electric Current - An Empirical Study

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Abstract

This paper looks at Kirchhoff's rules for circuit analysis are applications of conservation laws to circuits. The first rule is the application of conservation of charge, while the second rule is the application of conservation of energy. Conservation laws, even used in a specific application, such as circuit analysis, are so basic as to form the foundation of that application. *Current source* is a circuit element that maintains a constant current flow regardless of the voltage developed across its terminals as this voltage is determined by other circuit elements. That is, an ideal constant current source continually provides a specified amount of current regardless of the impedance that it is driving and as such, an ideal current source could, in theory, supply an infinite amount of energy. Ideal current source is called a "constant current source" as it provides a constant steady state current independent of the load connected to it producing an I-V characteristic represented by a straight line. As with voltage sources, the current source can be either independent (ideal) or dependent (controlled) by a voltage or current elsewhere in the circuit, which itself can be constant or time-varying. Ideal constant current sources are represented in a similar manner to voltage sources, but this time the current source symbol is that of a circle with an arrow inside to indicates the direction of the flow of the current. The direction of the current will correspond to the polarity of the corresponding voltage, flowing out from the positive terminal. However as practical current sources have an internal source resistance, this takes some of the current so the characteristic of this practical source is not flat and horizontal but will reduce as the current is now splitting into two parts, with one part of the current flowing into the parallel resistance, R_P and the other part of the current flowing straight to the output terminals.

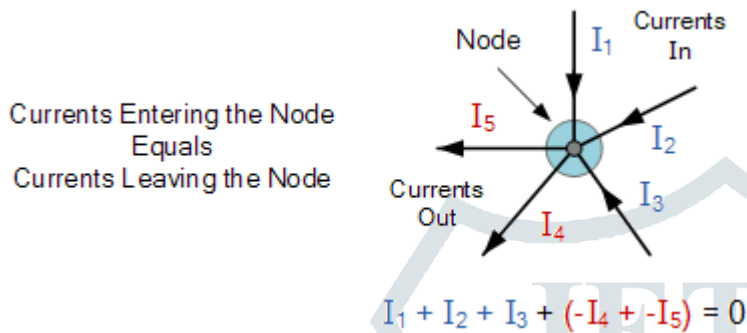
Ohms law tells us that when a current, (i) flows through a resistance, (R) a voltage drop is produce across the same resistance. The value of this voltage drop will be given as $i \cdot R_P$. Then V_{OUT} will be equal to the voltage drop across the resistor with no load attached. We remember that for an ideal source current, R_P is infinite as there is no internal resistance, therefore the terminal voltage will be zero as there is no voltage drop. In 1845, a German physicist, **Gustav Kirchhoff** developed a pair or set of rules or laws which deal with the conservation of current and energy within electrical circuits. These two rules are commonly known as: *Kirchhoffs Circuit Laws* with one of Kirchhoffs laws dealing with the current flowing around a closed circuit, **Kirchhoffs Current Law, (KCL)** while the other law deals with the voltage sources present in a closed circuit, **Kirchhoffs Voltage Law, (KVL)**.

Key words: Gustav Kirchhoff, Kirchhoff's Current Law, Kirchhoff's Voltage Law Ohms law.

Introduction

Kirchhoffs First Law states that the “total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node“. In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero, $I_{(\text{exiting})} + I_{(\text{entering})} = 0$. This idea by Kirchhoff is commonly known as the **Conservation of Charge**.

Kirchhoffs Current Law

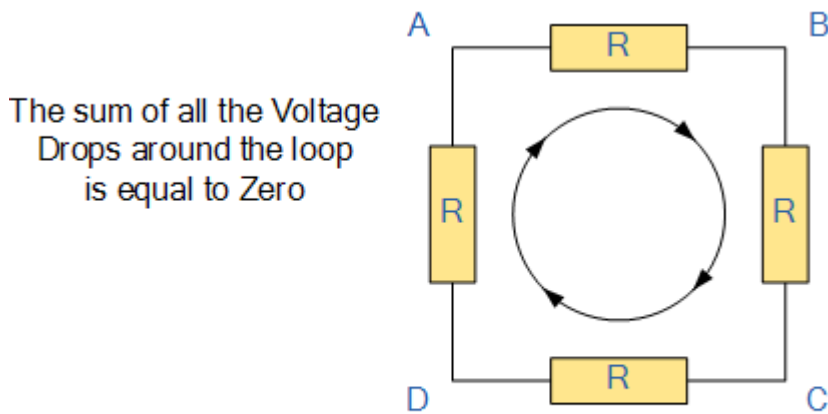


Here, the three currents entering the node, I_1 , I_2 , I_3 are all positive in value and the two currents leaving the node, I_4 and I_5 are negative in value. Then this means we can also rewrite the equation as;

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

The term **Node** in an electrical circuit generally refers to a connection or junction of two or more current carrying paths or elements such as cables and components. Also for current to flow either in or out of a node a closed circuit path must exist. We can use Kirchhoff's current law when analysing parallel circuits.

