TWO UNIT WARM STANDBY SYSTEM WITH TWO TYPES FAILUR, FAULT DETECTION AND INSPECTION

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ABSTRACT

The Present paper deals with the analysis of a two unit warm standby system in which the unit can fail two types (caused by machine defect and critical human error). In this system after the failure of an operative unit the failed unit is sent for fault detection to decide about the types of failure so that the appropriate repair accordingly is to be provided. After each repair, the repaired unit is sent for inspection to decide whether the repair is perfect or not. It the repair is perfect then the unit becomes operative or warm standby. Otherwise, it is sent for post repair. Using the regenerative point technique the various reliability characteristics of the system model under study are obtained.

Kew words : Fault detection, availability, Markov Process, Busy period.

INTRODUCTION

Various authors including [1, 2, 6, 12] working in the field of reliability have analysed many engineering systems with the assumption that the failed unit is sent for repair without knowing the type of failure. Also after each repair the unit becomes operative or standby assuming that the repair is perfect. But in real practical situation there exist many engineering systems in which it will be better for system designers to make the system more effective is to send the failed unit for fault detection analysis to find out the type of failure so that appropriate repair accordingly to the fault is to be provided. Similarly after each repair the repaired unit should be sent for inspection to decide whether the repair is perfect or not. If the repair is imperfect then it is sent for post repair.

Keeping the above view in our mind we in this paper analysed a two unit warm standby system in which the unit can fail with two types (caused by machine defect and critical human error). In this system, if the operative unit fails then it is sent for fault detection to decide about the type of failure and each repaired unit is sent for inspection to decide whether the repair is perfect or not. If the repair is perfect the unit becomes operative or waram standby. Otherwise. It is sent for post repair.

By using the regenerative point technique in Markov renewal process for analysing the system and the following effective measures for the system model are obtained.

- (i) Steady state transition Probabilities.
- (ii) Mean sojourn time
- (iii) Mean time to system failure
- (iv) Pointwise and steady state availability of the system.
- (v) Expected busy period of the repairman in time interval (0, t]
- (vi) Expected number of visits by the repairman in time interval (0, t]

(vii) Profit analysis of the system.

MODEL DESCRIPTION AND ASSUMPTIONS

- The system consists of two identical units. Initially, one unit is operative and the other is on warm standby.
- (ii) Upon failure of an operative unit the warm standby unit becomes operative instantaneously.
- (iii) Upon failure of an operative unit it is sent for fault detection to decide whether the unit failed by machine defect or critical human error before sending it for repair.
- (iv) The warm standby unit can only fail due to machine defect, so there is no need to send it for fault detection analysis.
- (v) After the repair, a unit goes for inspection to decide whether the repair is perfect or not. If the repair of a unit is found to be perfect then the repaired unit becomes operational, otherwise it is sent for post repair. The probability of having perfect repair is fixed.
- (vi) Failure rater of both the units are constant but non-identical and the distribution of time to machine repair, C.H.E. repair and post repair are general.
- (vii) Rates of fault detection and inspection are constant.
- (viii) A single server facility with discipline 'FCFS' is used for repair, Fault detection, inspection and post repair.

NOTATIONS AND STATES OF THE SYSTEM

- α Constant failure rate of operative unit
- β Constant failure rate of warm standby unit.
- φ Constant rate of fault detection
- δ Constant rate of inspection.
- p Probability of commenting machine repair.
- q Probability of committing C.H.E. repair.
- a Probability that the repair is perfect after inspection.
- b Probability that the repair is imperfect after inspection.
- f(.), F(.) P.d.t and c.d.f. of machine repair
- g(.), G (.) P.d.f and c.d.f of C.H.E. repair
- h(.), H(.) P.d.f and c.d.f of post repair time
- m'_1 , m'_2 , m'_3 , Mean time for machine repair, C.H.E. repair,
- m_1 and m_2 post repair, inspection and fault detection.
 - *No* Normal unit kept as operative
 - N_S Normal unit kept as warm standby.
 - F_f Failed unit under fault detection analysis.
 - F_{rm} Failed unit under machine repair.
 - F_{rc} Failed unit under C.H.E repair.
 - F_F Fault detection is continued from earlier state.
 - F_{wf} Failed unit is waiting for fault detection

- F_I Repaired unit under inspection.
- F_{Ic} Inspection of repaired unit is continued from earlier state.
- F_P Failed unit under post repair.
- F_{PC} Post repair of a failed unit is continued from earlier state.
- F_{RM} Machine repair of a failed unit is continued from earlier state.
- F_{RC} C.H.E. repair of a failed unit is continued from earlier state.

