A NOTE ON SUM SQUARE PRIME LABELING

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Abstract*— Sum square prime labeling of a graph is the labeling of the vertices with {0,1,2-------,p-1} and the edges with square of the sum of the labels of the incident vertices. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor (gcd) of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits sum square prime labeling. Here we identify some graphs for sum square prime labeling.

***Index Terms*— Graph labeling, greatest common incidence number, prime labeling, sum square.**

I. INTRODUCTION

All graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In [5], we introduced the concept of sum square prime labeling and proved that some path related graphs admit sum square prime labeling. In [6], [7] and [8], we proved some tree related graphs, snake related graphs and cycle related graphs also admit sum square prime labeling. In this paper we investigated sum square prime labeling of some other graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection
f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{ssp}^* : E(G) → set of natural numbers N by $f_{ssp}^*(uv) = \{f(u) + f(v)\}^2$. The induced function f_{ssp}^* is said to be a sum square prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits sum square prime labeling is called a sum square prime graph.

Theorem 2.1 Triangular belt TB(↑↑ − − −↑) admits sum square prime labeling.

Proof: Let G = TB(↑↑ − − −↑) and let v_1, v_2,----------------,v_{2n} are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 4n-3
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_i\, v_{i+1})$	$= (2i-1)^2$,	i = 1,2,------------,2n
$f_{ssp}^*(v_{2i-1}\, v_{2i+1})$	$= (4i-2)^2$,	i = 1,2,------------,n-1
$f_{ssp}^*(v_{2i}\, v_{2i+2})$	$= (4i)^2$,	i = 1,2,------------,n-1

Clearly f_{ssp}^* is an injection.

gcin of (v_1) = gcd of $\{f_{ssp}^*(v_1\, v_2) , f_{ssp}^*(v_1\, v_3)\}$
= gcd of { 1, 4}
= 1.

gcin of (v_{i+1}) = gcd of $\{f_{ssp}^*(v_i\, v_{i+1}) , f_{ssp}^*(v_{i+1}\, v_{i+2})\}$
= gcd of $\{ (2i-1)^2, (2i+1)^2\}$
= gcd of {2i-1 , 2i+1} = 1, i = 1,2,------------,2n-2

gcin of (v_{2n}) = gcd of $\{f_{ssp}^*(v_{2n-1}\, v_{2n}) , f_{ssp}^*(v_{2n-2}\, v_{2n})\}$
= gcd of $\{ (4n-3)^2, (4n-4)^2\}$
= gcd of { (4n-3), (4n-4)}
= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence TB(↑↑ − − −↑) , admits sum square prime labeling. ■

Example 2.1 G = TB(↑↑↑↑)

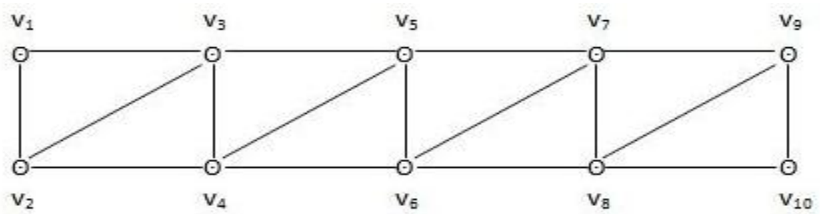

fig – 2.1

Theorem 2.2 Jelly fish graph JF(n,m) admits sum square prime labeling.

Proof: Let G = JF(n,m) and let v_1,v_2,----------------,v_{n+m+4} are the vertices of G

Here |V(G)| = n+m+4 and |E(G)| = n+m+5

Define a function f : V → {0,1,2,3,----------------,m+n+3 } by

$f(v_i) = i-1$, i = 1,2,------,m+n+4

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_{n+1}\ v_i)$ = $(n+i-1)^2$, i = 1,2,------------,n

$f_{ssp}^*(v_{n+1}\ v_{n+2})$ = $(2n+1)^2$.

