FORMALISED STRUCTURE OF LOWER PREDICATE CALCULUS.

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ABSTRACT :- The object of this paper is to discuss the brief structure of lower predicate calulus in the language structure . It is mainly devoted towards the study logical formulations of mathematical structures which is the backbone of all other disciplines.

Key-Words:- Predicate calculus, Modals, Atomic formula, Brouwer fixed point.

1. INTRODUCTION :- In this paper we define the logical connectives and its application in developing model structure and its operational applications in establishing some .intresting theorems connected with Russel's results in this direction which were further developed by skolem . we have also discusses the relations between the sentences of the formal langnage and the mathematical structures in which they hold.

2.1 The Language - L

(1) Binary predicate constant symbol $\in$ and $=$:

Individual Variable $\rightarrow V_0, V_1, V_2$.

Propositional connectives $\rightarrow$

(Unary), $\rightarrow$ $\wedge$, $\vee$, $\leftrightarrow$

Binary quantities $\exists$ and $\forall$.

Improper symbol (,)

All formulas in L can be constructed like this.

We may write U (x_1, ... x_m) to indicate that the free variables of the formula U are among the variables x_1 .... x_m.

A sentence is a formula with no free variables.

We always regard the Language L as fixed, Independent of the number of kind of truth values.

2.2 Interpretations of the logical connectives.

Let x = [0, 1] be the closed real unit interval. The function $\rightarrow$ is defined on X to X as follows.

for all x in x.

$\rightarrow x = 1 - x$.

The function $\rightarrow$ ' $\wedge$ ' $\vee$, $\leftrightarrow$ are defined On X × X to X as follows :

For all x, y in X.

$x -> y = \text{Min}\,(1, 1 - x + y)$,

$x \wedge y = \text{Min}\,(x, y)$

$x \vee y = \text{Min}\,(x, y)$

$x <-> y = 1 - |x - y|$.

The function $\exists$ and $\forall$ are natural generalization of the valued quantifiers and they are defined on the set of all non-empty subsets of X to X as follows :

For all non-empty $Y \subset X$ $\exists Y = \sup Y$ and $\forall Y = \inf y$.

In case of endowing with the natural topology, then each of the functions $\rightarrow$, $\rightarrow$, $\wedge$, $\vee$, $\leftrightarrow$ is continuous. Furthermore the function $\exists$ and $\forall$ are continuous with respect to a natural topology on the set of all non-empty subsets of X.

It shows that the seven functions defined above can be defined from $\rightarrow$, $\rightarrow$, and $\exists$.

2.3. The set X is referred to as the set of truth values of the infinite values logic. In order to discuse finite valued logices, we consider the following finite subsets of X,

For each $n \geq 2$ let

$$X_n = \left[0,\ \frac{1}{n-1},\ \frac{2}{n-1} \ldots \ldots \ldots \ldots, 1\right].$$

Each set X_n is referred to as the set of truth values of n - valued logic. Each one of the seven functions introduced above. When restricted to x_n, or $x_n \times x_n$ or the set of all non-empty subsets of x_n, yields values in x_n. In case n = 2, all functions have their standard 2 -valued meanings.

2.4 Modals.

A model M for L is a triple M = (A, E, I) where A is a non-empty set, E is a mapping of A × A into X, and I is the mapping of A × A into X such that for all a, b in A.

$I(a, b) = 0$ if $a \neq b$

$I(a, b) = 1$ if $a = b$.

Let M be the class of all such models. For the finite valued modals, for each $h \geq 2$. Let M_x be the class of all modals.

M = < A, E, I > In M such that the language of E is a subset of X_n. Members of M_n are referred to as n-valued modals for L.

Thus for all m, $n \geq 2$,

$Mn \subset M$ and

$Mn \subset Mm$ if and only if $(n - 1)$ divides $(m - 1)$.

The class M_2 is simple the familiar class of all 2-valued models for L.

2.5. The value function U_M for each formula $U(x_1, \ldots\ldots\ldots\ldots, x_m)$.

Model M = < A, E, I > and interpretation of $x_1, \ldots\ldots\ldots, x_m$ as elements $a_i, \ldots\ldots\ldots\ldots\ldots a_m$ in A. We detine a real number $U_m(a_1, \ldots\ldots\ldots\ldots a_m)$ in x by induction on the formulas.

We give the complete definition (when the model M is understood, we simply write $U(a_1, \ldots\ldots\ldots\ldots a_m)$ for $U_m(a_1, \ldots\ldots\ldots\ldots a_m)$

(i) If U is the atomic formula

$x_i = x_j$, where $i \leq 1$, $j \leq m$,

define $U(a_1 \ldots\ldots\ldots\ldots a_m) = T(a_i, a_j)$.

(ii) If U is the atomic formula

$x_i \in x_j$, where $i \leq 1$, $j \leq m$,

define $U(a_1 \ldots\ldots\ldots\ldots a_m) = E(a_i, a_j)$

(iii) If U, V are formulas such that $U(a_1 \ldots\ldots\ldots\ldots a_m)$ and $V(a_1 \ldots\ldots\ldots\ldots a_m)$ are defined, then define.