Considering these notations the possible states of the system are:

Up States

$$S_o = (N_o, N_s)$$
, $S_1 = (N_o, F_I)$, $S_2 = (N_o, F_{rm})$

$$S_3 = (N_o, F_P)$$
, $S_4 = (F_r, N_o)$, $S_6 = (F_{rc}, N_o)$

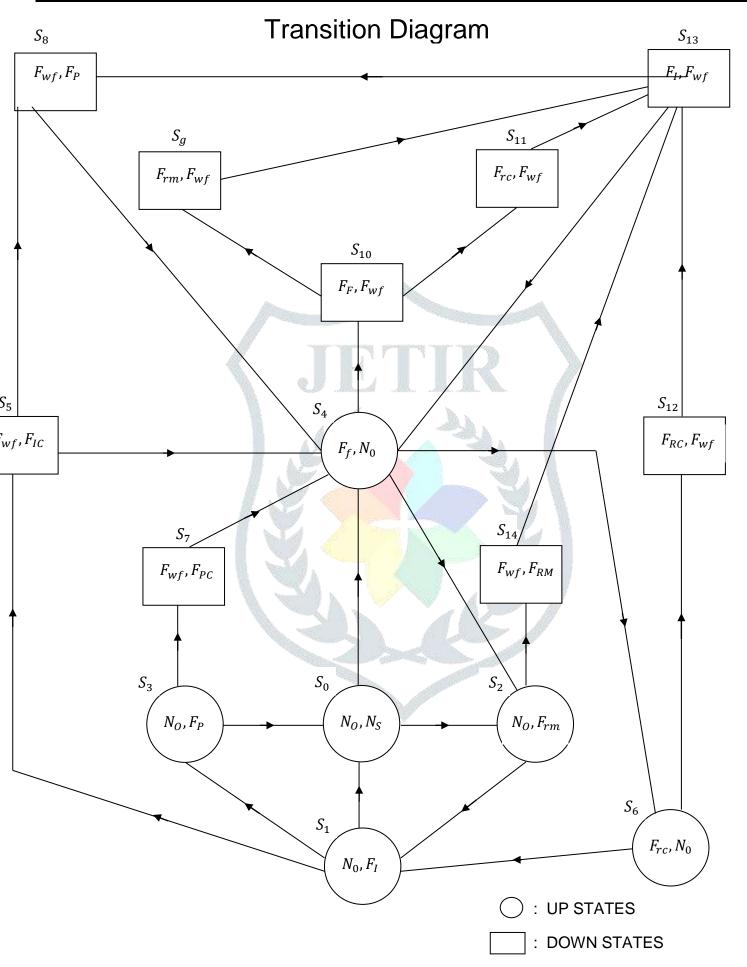
Down States

$$S_5 = (F_{wf}, F_{IC})$$
, $S_7 = (F_{wf}, F_{Pc})$, $S_8 = (F_{Wf}, F_P)$

$$S_9 = (F_{rm}, F_{wf}), \quad S_{10} = (F_F, F_{wf}), \quad S_{11} = (F_{rc}, F_{wf})$$

$$S_{12} = (F_{RC}, F_{wf})$$
, $S_{13} = (F_I, F_{wf})$, $S_{14} = (F_{wf}, F_{RM})$

The states S_0 , S_1 , S_2 , S_3 , S_4 , S_6 , S_8 , S_9 , S_{11} and S_{13} are regenerative while S_5 , S_7 , S_{10} , S_{12} and S_{14} are non-regenerative states. The possible transitions between the states are shown in the following figure.