$f_{ssp}^*(v_{n+1}\ v_{n+3})$ = $(2n+2)^2$.

$f_{ssp}^*(v_{n+3}\ v_{n+2})$ = $(2n+3)^2$.

$f_{ssp}^*(v_{n+4}\ v_{n+2})$ = $(2n+4)^2$.

$f_{ssp}^*(v_{n+4}\ v_{n+3})$ = $(2n+5)^2$.

$f_{ssp}^*(v_{n+4}\ v_{n+4+i})$ = $(2n+6+i)^2$, i = 1,2,------------,m

Clearly f_{ssp}^* is an injection

gcin of (v_{n+2}) = gcd of $\{f_{ssp}^*(v_{n+2}\ v_{n+3}), f_{ssp}^*(v_{n+2}\ v_{n+4})\}$
= gcd of { $(2n+3)^2$,$(2n+4)^2$}
= gcd of { 2n+3, 2n+4}
= 1.

gcin of (v_{n+1}) = gcd of $\{f_{ssp}^*(v_{n+1}\ v_1), f_{ssp}^*(v_{n+1}\ v_2)\}$
= gcd of { n^2,$(n+1)^2$}
= gcd of { n, n+1}
= 1.

gcin of (v_{n+3}) = gcd of $\{f_{ssp}^*(v_{n+1}\ v_{n+3}), f_{ssp}^*(v_{n+2}\ v_{n+3})\}$
= gcd of { $(2n+2)^2$,$(2n+3)^2$}
= gcd of { 2n+2, 2n+3}
= 1.

gcin of (v_{n+4}) = gcd of $\{f_{ssp}^*(v_{n+4}\ v_{n+5}), f_{ssp}^*(v_{n+4}\ v_{n+6})\}$
= gcd of { $(2n+7)^2$,$(2n+8)^2$}
= gcd of { 2n+7, 2n+8}
= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence JF(n,m), admits sum square prime labeling. ■

Example 2.2 G = JF(3,4)

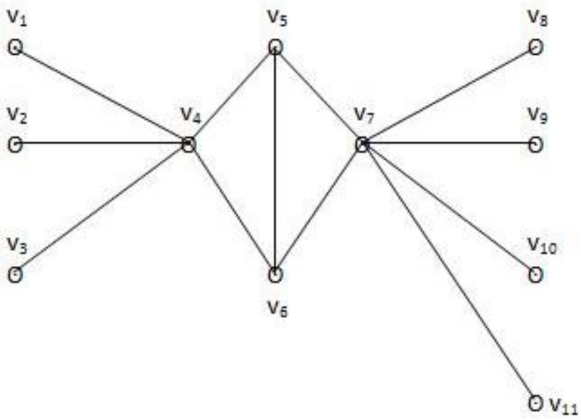

fig – 2.2

Definition 2.1 G be the graph obtained by adding pendant edges alternately to the vertices of path P_n . G is denoted by the symbol $P_n \odot A(K_1)$.

Theorem 2.3 $P_n \odot A(K_1)$ admits sum square prime labeling, when n is odd and pendant edge starts from the first vertex.

Proof: Let G = $P_n \odot A(K_1)$ and let v_1,v_2,----------------,$v_{\frac{3n+1}{2}}$ are the vertices of G

Here |V(G)| = $\frac{3n+1}{2}$ and |E(G)| = $\frac{3n-1}{2}$

Define a function f : V → {0,1,2,3,----------------, $\frac{3n-1}{2}$ } by

$f(v_i) = i-1$, i = 1,2,------, $\frac{3n+1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_{3i-2}\ v_{3i-1})$ = $(6i-5)^2$, i = 1,2,------------,$\frac{n+1}{2}$

