# M/G/1 vacation queueing system with breakdown, repair and server timeout

**[1]Y.Saritha, [2]K. Sathish Kumar, [3]V N Ramadevi, [4]K.Chandan**
[1]Research Scholar, [2]Research Scholar, 3Associate Professor [4]Professor
[1,2 and 4]Department of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh, India
[3]Associate Professor, GokarajuRangaraju Instsitute of Engineering and Technology, Hyderabad, Telangana,India

***Abstract*—M/G/1 vacation queueing system with server breakdown, repair and server timeout is analyzed. When the system is empty, the server waits for fixed time 'c'. At the expiration of this time, if no one arrives into the system then server takes vacation. If anyone arrived into the system during the timeout period as well as in vacation the server commences the service. If the system had breakdown, just after a breakdown the server undergoes for repair. After repair process is completed the server restarts the service to the arrived customer. Here, we obtained an expression for expected system length and also the numerical results are illustrated.**

***Index Terms*—Vacation queueing system, expected system length, breakdown, repair, and Server timeout.**

## I. INTRODUCTION

Queueing systems with server timeout is one in which server plays a vital role in single service system with breakdown, repair and vacations. A variety of Problems can be modeled by vacation queueing system. This includes Machine breakdown, maintenance in communication and computer systems, and token passing local area networks have been studied over a few decennary or decades and also successfully adapted.

In vacation queuing model the server completely stops service when it is on vacation. Queueing system with server vacations idea was first discussed by Levy and Yechiali [6] they introduced the utilization of idle time in an M/G/1 queueing system. Several excellent surveys on the vacation models were done by B.T. Doshi [1]. Later on Jau-chauank, Chian-Hangwu and Zhe George Zhang [3] also did a short survey on various kinds of vacation models and they intend to provide a brief compact of the past 10years.

When the system is empty, the server will wait for fixed time known as server timeout. Oliver C.Ibe [7], [8] derived an expression for the mean waiting time of a vacation queueing system in which server does not take frequently another vacation upon returning from a vacation and finding system empty, as in the multiple vacation scheme or wait indefinitely for a customer to arrive and also he extended this model for N-policy. E.Ramesh Kumar and Y. Praby Loit [2] derived an expression for the mean waiting time of a vacation queueing system in which server does not take immediately another vacation upon returning from a vacation and finding system empty, as in multiple vacation scheme or wait indefinitely for a customer to arrive. Y.Saritha, K.Satish Kumar and K.Chandan [11] derived the expected system length using different bulk size distributions for $M^X$/G/1 vacation Queueing system with server timeout.

Single Queue subjected to breakdown and repair has been studied by number of authors. By using supplementary variable technique Kailash C. Madan [4] derived an explicitly of the probability generating function of the number in the system and mean number in the system for M/G/1 type queue with time homogeneous breakdown and deterministic repair. Kuo-Hsiung Wang, Dong-Yuh-Yang and W.L. Pearn [5] investigated the optimal (T, P)-Policy and the optimal (p, N)-Policy M/G/1 queue with SOS, server breakdown and general startup times. S.Pazhani Bala Murugan and K.Santhi [9] studied a non-Morkovian queue with Multiple Working vacation and random breakdown. Tao Li and Liyuan Zhang [10] derived M/G/1 retrial G-queue with general retrial times, in which the server is subject to working breakdowns and repair.

The objective of this paper is to derive the expected system length for M/G/1 vacation queueing system with breakdown, repair and server timeout. Numerical solution for various parameters to the single server vacation queueing system is presented.

## II. MODEL DESCRIPTION

Let us consider M/G/1 queueing system with breakdown, repair and server timeout. Here customers are assumed to arrive according to a Poisson process with rate $\lambda$. Here the service times assumed to follow the general distribution with rate $\mu$ and commences the service in FIFS discipline. Whenever the system becomes empty the server waits for fixed time 'c', is called server timeout. At this time if a customer arrive then the server return to the system and do service. At the expiration of the time if no customer arrive then the server takes vacation. During vacation, if the server finds customer is waiting in the queue then the server return to the system and commence service to the waiting or arrived customer exhaustively; otherwise it take another vacation. However if the server works continuously, service is interrupted due to Server breakdown. Here the server is unable to work unless the machine gets repaired, therefore the server should undergo for repair process. When the repair work is completed the server immediately returns to the service system and does the service for the waiting customer as well as arrived customer in a queue.

The system performance has been shown in below Figure 1:


Figure 1: M/G/1 Vacation Queueing System with Breakdown, Repair and Server Timeout

## III. ANALYSIS OF THE MODEL

The customer arrivals occur according to a Poisson process with arrival rate 'λ'. The number of arriving customers joins in a single waiting line. The variable X, time to serve a customer has general distribution with cumulative distribution function (CDF) $F_X(x)$ with mean E(X) and its second moment E($X^2$). The S-transform of the probability density function (denoted by $f_X(x)$) of X; is $M_X(s)$ and is defined by

$$M_X(s) = E(e^{-SX}) = \int e^{-sx} f_X(x)dx \qquad (1)$$

In vacation queuing system the server may become unavailable for a random period of time, called vacation time if the system is empty. Let V be a vacation time which is assumed to have a general distribution with CDF $F_V(v)$, mean E (V), second moment E ($V^2$) and its S-transform, denoted by $M_V(s)$ and is given by

$$M_V(s) = \frac{\gamma}{s+\gamma} \qquad (2)$$

Here X and V are mutually independent variables.