Kirchhoffs Second Law states that “in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop” which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchhoff is known as the **Conservation of Energy**.



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

Starting at any point in the loop continue in the **same direction** noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero. We can use Kirchoff's voltage law when analysing series circuits.

When analysing either DC circuits or AC circuits using **Kirchoffs Circuit Laws** a number of definitions and terminologies are used to describe the parts of the circuit being analysed such as: node, paths, branches, loops and meshes. These terms are used frequently in circuit analysis so it is important to understand them.

Common DC Circuit Theory Terms:

- • Circuit – a circuit is a closed loop conducting path in which an electrical current flows.
- • Path – a single line of connecting elements or sources.
- • Node – a node is a junction, connection or terminal within a circuit where two or more circuit elements are connected or joined together giving a connection point between two or more branches. A node is indicated by a dot.
- • Branch – a branch is a single or group of components such as resistors or a source which are connected between two nodes.
- • Loop – a loop is a simple closed path in a circuit in which no circuit element or node is encountered more than once.
- • Mesh – a mesh is a single closed loop series path that does not contain any other paths. There are no loops inside a mesh.

Objective:

This paper intends to explore Kirchhoff laws which are essential for resistor network theory. They were formulated by the German scientist Gustav Kirchhoff in 1845. The laws describe the conservation of energy and charge in electrical networks.

Kirchhoff's Circuit Laws and application

To determine the amount or magnitude of the electrical current flowing around an electrical or electronic circuit, we need to use certain laws or rules that allows us to write down these currents in the form of an equation. The network equations used are those according to Kirchhoff's laws, and as we are dealing with circuit currents, we will be looking at Kirchhoff's current law, (KCL).

Gustav Kirchhoff's Current Law is one of the fundamental laws used for circuit analysis. His current law states that for a parallel path **the total current entering a circuits junction is exactly equal to the total current leaving the same junction**. This is because it has no other place to go as no charge is lost.

In other words the algebraic sum of ALL the currents entering and leaving a junction must be equal to zero as: $\Sigma I_{IN} = \Sigma I_{OUT}$.

Source and linear-circuit elements are ideal circuit elements. One central notion of circuit theory is combining the ideal elements to describe how physical elements operate in the real world. A circuit connects circuit elements together in a specific configuration designed to transform the source signal (originating from a voltage or current source) into another signal—the output—that corresponds to the current or voltage defined for a particular circuit element. What we need to solve in every circuit problem are mathematical statements that express how the circuit elements are interconnected. This implies that we need the laws that govern the electrical connection of circuit elements. Kirchhoff's laws, one for voltage and one for current, determine what a connection between circuit elements means. These laws can help us analyze this circuit. The places where circuit elements attach to each other are called nodes.

• Kirchhoff's current law

At every node, the sum of all currents entering a node must equal zero. What this law means physically is that charge cannot accumulate in a node; what goes in must come out.

• Kirchhoff's voltage law

The voltage law says that the sum of voltages around every closed loop in the circuit must equal zero. A closed loop has the following obvious definition: starting at a node, trace a path through the circuit that returns you to the origin

node. Kirchhoff's voltage law expresses the fact that electric fields are conservative: the total work performed in moving a test charge around a closed path is zero.

Resistors in Parallel

Let's look how we could apply Kirchhoff's current law to resistors in parallel, whether the resistances in those branches are equal or unequal. Consider the following circuit diagram:

In this simple parallel resistor example there are two distinct junctions for current. Junction one occurs at node B, and junction two occurs at node E. Thus we can use Kirchhoff's Junction Rule for the electrical currents at both of these two distinct junctions, for those currents entering the junction and for those currents flowing leaving the junction.

Applying KCL to complex circuits.

We can use Kirchhoff's current law to find the currents flowing around more complex circuits. We hopefully know by now that the algebraic sum of all the currents at a node (junction point) is equal to zero and with this idea in mind, it is a simple case of determining the currents entering a node and those leaving the node. We can confirm by analysis that Kirchhoff's current law (KCL) which states that the algebraic sum of

Gustav Kirchhoff's Voltage Law is the second of his fundamental laws we can use for circuit analysis. His voltage law states that for a closed loop series path **the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero**. This is because a circuit loop is a closed conducting path so no energy is lost.