$f_{ssp}^*(v_{3i-2}\ v_{3i})$ = $(6i-4)^2$, i = 1,2,------------,$\frac{n-1}{2}$

$f_{ssp}^*(v_{3i+1}\ v_{3i})$ = $(6i-1)^2$, i = 1,2,------------,$\frac{n-1}{2}$

Clearly f_{ssp}^* is an injection

gcin of (v_{3i}) $= \text{gcd of } \{f_{ssp}^*(v_{3i-2}\, v_{3i}), f_{ssp}^*(v_{3i}\, v_{3i+1})\}$
$= \text{gcd of } \{ (6i-4)^2, (6i-1)^2\},$
$= \text{gcd of } \{ 6i-4, 6i-1\}$
$= \text{gcd of } \{ 6i-4, 3\}$
$= 1,$ $i = 1,2,\text{------------},\frac{n-1}{2}$

gcin of (v_{3i-2}) $= \text{gcd of } \{f_{ssp}^*(v_{3i-2}\, v_{3i}), f_{ssp}^*(v_{3i-2}\, v_{3i-1})\}$
$= \text{gcd of } \{ (6i-4)^2, (6i-5)^2\}$
$= \text{gcd of } \{ 6i-4, 6i-5\}$
$= \text{gcd of } \{ 6i-5, 1\}$
$= 1,$ $i = 1,2,\text{------------},\frac{n-1}{2}$

gcin of $(v_{\frac{3n-1}{2}})$ $= \text{gcd of } \{f_{ssp}^*(v_{\frac{3n-3}{2}}\, v_{\frac{3n-1}{2}}), f_{ssp}^*(v_{\frac{3n+1}{2}}\, v_{\frac{3n-1}{2}})\}$
$= \text{gcd of } \{ (3n-4)^2, (3n-2)^2\}$
$= \text{gcd of } \{ 3n-4, 3n-2\}$
$= \text{gcd of } \{ 3n-4, 2\} = 1.$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \odot A(K_1)$, admits sum square prime labeling. ■

Example 2.3 $G = P_7 \odot A(K_1)$

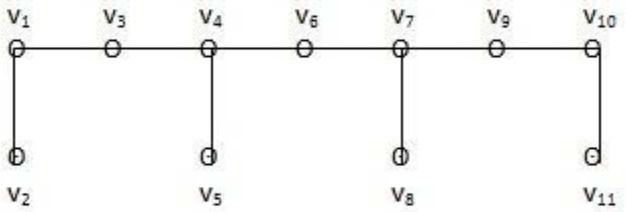

fig – 2.3

Theorem 2.4 $P_n \odot A(K_1)$ admits sum square prime labeling, when n is odd and pendant edge starts from the second vertex.

Proof: Let $G = P_n \odot A(K_1)$ and let $v_1, v_2, \text{----------------}, v_{\frac{3n-1}{2}}$ are the vertices of G

Here $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = \frac{3n-3}{2}$

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------}, \frac{3n-3}{2}\}$ by

$f(v_i) = i-1$, $i = 1,2,\text{------}, \frac{3n-1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_{3i-2}\, v_{3i-1}) = (6i-5)^2,$ $i = 1,2,\text{------------},\frac{n-1}{2}$

$f_{ssp}^*(v_{3i-2}\, v_{3i}) = (6i-4)^2,$ $i = 1,2,\text{------------},\frac{n-3}{2}$

$f_{ssp}^*(v_{3i+1}\, v_{3i}) = (6i-1)^2,$ $i = 1,2,\text{------------},\frac{n-3}{2}$