If the system works continuously the server is subjected to have breakdown with rate **α**, and it has failure time distribution as exponential with mean $\frac{1}{\alpha}$. Then the system instantly undergoes for repair process and the repair times are deterministic of constant (fixed) duration d (>0). After repair process completed the server restarts the service to the customer. Let P (z) denotes the probability generating function of the number in the system, whose z-transformation is denoted as $G_{L(M/G/1)}(z)$ is given from [Ref 4]

$$G_{L(M/G/1)}(Z) = \frac{(\lambda+\alpha-\lambda z)M_x(\lambda+\alpha-\lambda z)(z-1)(1+\alpha z)\left[\dfrac{\alpha M_x(\alpha)-(1-M_x(\alpha))(\lambda+\lambda\alpha d)}{\alpha(\alpha+1)M_x(\alpha)}\right]}{\lfloor\lambda+\alpha-\lambda z\rfloor\lfloor z-M_x(\lambda+\alpha-\lambda z)\rfloor-[1-M_x(\lambda+\alpha-\lambda z)]\alpha z e^{-\lambda d(1-z)}} \qquad (3)$$

Let, the random variable A denotes the number of customers in the system at the beginning of busy period. The probability mass function of A is $P_A$ (a) =P [A=a], whose z-transformation $G_A$ (z) is given by

$$G_A(z) = E(z^A) = \sum_{a=1}^{\infty} z^a p_A(a) \qquad (4)$$

The mean of A is E (A) and its second moment is E ($A^2$).

Let, the random variable B denotes the number of customers left by an arbitrary departing customer. The PMF of B is $P_B(b)$, whose z-transform is given by

$$G_B(Z) = \frac{1-G_A(z)}{(1-z)E(A)} \qquad (5)$$

The mean of B is E (B) and its second moment is E ($B^2$).

Let, L denote the number of customers in the system at an arbitrary point in time. The PMF of L is $P_L(l)$, whose z-transform is given by

$$G_L(Z) = G_B(Z).G_{L(M/G/1)}(Z) \qquad (6)$$

The mean of L is E (L) and its second moment is E ($L^2$). Let $W_q$ denotes the waiting time in the system. By applying little's law we obtain the expected system length of the customer is from [Ref 11]

$$E(W_q) = \frac{1}{\lambda}\left.\frac{dG_L(z)}{dz}\right|_{Z=1} - E(X) \qquad (7)$$

$$\lambda(E(W_q)+E(X)) = \left.\frac{dG_L(z)}{dz}\right|_{Z=1}$$

$$\lambda(E(W_q)+E(X)) = \frac{d}{dz}G_L(z)\Big|_{Z=1} = E(L) \qquad (7(a))$$

Let $G_A$ (z) is the Laplace –Stieltjes transform of Expected system length [Ref 11] and is given by

$$G_A(z) = zp_2 + \frac{M_v(\lambda - \lambda z) - M_v(\lambda)}{1 - M_v(\lambda)} p_3 \qquad (8)$$

Where $p_2 = \dfrac{M_V(\lambda)\{1-e^{-\lambda c}\}}{(1-e^{-\lambda c})M_V(\lambda)}$ and $p_3 = \dfrac{1-M_V(\lambda)}{(1-e^{-\lambda c})M_V(\lambda)}$

From equation (8), we obtain mean and second moment of $G_A(z)$

$$G_A^{'} = E(A) = \frac{\gamma^2(1-e^{-\lambda c}) - \lambda(\lambda+\gamma)}{((\lambda+\gamma) - e^{-\lambda c}\gamma)\gamma} \qquad (9)$$

And
$$G_A^{''} = E(A^2) = \frac{2\lambda^2(\lambda+\gamma)}{((\lambda+\gamma) - e^{-\lambda c}\gamma)\gamma^2} \qquad (10)$$

By assuming equation (2), we can derive $M_V(s) = \dfrac{\gamma}{s+\gamma} \Rightarrow M_V(\lambda) = \dfrac{\gamma}{\lambda+\gamma}$ (11)

By substituting equation (3), (5) in equation (6), we get an expression

$$G_L(z) = \frac{1-G_A(z)}{(1-z)E(A)} \frac{(\lambda+\alpha-\lambda z)M_x(\lambda+\alpha-\lambda z)(z-1)(1+\alpha z)\left[\dfrac{\alpha M_x(\alpha) - (1-M_x(\alpha))(\lambda+\lambda\alpha d)}{\alpha(\alpha+1)M_x(\alpha)}\right]}{\lfloor \lambda+\alpha-\lambda z \rfloor \lfloor z - M_x(\lambda+\alpha-\lambda z)\rfloor - [1-M_x(\lambda+\alpha-\lambda z)]\alpha z e^{-\lambda d(1-z)}} \qquad (12)$$