$(-> U)(a_1 \ldots\ldots\ldots\ldots a_m) = \rightarrow (U(a_1 \ldots\ldots\ldots\ldots a_m))$

$(U -> V)(a_1 \ldots\ldots\ldots a_m) = U(a_1 \ldots\ldots\ldots a_m) -> V(a_1 \ldots\ldots\ldots a_m)$

$(U \wedge V)(a_1 \ldots\ldots\ldots a_m) = U(a_1 \ldots\ldots\ldots a_m) \wedge \vee (a_1 \ldots\ldots\ldots a_m)$

$(U \vee V)(a_1 \ldots\ldots\ldots a_m) = U(a_1 \ldots\ldots\ldots a_m) \vee \vee (a_1 \ldots\ldots\ldots a_m)$

$(U <-> V)(a_1 \ldots\ldots\ldots a_m) = U(a_1 \ldots\ldots\ldots a_m) <-> V(a_1 \ldots\ldots\ldots a_m)$

(iv) If $U(x_1, \ldots\ldots\ldots\ldots, x_{m+1})$ is a formula such that $U(a_1 \ldots\ldots\ldots\ldots a_{m+1})$ is defined for all $a_1, \ldots\ldots\ldots\ldots, b_{m+1}$ in A. Then we have -

$(\exists x_{m+1} U)(a_1 \ldots\ldots\ldots a_m) = (\exists\{U(a_1 \ldots\ldots\ldots\ldots a_{m+1}): a_{m+1}$ in A$\})$,

$(\forall x_{m+1} U)(a_1 \ldots\ldots\ldots a_m) = \forall \{U(a_1 \ldots\ldots\ldots\ldots a_{m+1}): a_{m+1}$ in A$\}$,

Thus $U_m(a_1 \ldots\ldots\ldots\ldots a_m)$ is a uniquely defined real number in x, Furthermore, for each $n \geq 2$

If M is in M_n, then $U_m(a_1, a_2, \ldots\ldots\ldots\ldots, am)$ is in X_n.

In case U is a sentence, the value of $U_M(a_1, a_2, ... , a_m)$ is independent of the elements $a_1, ... a_m$ in A and we write here U_M for U_M $(a_1, ... ,a_m)$. The real number M_m in case U is a sentance, is referred to as the value of the sentences U on the model M.

U_M may be defined as a function mapping the class

(The set of all sentences) × M

into the set X. Also, if M is in M_n, then U_m is in X_n for every sentence U. It was evident that in case n = 2.

We have the Usual value function U_M as M ranges in M_2 and U ranges over all sentences.

3.1 The operations Mod and Theorem

Let E be a set of sentences of L, K be a class of models in M, and Y be a subsets of X. Let us define

Mod $(\Sigma , y) = \{M \text{ in } M_n : U_M \text{ is in } y \text{ for every } U \text{ in } \Sigma \}$,

Th $(K, Y) = \{U \text{ sentences of } L : U_M \text{ is in } y \text{ for every } M \text{ in } K\}$.

for each $n \geq 2$, we define

Modn $(\Sigma , y) = \{M \text{ in } M_n : U_M \text{ is in } y \text{ for every } U \text{ in } E\}$

We find that Mod_2 $((\Sigma \{1\})$ is an elementary class in 2 - valued model theory, and Th $(Mod_2$ E { (1)}, {(1)}) is the set of all semantical consequences of Σ in 2 – valued logic.

On the basis of these assumptions change and k either proved some interesting theorem of Model theoretical structure.

3.2 The axiom scheme of comprehension in many valued logic. Let Σ_0 be the set of all sentences of L of the form

$$\forall x_1 \ .... x_m \exists y \forall \ (t \in y \leftrightarrow U (t, x_1 .... x_n))$$

Where the formula U doesnot contain y free.

The free variables of the parameters of the formula U. We many refer to $\sum_0$ the axiom a scheme of comprehension.

Russell introduced an interesting results from the set of axioms denoted by $\sum_0$.

$Mod_1 (\sum_0, \{1\}) = 0$

In its extended form we may write to as $Mod_n (\sum_a, \{1\}) = 0$ for each $n \geq 2$.

Skolem and Tarski also give similar results. We may further express in the form

If n is odd, then $Mod_n (\sum_0, [1/2, 1]) = 0$ and $Mod_n (\sum_0, [1/2, 1]) \neq 0$. If n is even let p be the rational $n - \frac{2}{(2,\ n-2)}$, then

$$Mod_n (\sum_0, [P, 1]) = 0 \text{ and } Modn (\sum_0, [P, 1]) \neq 0$$

3.3 The problem of the consistency of the axiom schemes of comprehension in infinite valued logic is to decide whether or not

$Mod (\sum_0, \{(1\}) \neq 0$. we discuss below some of its interpretations which is still open question.

Let $\sum_p$ be the set of all sentences of L of the form where $U (1, x_1 .... x_m)$ contains no bound variables. Skolem proved earlier that

$Mod (\sum_1. .... (1)) \neq 0$.

We find that it is slight departure from the finite valued results earlier. Since each of these results holds with $\sum_0$ replaced by $\sum_1$.

Leter on skolem improved in earlier theorems. Let $\sum_2$ be the set of all sentences of L of the form where $U(t, x_1 .... x_n)$ does not contain any parameters $x_1 .... x_m$ but may contain arbitrary bound variables.

$$Mod (\sum_2, \{1\}) \neq 0$$

Let $\sum_3$ be the set of all sentences of L of the form (*) where $U (t, x_1, ... x_m)$ may contain parameters $(x_1 .... x_m)$, but each bound variables of U is restricted to occur only in the second space in atomic formula of the form v, w.

$$\text{Mod}(\sum_3, \{1\}) \neq 0$$

The last result is an improvement of skolem's result earlier proved, Since $\sum_1 \subset \sum_3$.

Fenstal obtained another result in this direction. Let $\sum_4$ be the set of all sentences of L of the form * Where $U(t, x_1 \ldots\ldots\ldots x_n)$ carries the single restriction that the free variable t is allowed to occur only in the first place. In atomic formula of the form $V \in W$

$$\text{Mod}(\sum_4, \{1\}) \neq 0.$$

CONCLUSION :- We have proved all the abvoe results based on the original method of skolem using the Brouwer fixed point theorem for the space X^n and also we have mentioned some interpretation which is still open for the further research work.

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