$f_{ssp}^*(v_1\, v_{\frac{3n-1}{2}}) = (\frac{3n-3}{2})^2.$

$f_{ssp}^*(v_{\frac{3n-7}{2}}\, v_{\frac{3n-3}{2}}) = (3n-7)^2.$

Clearly f_{ssp}^* is an injection

gcin of (v_{3i}) $= 1,$ $i = 1,2,\text{------------},\frac{n-3}{2}$

gcin of (v_{3i+1}) $= \text{gcd of } \{f_{ssp}^*(v_{3i+1}\, v_{3i}), f_{ssp}^*(v_{3i+1}\, v_{3i+2})\}$
$= \text{gcd of } \{ (6i-1)^2, (6i+1)^2\}$
$= \text{gcd of } \{ 6i-1, 6i+1\}$
$= \text{gcd of } \{ 6i-1, 2\} = 1,$ $i = 1,2,\text{------------},\frac{n-3}{2}$

gcin of (v_1) $= \text{gcd of } \{f_{ssp}^*(v_1\, v_2), f_{ssp}^*(v_1\, v_3)\}$
$= \text{gcd of } \{ 1,4\} = 1.$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \odot A(K_1)$, admits sum square prime labeling. ■

Example 2.4 $G = P_7 \odot A(K_1)$

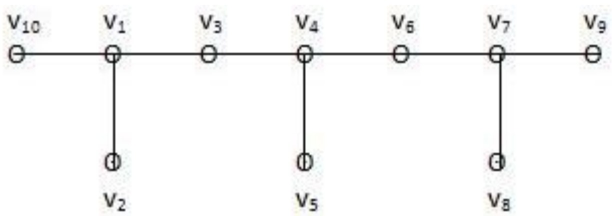

fig – 2.4

Theorem 2.5 $P_n \odot A(K_1)$ admits sum square prime labeling, when n is even.

Proof: Let $G = P_n \odot A(K_1)$ and let $v_1, v_2, \text{---------------}, v_{\frac{3n}{2}}$ are the vertices of G

Here $|V(G)| = \frac{3n}{2}$ and $|E(G)| = \frac{3n-2}{2}$

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------}, \frac{3n-2}{2}\}$ by

$f(v_i) = i-1$, $\quad i = 1,2,\text{------}, \frac{3n}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_{3i-2}\, v_{3i-1}) = (6i-5)^2$, $\quad i = 1,2,\text{------------}, \frac{n}{2}$

$f_{ssp}^*(v_{3i-2}\, v_{3i}) = (6i-4)^2$, $\quad i = 1,2,\text{------------}, \frac{n}{2}$

$f_{ssp}^*(v_{3i+1}\, v_{3i}) = (6i-1)^2$, $\quad i = 1,2,\text{------------}, \frac{n-2}{2}$

Clearly f_{ssp}^* is an injection

gcin of $(v_{3i}) = 1$, $\quad i = 1,2,\text{------------}, \frac{n-2}{2}$

gcin of $(v_{3i-2}) = 1$, $\quad i = 1,2,\text{------------}, \frac{n}{2}$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \Theta A(K_1)$, admits sum square prime labeling. ■

Example 2.5 $G = P_7 \Theta A(K_1)$

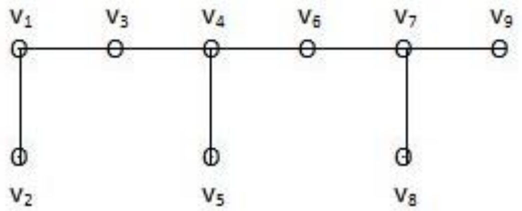

fig – 2.5

Theorem 2.6 Tensor product of star $K_{1,n}$ and path P_2 admits sum square prime labeling.

Proof : Let $G = K_{1,n} \otimes P_2$ and let a, b, $v_1,v_2,\text{----------------},v_n$, $u_1,u_2,\text{----------------},u_n$ are the vertices of G.