In other words the algebraic sum of ALL the potential differences around the loop must be equal to zero as: $\sum V = 0$. Note here that the term "algebraic sum" means to take into account the polarities and signs of the sources and voltage drops around the loop. This idea by Kirchhoff is commonly known as the **Conservation of Energy**, as moving around a closed loop, or circuit, you will end up back to where you started in the circuit and therefore back to the same initial potential with no loss of voltage around the loop. Hence any voltage drops around the loop must be equal to any voltage sources met along the way.

So when applying Kirchhoff's voltage law to a specific circuit element, it is important that we pay special attention to the algebraic signs, (+ and -) of the voltage drops across elements and the emf's of sources otherwise our calculations may be wrong.

But before we look more closely at Kirchhoff's voltage law (KVL) lets first understand the voltage drop across a single element such as a resistor.

A Single Circuit Element

For this simple example we will assume that the current, I is in the same direction as the flow of positive charge, that is conventional current flow.

Here the flow of current through the resistor is from point A to point B, that is from positive terminal to a negative terminal. Thus as we are travelling in the same direction as current flow, there will be a *fall* in potential across the resistive element giving rise to a $-IR$ voltage drop across it.

If the flow of current was in the opposite direction from point B to point A, then there would be a *rise* in potential across the resistive element as we are moving from a $-$ potential to a $+$ potential giving us a $+IR$ voltage drop.

Thus to apply Kirchhoff's voltage law correctly to a circuit, we must first understand the direction of the polarity and as we can see, the sign of the voltage drop across the resistive element will depend on the direction of the current flowing through it. As a general rule, you will lose potential in the same direction of current across an element and gain potential as you move in the direction of an emf source.

The direction of current flow around a closed circuit can be assumed to be either clockwise or anticlockwise and either one can be chosen. If the direction chosen is different from the actual direction of current flow, the result will still be correct and valid but will result in the algebraic answer having a minus sign.

A Single Circuit Loop

Kirchhoff's voltage law states that the algebraic sum of the potential differences in any loop must be equal to zero as: $\Sigma V = 0$. Since the two resistors, R_1 and R_2 are wired together in a series connection, they are both part of the same loop so the same current must flow through each resistor.

Thus the voltage drop across resistor, $R_1 = I \cdot R_1$ and the voltage drop across resistor, $R_2 = I \cdot R_2$ giving by KVL:

We can see that applying Kirchhoff's Voltage Law to this single closed loop produces the formula for the equivalent or total resistance in the series circuit and we can expand on this to find the values of the voltage drops around the loop.

Kirchhoff's Circuit Loop

We have seen here that Kirchhoff's voltage law, KVL is Kirchhoff's second law and states that the algebraic sum of all the voltage drops, as you go around a closed circuit from some fixed point and return back to the same point, and taking polarity into account, is always zero. That is $\Sigma V = 0$

The theory behind Kirchhoff's second law is also known as the law of conservation of voltage, and this is particularly useful for us when dealing with series circuits, as series circuits also act as voltage dividers and the voltage divider circuit is an important application of many series circuits. By applying Kirchhoff's rules, we generate equations that allow us to find the unknowns in circuits. The unknowns may be currents, emfs, or resistances. Each time a rule is applied, an equation is produced. If there are as many independent equations as unknowns, then the problem can be solved. There are two decisions you must make when applying Kirchhoff's rules. These decisions determine the signs of various quantities in the equations you obtain from applying the rules.

1. When applying Kirchhoff's first rule, the junction rule, label the current in each branch and decide in what direction it is going.
2. When applying Kirchhoff's second rule, the loop rule, you must identify a closed loop and decide in which direction to go around it, clockwise or counterclockwise.

Conclusion

Kirchhoff's Circuit laws are now known as Kirchhoff's Voltage and Current Laws. Since these laws apply to all electric circuits, understanding their fundamentals is paramount in the understanding of how an electronic circuit functions. These pair of laws that deal with the conservation of current and energy within electrical circuits. These two laws are commonly known as Kirchhoff's Voltage and Current Law. These laws help in calculating the electrical resistance of a complex network or impedance in case of AC and the current flow in different streams of the network. In the next section, let us look at what these laws state. The voltage arrows and polarity signs are just reference directions for voltage. When the circuit analysis is complete, one or more of the element voltages around the loop will be negative with respect to its voltage arrow. The signs of the actual voltages always sort themselves out during calculations.

Although these laws have immortalised Kirchhoff in the field of Electrical Engineering, he has additional discoveries. He was the first person to verify that an electrical impulse travelled at the speed of light. Furthermore, Kirchhoff made a major contribution to the study of spectroscopy and he advanced the research into blackbody radiation.

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