Transition Probability and Sojourn Times

The non zero elements of the transition probability, P = (Pij) are siven below

$$P_{02} = \frac{\beta}{\alpha + \beta}, \quad P_{04} = \frac{\alpha}{\alpha + \beta}, \quad P_{10} = \frac{\alpha\delta}{\alpha + \delta}$$

$$P_{13} = \frac{b\delta}{\alpha + \delta}, \quad P_{15} = \frac{\alpha}{\alpha + \delta}, \quad P_{14}^{(5)} = \frac{a\alpha}{\alpha + \delta}$$

$$P_{18}^{(5)} = \frac{b\alpha}{\alpha + \delta}, \quad P_{21} = f^{*}(\alpha)$$

$$P_{2,14} = 1 - f^{*}(\alpha) = P_{2,13}^{(14)}, \quad P_{30} = h^{*}(\alpha)$$

$$P_{37} = 1 - h^{*}(\alpha) = P_{34}^{(7)}, \quad P_{42} = \frac{p\phi}{\alpha + \phi}$$

$$P_{46} = \frac{q\phi}{\alpha + \phi}, \quad P_{49}^{(10)} = P - \frac{P\phi}{\alpha + \phi}$$

$$P_{4,10} = \frac{\alpha}{\alpha + \phi}, \quad P_{4,11}^{(10)} = q - \frac{q\phi}{\alpha + \phi}$$

$$P_{54} = \alpha = P_{13,8}, \quad P_{58} = b = P_{13,4}$$

$$P_{61} = g^{*}(\alpha), \quad P_{6,12} = 1 - g^{*}(\alpha) = P_{6,13}^{(12)}$$

$$P_{7,14} = 1 = P_{84} = P_{9,13} = P_{12,13}, = P_{14,13} = P_{11,13}$$

 $P_{10,9} = p$, $P_{10,11} = q$

The above probabilities satisfies the following relations:

$$P_{02} + P_{04} = 1 = P_{10} + P_{13} + P_{15} = P_{10} + P_{13} + P_{14}^{(5)} + P_{18}^{(5)}$$

$$P_{21} + P_{2,14} = 1 = P_{21} + P_{2,13}^{(14)}$$

$$P_{30} + P_{37} = 1 = P_{30} + P_{34}^{(7)}$$

$$P_{42} + P_{46} + P_{4,10} = 1 = P_{42} + P_{46} + P_{49}^{(10)} + P_{4,11}^{(10)}$$

$$P_{54} + P_{58} = 1 = P_{61} + P_{6,12} = P_{61} + P_{6,13}^{(12)}$$

$$P_{10,9} + P_{10,11} = 1 = P_{13,4} + P_{13,8}$$

Als, the mean sojourn times are:

$$\mu_{0} = \frac{1}{\alpha + \beta} , \ \mu_{1} = \frac{1}{\alpha + \delta} , \ \mu_{2} = \frac{1 - f^{*}(\alpha)}{\alpha}$$

$$\mu_{3} = \frac{1 - h^{*}(\alpha)}{\alpha} , \ \mu_{4} = \frac{1}{\alpha + \phi} , \ \mu_{6} = \frac{1 - g^{*}(\alpha)}{\alpha}$$

$$\mu_{8} = \int_{0}^{\infty} t h(t) dt , \ \mu_{9} = \int_{0}^{\infty} t f(t) dt$$

$$\mu_{11} = \int_{0}^{\infty} t g(t) dt , \ \mu_{13} = \frac{1}{\delta}$$

and conditional mean sojourn times are

$$m_{02} = \frac{\beta}{(\alpha + \beta)^2}$$
, $m_{04} = \frac{\alpha}{(\alpha + \beta)^2}$, $m_{10} = \frac{a\delta}{(\alpha + \delta)^2}$

$$m_{13} = \frac{b\delta}{(\alpha + \delta)^2}$$
 , $m_{14}^{(5)} = \frac{\alpha}{\delta} - \frac{a\delta}{(\alpha + \delta)^2}$

$$m_{15} = \alpha / (\alpha + \delta)^2$$
 , $m_{18}^{(5)} = \alpha / \delta - \frac{b\delta}{(\alpha + \delta)^2}$

$$\begin{split} m_{21} &= \int t e^{-\alpha t} dt \quad , \quad m_{2,13}^{(14)} = \int t (1 - e^{-\alpha t}) f(t) dt \\ m_{2,14} &= \alpha \int t e^{-\alpha t} \bar{F}(t) dt \quad , \quad m_{30} = \int t e^{-\alpha t} h(t) dt \\ m_{34}^{(7)} &= \int t (1 - e^{\alpha t}) h(t) dt \quad , \quad m_{37} = \alpha \int t e^{-\alpha t} \bar{H}(t) dt \\ m_{42} &= \frac{P \Phi}{(\alpha + \Phi)^2} \quad , \quad m_{46} = \frac{q \Phi}{(\alpha + \Phi)^2} \\ m_{49}^{(10)} &= \frac{P}{\Phi} - \frac{P \Phi}{(\alpha + \Phi)^2} \quad , \quad m_{4,11}^{(10)} = \frac{q}{\Phi} - \frac{q \Phi}{(\alpha + \Phi)^2} \\ m_{61} &= \int t e^{-\alpha t} g(t) dt \quad , \quad m_{6,13}^{(12)} = \int t (1 - e^{-\alpha t}) g(t) dt \\ m_{84} &= \int t h(t) dt \quad , \quad m_{9,13} = \int t f(t) dt \\ m_{11,13} &= \int t g(t) dt \quad , \quad m_{13,4} = \frac{b}{\delta} \\ m_{13,8} &= \frac{a}{\delta} \end{split}$$