Here $|V(G)| = 2n+2$ and $|E(G)| = 2n$

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},2n+1\}$ by

$f(v_i) = n+i+1$, $\quad i = 1,2,\text{------},n$

$f(u_i) = i+1$, $\quad i = 1,2,\text{------},n$

$f(a) = 0$, $f(b) = 1$.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(a\, u_i) = (i+1)^2$, $\quad i = 1,2,\text{------------},n$

$f_{ssp}^*(b\, v_i) = (n+i+2)^2$, $\quad i = 1,2,\text{------------},n$

Clearly f_{ssp}^* is an injection

gcin of (a) = 1,

gcin of (b) = 1,

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $K_{1,n} \otimes P_2$ admits sum square prime labeling. ■

Example 2.6 $G = K_{1,4} \otimes P_2$

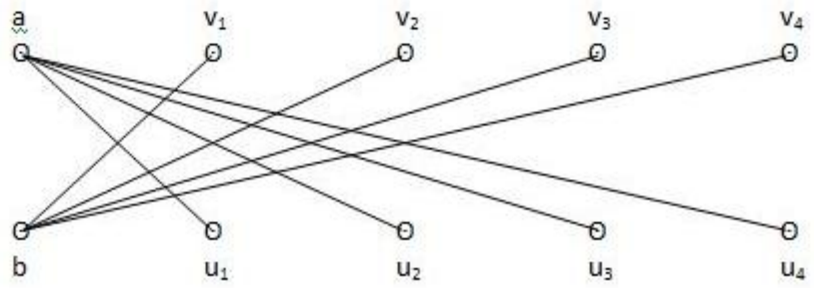

fig – 2.6

Theorem 2.7 Two copies of cycle C_n sharing a common vertex admits sum square prime labeling, when n is odd and $(n-2) \not\equiv 0 \pmod 3$.

Proof : Let $G = 2(C_n) - v$ and let $v_1,v_2,\text{----------------},v_{2n-1}$ are the vertices of G.

Here $|V(G)| = 2n-1$ and $|E(G)| = 2n$

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},2n-2\}$ by

$f(v_i) = i-1$, $\quad i = 1,2,\text{------},2n-1$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_i\, v_{i+1}) = (2i-1)^2$, $\quad i = 1,2,\text{------------},2n-2$

$f_{ssp}^*(v_1\, v_n) = (n-1)^2$.

$f_{ssp}^*(v_{2n-1}\, v_n) = (3n-3)^2$.

Clearly f_{ssp}^* is an injection

gcin of $(v_1) = $ gcd of $\{f_{ssp}^*(v_1\, v_2), f_{ssp}^*(v_1\, v_n)\}$

= gcd of $\{1,(n-1)^2\}$ = 1.

gcin of (v_{i+1}) = 1, i = 1,2,------------,2n-3

gcin of (v_{2n-1}) = gcd of $\{f_{ssp}^*(v_{2n-1}\ v_n)\ ,\ f_{ssp}^*(v_{2n-2}\ v_{2n-1})\ \}$

= gcd of $\{\ (3n-3)^2, (4n-5)^2\}$

= gcd of { (3n-3), (4n-5)}

= gcd of { (3n-3), (n-2)}

= gcd of { 3, (n-2)}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $2(C_n)$ - v admits sum square prime labeling. ■

Example 2.7 G = $2(C_7)$ - v

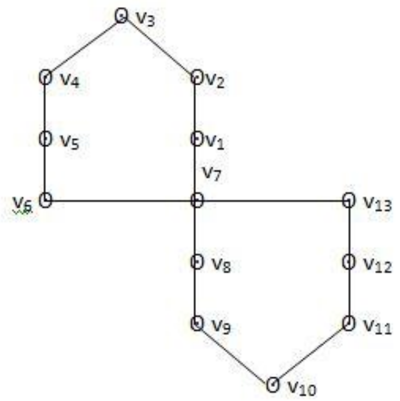

fig – 2.7

Theorem 2.8 Let G be the graph obtained by joining two copies of cycle C_n by path P_n . G admits sum square prime labeling, when n is odd and $(n+3) \not\equiv 0(\text{mod}10)$.

Proof : Let G = $2(C_n) \cup P_n$ and let v_1,v_2,----------------,v_{3n-2} are the vertices of G.