Put $K = \left[\dfrac{\alpha M_x(\alpha) - (1-M_x(\alpha))(\lambda+\lambda\alpha d)}{E(A)\alpha(\alpha+1)M_x(\alpha)}\right]$ in the above equation (12) then we get

$$G_L(z) = \frac{1-G_A(z)(\lambda+\alpha-\lambda z)M_x(\lambda+\alpha-\lambda z)(1+\alpha z)K}{[1-M_x(\lambda+\alpha-\lambda z)]\alpha z e^{-\lambda d(1-z)} - \lfloor \lambda+\alpha-\lambda z \rfloor \lfloor z - M_x(\lambda+\alpha-\lambda z)\rfloor} \qquad (13)$$

By differentiating the above equation (13) w.r.to z, and substitute z=1 then we get $G_L(1)$

$$G_L(1) = \frac{k.E(A)\alpha(\alpha+1)M_x(\alpha)}{\alpha M_x(\alpha) - (1-M_x(\alpha))(\lambda+\lambda\alpha d)} \qquad (14)$$

Now substituting K value in equation (14) we get

$$G_L(1) = 1 \qquad (15)$$

Again differentiating the above expression (13) w.r.to z, and substitute z=1. Then we get $G_L^{'}(1)$

$$G_L^{1}(1) = \frac{p_1 G_A^{''}(1)(1+\alpha)}{2(\mu-\lambda(1+\alpha d)} + \frac{2p_1 G_A^{'}(1)\alpha}{2(\mu-\lambda(1+\alpha d)} + \frac{2\lambda(1+\alpha d)}{2(\mu-\lambda(1+\alpha d))} + \frac{\alpha(\lambda d)^2}{2(\mu-\lambda(1+\alpha d))} \qquad (16)$$

Now substituting equations (9), (10) in above equation (16), we get

$$G_L^{1}(1) = E(L) = \frac{\lambda^2(\lambda+\gamma)}{\gamma^3(1-e^{-\lambda c}) + \lambda\gamma(\lambda+\gamma)} + \frac{\alpha}{1+\alpha} + \frac{\lambda(1+\alpha d)}{\mu-\lambda(1+\alpha d)} + \frac{\alpha(\lambda d)^2}{2(\mu-\lambda(1+\alpha d))} \qquad (17)$$

Thus by above expression, we obtain the expected system length.

**Particular Case**

If system suffers no breakdowns and repairs, then letting α=0 and d=0 in the above expression (in equation (17)) then the resulting expression is a known for the M/G/1 vacation queueing system with server Time Out (Reference [8]).

## IV. Numerical Illustrations

Thus by using equation (17) and varying different parameters, we get the following numerical illustration in Table 1 as given below:

**TABLE1: Effect of Different Variables (d, λ, μ, c, α and $\gamma$ ) on system length for fixed values of d=2, λ=1, μ=150, c=1, α=4 and $\gamma$ =5.5.**

| **TABLE1: Effect of Different Variables (d, λ, μ, c, α and $\gamma$ ) on system length for fixed values of d=2, λ=1, μ=150, c=1, α=4 and $\gamma$ =5.5.** | | | | | |
|---|---|---|---|---|---|
| **Parameter** | **Effect Of Parameters** | **E(L)** | **Parameter** | **Effect Of Parameters** | **E(L)** |
| d | 2 | 1.0142 | λ | 1 | 1.0142 |
| | 6 | 1.6696 | | 3 | 5.8931 |
| | 10 | 3.1047 | | 8 | 6.2669 |
| | 14 | 5.716 | | 11 | 19.3216 |
| | 18 | 10.257 | | 14 | 68.5207 |
| μ | 150 | 1.0142 | $\gamma$ | 5.5 | 1.0142 |

| | | | | | |
|---|---|---|---|---|---|
| | 200 | 0.9826 | | 6 | 0.9946 |
| | 250 | 0.9641 | | 6.5 | 0.9806 |
| | 300 | 0.9521 | | 7 | 0.9703 |
| | 350 | 0.9435 | | 7.5 | 0.9625 |
| α | 4 | 0.0142 | c | 1 | 1.0142 |
| | 5 | 1.0781 | | 1.5 | 0.9901 |
| | 6 | 1.1333 | | 2 | 0.9807 |
| | 7 | 1.1835 | | 2.5 | 0.9761 |
| | 8 | 1.2306 | | 3 | 0.9736 |

From the above Table1

As d, $\lambda$, $\gamma$ and α were increasing then expected system length E (L) is also increasing.

As μ and c are increasing then expected system length E (L) is decreasing.

## V Conclusion

In this model, we derived an expression of expected system length for M/G/1 vacation queueing system with breakdown, repair and server timeout. Numerical illustrations are given under a set of parameters that shows the impact of the timeout period on the expected system length.


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