It can easily verified that

$$m_{02} + m_{04} = \mu_0$$

$$m_{10} + m_{13} + m_{15} = \mu_1$$

$$m_{10} + m_{13} + m_{14}^{(4)} + m_{18}^{(5)} = \frac{1}{\delta} = m_1$$

$$m_{21} + m_{2,14} = \mu_2$$

$$\begin{split} m_{21} + m_{2,13}^{(14)} &= \int t f(t) dt = m'_1 \\ m_{30} + m_{37} &= \mu_3 \\ m_{30} + m_{34}^{(7)} &= \int t h(t) dt = m'_3 \\ m_{42} + m_{46} + m_{4,10} &= \mu_4 \\ m_{42} + m_{46} + m_{49}^{(10)} + m_{4,11}^{(10)} &= 1/_{\bigoplus} = m_2 \\ m_{61} + m_{6,12} &= \mu_6 , m_{61} + m_{6,13}^{(12)} &= \int t g(t) dt = m'_2 \\ m_{84} &= \int t h(t) dt = m'_3 , m_{9,13} &= \int t f(t) dt = m'_1 \\ m_{11,13} &= \int t g(t) dt = m'_2 \\ m_{13,4} + m_{13,8} &= 1/_{\delta} = \mu_{13} \end{split}$$

MEAN TIME TO SYSTEM FAILURE

To find MTSF, We consider that the failed states Sj (j=5, 7, 8, 9, 10, 11, 12, 13, 14) are absorbing. Using the simple probabilistic arguments, we get

$$\pi_0(t) = Q_{02}(t) \$ \pi_2(t) + Q_{04}(t) \$ \pi_4(t)$$

$$\pi_1(t) = Q_{10}(t) \$ \pi_0(t) + Q_{13}(t) \$ \pi_3(t) + Q_{15}(t)$$

$$\pi_2(t) = Q_{21}(t) \$ \pi_1(t) + Q_{2,14}(t)$$

 $\pi_3(t) = Q_{30}(t) \$ \pi_0(t) + Q_{37}(t)$

$$\pi_4(t) = Q_{42}(t) \$ \pi_2(t) + Q_{46}(t) \$ \pi_6(t) + Q_{4,10}(t)$$

$$\pi_6(t) = Q_{61}(t) \$ \pi_1(t) + Q_{6,12}(t)$$

(1-6)

Taking Laplace - Stieltjes transform of the of the relation (1 – 6) and solving for $\tilde{\pi}_0(s)$ and omitting the argument 'S' for brevity, we get

MTSF = E (T) =
$$\frac{d\tilde{\pi}_0(s)}{ds}\Big|_{s=0} = \frac{N_1}{D_1}$$
 (7)

Where

$$\begin{split} N_{1} &= \mu_{0} + \mu_{1} [P_{02} P_{21} + P_{04} P_{42} P_{21} + P_{04} P_{46} P_{61}] + \mu_{2} (P_{02} + P_{04} P_{42}) \\ &+ \mu_{3} [P_{02} P_{21} P_{13} + P_{04} P_{42} P_{21} P_{13} + P_{04} P_{46} P_{61} P_{13}] + \mu_{4} P_{04} \\ &+ \mu_{6} P_{04} P_{46} \end{split}$$

and

$$D_{1} = 1 - P_{02}P_{21}P_{10} - P_{02}P_{21}P_{13}P_{30} - P_{04}P_{42}P_{21}P_{10} - P_{04}P_{42}P_{21}P_{13}P_{30}$$

