A Study of Generalized Recurrent Kenmotsu Manifold

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Abstract

The object of the present paper is to study study of Generalized Recurrent Ken- motsu Manifold and some properties of quarter-symmetric non-metric connection.

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1 Introduction

Let (M_n, g) be a Riemannian manifold of dimension n. A linear connection ∇ in (M_n, g), whose torsion tensor T of type (1, 2) is defined as

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y], \tag{1.1}$$

for arbitrary vector fields X and Y, is said to be torsion free or symmetric if T vanishes, otherwise it is non-symmetric. If the connection ∇ satisfy $\nabla g = 0$ in (M_n, g), then it is called metric connection, otherwise it is non-metric.

In 1972, K. Kenmotsu [25] introduced a class of contact Riemann manifold known as Kenmotsu Manifold. He studied that if a Kenmotsu manifold satisfy the condition $R(X, Y)Z = 0$, then the manifold is of negative curvature -1, where R is the Riemannian curvature tensor of type (1, 3) and $R(X, Y)Z$ is derivative of tensor algebra at each point of the tangent space. Several properties of Kenmotsu Manifold have been studied by De and Pathak [8], De [9], Sinha and Srivastav [35], De and Pathak [8]and many others. Ozgur studied generalised recurrent Kenmotsu manifold and proved that if M be a generalised recurrent Kenmotsu manifold and generalised Ricci Recurrent Kenmotsu manifold then $\beta = \alpha$ holds on M. Sular studies the generalised recurrent and generalised Ricci recurrent Kenmotsu manifol with respect to semi symmetric metric connection and proved that $\beta = 2\alpha$ where α and β are smooth functions and M is generalised recurrent and generalised Ricci recurrent Kenmotsu manifold admitting a semi-symmetric connection. In the present chapter, we studied the properties of semi-symmetric non- metric connection in Kenmotsu manifold.

2 Generalized Recurrent Kenmotsu Manifold

Definition 2.1. A non-flat n-dimensional differentiable manifold M_n, ($n > 3$), is called generalized recurrent manifold if its curvature tensor K satisfies the condition

$$(D_X K)(Y, Z)W = A(X)K(Y, Z)W + B(X)\,[g(Z, W)Y - g(Y, W)Z], \qquad (2.1)$$

where A and $B \neq 0$ are 1-forms defined as

$$A(X) = g(X, \rho_1), \quad B(X) = g(X, \rho_2), \qquad (2.2)$$

for arbitrary vector fields X, Y, Z and W. Here ρ_1 and ρ_2 are the vector fields associated with the 1-forms A and B respectively.

Definition 2.2. A non-flat n-dimensional differentiable manifold M_n, ($n > 3$), is called generalized Ricci-recurrent if its Ricci tensor S satisfies the condition

$$(D_X S)(Y, Z) = A(X)S(Y, Z) + (n - 1)B(X)g(Y, Z), \qquad (2.3)$$

for arbitrary vector fields X, Y and Z, where A and B are defined as in (2.2).

In the similar fashion, we defined the following definitions:

Definition 2.3. A non-flat n-dimensional differentiable manifold M_n, ($n > 3$), is called generalized recurrent with respect to the semi-symmetric non-metric connection ∇ if its curvature tensor R satisfies the condition

$$(\nabla_X R)(Y, Z)W = A(X)R(Y, Z)W + B(X)\,[g(Z, W)Y - g(Y, W)Z], \qquad (2.4)$$

for arbitrary vector fields X, Y, Z and W.

Definition 2.4. A non-flat n-dimensional differentiable manifold M_n, ($n > 3$), is called generalized Ricci-recurrent with respect to the semi-symmetric non-metric connection ∇ if its Ricci tensor $\tilde{S}$ satisfies the condition

$$(\nabla_X \tilde{S})(Y, Z) = A(X)\tilde{S}(Y, Z) + (n - 1)B(X)g(Y, Z), \qquad (2.5)$$

for arbitrary vector fields X, Y and Z, where A and B are defined as in (2.2).

Now we consider the generalized recurrent and generalized Ricci-recurrent Kenmotsu manifold admitting the semi-symmetric non-metric connection ∇ and prove the following theorems:

Theorem 2.5. Let M_n be an n-dimensional generalized recurrent Kenmotsu manifold equipped with a semi-symmetric non-metric connection where A and B. Then $B = A$ holds on M_n where A and B are 1-forms on the manifold.

Theorem 2.6. If an n-dimensional generalized Ricci-recurrent Kenmotsu manifold M_n admitting a semi-symmetric non-metric connection ∇, then $B = A$ holds on M_n, where A and B are fundamental 1-forms on M_n.

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