Here |V(G)| = 3n-2 and |E(G)| = 3n-1

Define a function f : V → {0,1,2,3,----------------,3n-3} by

$f(v_i)$ = i-1 , i = 1,2,------,3n-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_i\ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,3n-3

$f_{ssp}^*(v_1\ v_n)$ = $(n-1)^2$.

$f_{ssp}^*(v_{2n-1}\ v_{3n-2})$ = $(5n-5)^2$.

Clearly f_{ssp}^* is an injection

gcin of (v_1) = 1.

gcin of (v_{i+1}) = 1, i = 1,2,------------,3n-4

gcin of (v_{3n-2}) = gcd of $\{f_{ssp}^*(v_{2n-1}\ v_{3n-2})\ ,\ f_{ssp}^*(v_{3n-2}\ v_{3n-3})\ \}$

= gcd of $\{\ (5n-5)^2, (6n-7)^2\}$

= gcd of { (5n-5), (6n-7)}

= gcd of { (n-2), (5n-5)}

= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $2(C_n) \cup P_n$ admits sum square prime labeling. ■

Example 2.8 G = $2(C_5) \cup P_5$

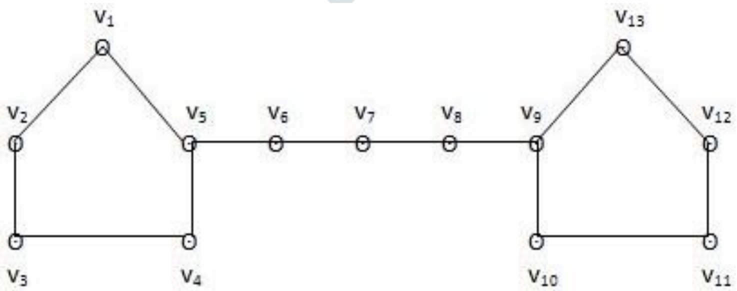

fig – 2.8

Theorem 2.9 Splitting graph of star $K_{1,n}$ admits sum square prime labeling, when n is even and $(n-2) \not\equiv 0(\text{mod}6)$.

Proof : Let G = $S'(K_{1,n})$ and let a, v_1,v_2,----------------,v_{2n+1} are the vertices of G.

Here |V(G)| = 2n+2 and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n+1} by

$f(v_i)$ = i , i = 1,2,------,2n+1

f(a) = 0.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(a\ v_i)$ = $(i)^2$, i = 1,2,------------,n

$f_{ssp}^*(v_{n+1}\, v_i)$ = $(n+i+1)^2$, i = 1,2,------------,n
$f_{ssp}^*(v_{n+1}\, v_{n+i+1})$ = $(2n+i+2)^2$, i = 1,2,------------,n

Clearly f_{ssp}^* is an injection
gcin of (a) = 1.
gcin of (v_{n+1}) = 1.
gcin of (v_i) = gcd of $\{f_{ssp}^*(av_i)\,,\, f_{ssp}^*(v_{n+1}\, v_i)\}$
= gcd of $\{(i)^2, (n+i+1)^2\}$
= gcd of {i , n+i+1} = 1, i = 1,2,------------,n
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $S'(K_{1,n})$ admits sum square prime labeling. ■

Example 2.9 G = $S'(K_{1,4})$

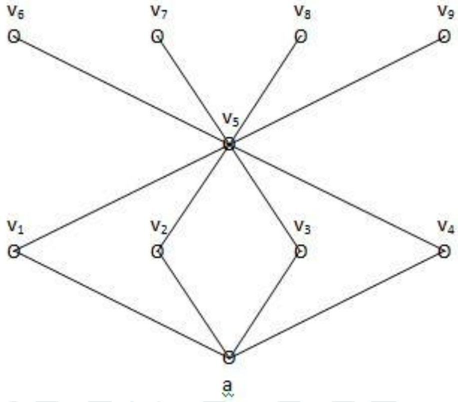

fig – 2.9

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