DETERMINATION OF CRITICAL OR EQUILIBRIUM POINT

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Introduction

Burglary is the most common property of crime. Further more, burglars are quite similar to thieves in a number of respects, so that these two classes provide leaves of the ordinary penal institution. Researchers found that burglars as a class were somewhat more intelligent than thieves but were less competent than embezzlers, forgers and robbers.

Burglary occurs everywhere. In recent decades it is a major issue in every big city. Some good neighborhoods in a city approximately free from burglary events. However there are some bad neighbourhoods where dense agglomerated of burglary or other crimes commit.

There is a spatio-temporal correlation between burglary and victims or their close neighbors. Some neighbors repeatedly victimized within short interval of time. Thus burglary often is clustered densely which tends to be spatial localized into a regions. These spatio-temporal clusters of burglary events occurrence are often referred to as burglary “hotspots”.

Burglary hotspots are observed to vary depending upon the particular geographic, economic or environmental conditions present. Moreover, hotspots are seen to emerge or diffuse depending on specific category of crime. The emergence of hotspots is connected to repeat victimization. A successful burglar have tendency to commit repeat burglary in the same house or nearby house.

Some specific theory has been discussed in this model to understand why hotspots increase in some locations rather than others. How they evolve and how their different sizes and lifetime features are connected the different behaviour of burglars, victims and cops on dots.

To discuss the model some specific theories and hypothesis like Routine activity theory, Crime pattern theory, rational choice theory, repeat and near repeat victimization theory and broken window theory are referred.

Crime pattern theory supposes that crime is not random. The location of a crime is likely near a criminal’s normal activity space. The normal activity space is the collection of areas where the individual most frequently comes into contact with others.

Neglecting the terms containing spatial laplacian we can write the spatial invariant system corresponding to the system equation as :

$$\frac{\partial R}{\partial t} = -\beta_1 \left(R - R_0 \right) + \alpha \rho R \qquad (1)$$

$$\frac{\partial \rho}{\partial t} = -\rho R + G \qquad (2)$$

For critical point $\left(R^*, \rho^* \right), \frac{\partial R}{\partial t} = 0$ and $\frac{\partial \rho}{\partial t} = 0$

Which gives the followings

$$0 = -\beta_1 \left(R^* - R_0 \right) + \alpha \rho^* R^*$$

and $0 = -\rho^* R^* + G$

solving these we have,

$$R^* = \frac{\beta_1 R_0 + \alpha G}{\beta_1}$$

and $$\rho^* = \frac{\beta_1 G}{\beta_1 R_0 + \alpha G}$$

Thus the fixed point is

$$\left(R^*, \rho^* \right) = \left(\frac{\beta_1 R_0 + \alpha G}{\beta_1}, \frac{\beta_1 G}{\beta_1 R_0 + \alpha G} \right)$$

Analysis of stability

The system equation can be written as :

$$\frac{\partial R}{\partial t} = -\beta_1 \left(R - R_0 \right) + \alpha \rho R = f \left(R, \rho \right), \qquad \text{say}$$

$$\frac{\partial \rho}{\partial t} = -\rho R + G = g(R, \rho), \quad \text{say}$$

Now,

$$\frac{\partial f}{\partial R} = -\beta_1 + \alpha\rho, \ \frac{\partial f}{\partial \rho} = \alpha R$$

$$\frac{\partial g}{\partial R} = -\rho, \ \frac{\partial g}{\partial \rho} = -R$$

Jacobian matrix J_* at the critical point (R^*, ρ^*) is given by

$$J_{*(R^*,\rho^*)} = \begin{pmatrix} \frac{\partial f}{\partial R} & \frac{\partial f}{\partial \rho} \\ \frac{\partial g}{\partial R} & \frac{\partial g}{\partial \rho} \end{pmatrix}_{(R^*,\rho^*)}$$

$$= \begin{pmatrix} -\beta_1 + \alpha\rho & \alpha R \\ -\rho & -R \end{pmatrix}_{(R^*,\rho^*)}$$

$$= \begin{pmatrix} -\beta_1 + \alpha\rho^* & \alpha R^* \\ -\rho^* & -R^* \end{pmatrix}$$

$$\det(J_* - \lambda I) = \det\left[\begin{pmatrix} (-\beta_1 + \alpha\rho^*) - \lambda & \alpha R^* \\ -\rho^* & -R^* - \lambda \end{pmatrix}\right]$$

So, the characteristic equation for the eigenvalues λ is given by

$$\det\left\{\begin{pmatrix} (-\beta_1 + \alpha\rho^*) - \lambda & \alpha R^* \\ -\rho^* & -R^* - \lambda \end{pmatrix}\right\} = 0$$

$$\Rightarrow \quad \lambda^2 + (\beta_1 - \alpha\rho^* + R^*)\lambda + \beta_1 R^* = 0$$

$$\Rightarrow \quad \lambda = \frac{-(\beta_1 - \alpha\rho^* + R^*) \pm \sqrt{(\beta_1 - \alpha\rho^* + R^*)^2 - 4\beta_1 R^*}}{2}$$

For real roots, $\left(\beta_1 - \alpha\rho^* + R^*\right)^2 - 4\beta_1 R^* > 0$

Here, suppose

$$\lambda_1 = \frac{-\left(\beta_1 - \alpha\rho^* + R^*\right) + \sqrt{(\beta_1 - \alpha\rho^* + R^*)^2 - 4\beta_1 R^*}}{2}$$

and $$\lambda_2 = \frac{-\left(\beta_1 - \alpha\rho^* + R^*\right) - \sqrt{(\beta_1 - \alpha\rho^* + R^*)^2 - 4\beta_1 R^*}}{2}$$

are two real eigenvalues

let, $\lambda_1 < 0$

$$\Rightarrow \quad -\left(\beta_1 - \alpha\rho^* + R^*\right) + \sqrt{(\beta_1 - \alpha\rho^* + R^*)^2 - 4\beta_1 R^*} < 0$$

$$\Rightarrow \quad \left(\beta_1 - \alpha\rho^* + R^*\right)^2 - 4\beta_1 R^* < \left(\beta_1 - \alpha\rho^* + R^*\right)^2$$

$$\Rightarrow \quad -4\beta_1 R^* < 0$$

$$\Rightarrow \quad \beta_1 R^* > 0$$

$$\Rightarrow \quad \beta_1 R_0 + \alpha G > 0$$

Since $R^* = \dfrac{\beta_1 R_0 + \alpha G}{\beta_1}$

Which is true for α, β_1, R_0 and G are positive. Hence our assumption that $\lambda_1 < 0$ is true. But obviously, $\lambda_2 < 0$. Hence the two eigenvalues λ_1 and λ_2 of the jacobian matrix is negative and

Hence the system is stable for the fixed point

$$(R^*, \rho^*) = \left(\frac{\beta_1 R_0 + \alpha G}{\beta_1}, \frac{\beta_1 G}{\beta_1 R_0 + \alpha G} \right)$$

Conclusion

The model described that the areas are **Risk of victimization** areas where burglars live, work, or spend free time. The movements of burglars are considered as a biased random walk towards the place of **Risk of victimization**. This model gives a primary conception to describe dynamics of burglary.

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