$$-P_{04}P_{46}P_{61}P_{10} - P_{04}P_{46}P_{61}P_{13}P_{30}$$

AVAILABILITY ANALYSIS

Define $M_i(t)$, as the probability that the system starting from up state $S_i \in E$ remains up till time t without passing through any regenerative state Thus, we have

$$M_{\alpha}(t) = e^{-(\alpha+\beta)t}$$
, $M_{1}(t) = e^{-(\alpha+\delta)t}$

$$M_2(t) = e^{-\alpha t} \overline{F}(t)$$
, $M_3(t) = e^{-\alpha t} \overline{H}(t)$

$$M_4(t) = e^{-(\alpha + \phi)t}$$
, $M_6(t) = e^{-\alpha t} \bar{G}(t)$

Using the theory of regenerative process, the set of recursive relation of point wise availability $A_i(t)$ are

$$A_o(t) = M_0(t) + q_{02}(t) \odot A_2(t) + q_{04}(t) \odot A_4(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{13}(t) \odot A_3(t) + q_{14}^{(5)}(t) \odot A_4(t)$$

$$+q_{18}^{(5)}(t)$$
© $A_8(t)$

$$A_{2}(t) = M_{2}(t) + q_{21}(t) \odot A_{1}(t) + q_{2,13}^{(14)}(t) \odot A_{13}(t)$$

$$A_{3}(t) = M_{3}(t) + q_{30}(t) \odot A_{0}(t) + q_{34}^{(7)}(t) \odot A_{4}(t)$$

$$A_4(t) = M_4(t) + q_{42}(t) \otimes A_2(t) + Q_{46}(t) \otimes A_6(t) + q_{49}^{(10)}(t) \otimes A_9(t)$$

 $+Q_{4,11}^{(10)}(t) \odot A_{11}(t)$

$$A_6(t) = M_6(t) + q_{61}(t) \otimes A_1(t) + q_{6,13}^{(12)}(t) \otimes A_{13}(t)$$

$$A_8(t) = q_{84}(t) \odot A_4(t)$$

$$A_9(t) = q_{9,13}(t) \odot A_{13}(t)$$

$$A_{11}(t) = q_{11,13}(t) \odot A_{13}(t)$$

$$A_{13}(t) = q_{13,4}(t) \odot A_4(t) + q_{13,8}(t) \odot A_8(t)$$
 (8-17)

Taking Laplace transform of relation (8 - 17) and solving for $A_0^*(s)$ by omitting the argument 's' for brevity, we have

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$
(18)

The steady state availability, when the system starts operation from S_0 is obtained as follows

$$A_0 = \lim_{s \to 0} s A_0^*(s) = \frac{N_2(0)}{D'_2(0)} = \frac{N_2}{D_2}$$
(19)

Where

$$N_2 = (P_{46}P_{61} + P_{42}P_{21}) \left[\mu_0(P_{10} + P_{13}P_{30}) + \mu_1 \{1 - P_{02}P_{21} + 2P_{02}P_{21} \times P_{02}P_{21} + P_{02}P_{21} + 2P_{02}P_{21} + 2P_{02}P_{21} \times P_{02}P_{21} + 2P_{02}P_{21} + 2P_{02}P_{22} + 2P_$$

$$(P_{10} + P_{13}P_{30})\} + \mu_2 P_{02}(P_{10} + P_{13}P_{30}) + \mu_3 \{P_{13}(1 - P_{02}P_{21})\}$$

$$+2P_{02}P_{21}P_{13}(P_{10}+P_{13}P_{30})\}]+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{02}P_{21}+P_{13}P_{13}P_{13})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{21})\}\{1-P_{12}P_{21}+P_{13}P_{21})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{22})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{22})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{22})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{22})\}+\{\mu_4+(P_{46}P_{61}+P_{42}P_{22})\}+\{\mu_4+(P_{46}P_{61}+P_{42})+\{\mu_4+(P_{46}P_{61}+P_{42})+\{\mu_4+(P_{46}P_{61}+P_{42})+\{\mu_4+(P_{46}P_{61}+P_{42})+$$

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 $+P_{02}P_{21}(P_{10}+P_{13}P_{30})\}$

And

$$D_2 = (P_{46}P_{61} + P_{42}P_{21})\{\mu_0(P_{10} + P_{13}P_{30}) + m_1\}$$

$$+m_{2}\{1-P_{02}P_{21}(P_{10}+P_{13}P_{30})\}+m'_{1}\left\{P_{42}+P_{49}^{(10)}+(P_{02}P_{46}P_{61}+P_{13}P_{13})+m'_{1}\right\}$$

$$-P_{02}P_{21}P_{49}^{(10)})(P_{10} + P_{13}P_{30})\} + m'_{2}\left\{P_{46} + P_{4,11}^{(10)} - (P_{02}P_{46}P_{61} + P_{13}P_{30})\right\} + m'_{2}\left\{P_{46} + P_{4,11}^{(10)} - (P_{10}P_{46}P_{61} + P_{13}P_{61})\right\} + m'_{2}\left\{P_{46} + P_{4,11}^{(10)} - (P_{10}P_{46}P_{61} + P_{13}P_{61})\right\} + m'_{2}\left\{P_{46} + P_{4,11}^{(10)} - (P_{10}P_{46}P_{61} + P_{13}P_{61})\right\} + m'_{2}\left\{P_{46} + P_{4,11}^{(10)} - (P_{10}P_{46}P_{61} + P_{14}P_{61})\right\} + m'_{2}\left\{P_{46} + P_{4,11}^{(10)} - (P_{10}P_{46}P_{61} + P_{14}P_{61})\right\} + m'_{2}\left\{P_{46} + P_{4,11}^{(10)} - (P_{10}P_{46}P_{61} + P_{14}P_{61})\right\}$$

$$P_{02}P_{21}P_{4,11}^{(10)})(P_{10} + P_{13}P_{30})\} + m'_{3}\{P_{13,8} + (P_{46}P_{61} + P_{42}P_{21}) \times$$

$$\left(P_{13} + P_{14}^{(5)} - P_{13,8}\right) + \mu_{13} \{1 - P_{42}P_{21} - P_{46}P_{61}(1 - P_{02}P_{10})\}$$

$$-P_{02}P_{21}(1-P_{42})(P_{10}+P_{13}P_{30})\}$$

BUSY PERIOD ANALYSIS

 $B_i(t)$ is defined as the probability that the repairman is busy at epoch t starting from the state $S_i \in E$. By using the probabilistic arguments, we get

$$B_{0}(t) = q_{02}(t) \odot B_{2}(t) + q_{04}(t) \odot B_{4}(t)$$

$$B_{1}(t) = W_{1}(t) + q_{10}(t) \odot B_{0}(t) + q_{13}(t) \odot B_{3}(t) + q_{14}^{(5)}(t) \odot B_{4}(t)$$

$$+ q_{18}^{(5)}(t) \odot B_{8}(t)$$

$$B_{2}(t) = W_{2}(t) + q_{21}(t) \odot B_{1}(t) + q_{2,13}^{(14)}(t) \odot B_{13}(t)$$

$$B_{3}(t) = W_{3}(t) + q_{30}(t) \odot B_{0}(t) + q_{34}^{(7)}(t) \odot B_{4}(t)$$

$$B_4(t) = W_4(t) + q_{42}(t) \otimes B_2(t) + q_{46}(t) \otimes B_6(t) + q_{49}^{(10)}(t) \otimes B_9(t)$$

 $+q_{4,11}^{(10)}(t) @B_{11}(t)$

$$B_6(t) = W_6(t) + q_{61}(t) \odot B_1(t) + q_{6,13}^{(12)}(t) \odot B_{13}(t)$$

$$B_8(t) = W_8(t) + q_{84}(t) \odot B_4(t)$$

$$B_9(t) = W_9(t) + q_{9,13}(t) \odot B_{13}(t)$$

$$B_{11}(t) = W_{11}(t) + q_{11,13}(t) \otimes B_{13}(t)$$

$$B_{13}(t) = W_{13}(t) + q_{13,4}(t) \odot B_4(t) + q_{13,8}(t) \odot B_8(t)$$

(20-29)

Where

$$W_1(t) = e^{-\delta t} = W_{13}(t)$$
, $W_2(t) = \overline{F}(t) = W_9(t)$

$$W_3(t) = \overline{H}(t) = W_8(t)$$
, $W_4(t) = e^{-\varphi t}$

 $W_6(t) = \bar{G}(t) = W_{11}(t)$

Taking Laplace transform of the relation (20 - 29) and solving them for $B_0^*(s)$. By

omitting the argument 's' for brevity, we get

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)}$$
(30)

The steady busy period, when the system starts from S_i , is obtained as

$$B_0 = \lim_{s \to 0} s B_0^*(s) = \frac{N_3}{D_2}$$
(31)

Where

$$N_3 = m_1 [P_{02}(P_{10} + P_{13}P_{30})(P_{46}P_{61} + P_{42}P_{21} - 1)] + (m_1 + m_2) \times$$

$$[P_{04} + P_{02}P_{21}(1 - P_{10} - P_{13}P_{30})]$$

$$-m'_{1}[P_{02}(P_{10}+P_{13}P_{30})(1-P_{46}P_{61}-P_{42}P_{21})]+m'_{2}(1-P_{49}^{(10)})\times$$

$$[P_{04} + P_{02}P_{21}(1 - P_{10} - P_{13}P_{30})] + m'_{3}[\{P_{04} + P_{02}P_{21}(1 - P_{10} - P_{13}P_{30})\} \times$$

$$\left\{ (P_{46}P_{61} + P_{42}P_{21}) \left(P_{13} + P_{18}^{(5)} + P_{13,8} \right) - P_{13,8} \right\}$$

$$-(P_{10} + P_{13}P_{30})(1 - P_{46}P_{61} - P_{42}P_{21})\left\{P_{02} - P_{02}P_{21}(P_{10} + P_{14}^{(5)})\right\}\right]$$

EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

 $V_i(t)$ be the expected number of visits by the repairman in the time interval (0, t] given that the system entered into a regenerative state S_i at t = 0. Therefore, by using the probabilistic arguments, we have the following recursive relations :

$$V_0(t) = Q_{02}(t) \$ [1 + V_2(t)] + Q_{04}(t) \$ [1 + V_4(t)]$$

$$V_{1}(t) = Q_{10}(t) \$ V_{0}(t) + Q_{13}(t) \$ V_{3}(t) + Q_{14}^{(5)}(t) \$ V_{4}(t) + Q_{18}^{(5)}(t) \$ V_{8}(t)$$

$$V_2(t) = Q_{21}(t) \$ V_1(t) + Q_{2,13}^{(14)}(t) \$ V_{13}(t)$$

$$V_{3}(t) = Q_{30}(t) \$ V_{0}(t) + Q_{34}^{(7)}(t) \$ V_{4}(t)$$

$$V_4(t) = Q_{42}(t) \$ V_2(t) + Q_{46}(t) \$ V_6(t) + Q_{49}^{(10)}(t) \$ V_9(t) + Q_{4,11}^{(10)}(t) \$ V_{11}(t)$$

$$V_6(t) = Q_{61}(t) \$ V_1(t) + Q_{6,13}^{(12)}(t) \$ V_{13}(t)$$

$$V_8(t) = Q_{84}(t) \$ V_4(t)$$

$$V_9(t) = Q_{9,13}(t) \$ V_{13}(t)$$

$$V_{11}(t) = Q_{11,13}(t) \$ V_{13}(t)$$

$$V_{13}(t) = Q_{13,4}(t) \$ V_4(t) + Q_{13,8}(t) \$ V_8(t)$$

(32-41)

Taking the Laplace – Stieltjes transform of the relations (32 - 41) and solving them for $\tilde{V}_0(s)$. By omitting the argument 's' for brevity, we get

$$\tilde{V}_0(s) = \frac{N_4(s)}{D_2(s)}$$
(42)

In the steady state the number of visits per unit is given by.

$$V_0 = \lim_{t \to 0} \left[\frac{V_0(t)}{t} \right] = \lim_{s \to 0} s. \tilde{V}_0(s) = \frac{N_4}{D_2}$$
 (43)

Where

$$N_4 = (P_{46}P_{61} + P_{42}P_{21})(P_{10} + P_{13}P_{30})$$

And D_2 is same as defined in availability.

PROFIT ANALYSIS

The profit obtained to the system model in steady state can be obtained as given by

$$P = L_0 A_0 - L_1 B_0 - L_2 V_0 \tag{44}$$

Where L_0, L_1 and L_2 be the revenue per unit up time of the system, cost per unit for which the repairman is busy and cost per visit by the repairman respectively.

CONCLUSION

In the present study for making the system more effective, the concept of fault detection is used to decide the types of failure, inspection is used to decide whether the repair is prefect or not and post repair is used to repair the unit again if repair of the unit is imperfect. The optimum results of the reliability measures are obtained which are shown in equations (7), (19), (31), (43), and (44).

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