A Method for Solving Integer(each non zero) Linear Fractional Programming Problems with Triangular Fuzzy Variables

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Abstract

A method for solving integer linear fractional programming problems with fuzzy constraint parameters by using classical integer linear fractional programming has been proposed in this paper. In the method, ranking functions are not used, the proposed method can serve managers by providing the best solution to a variety of integer linear fractional programming problems with fuzzy constraint parameters in a simple and effective manner. With the help of numerical examples, the method is illustrated. In the fractional programming it is an essential condition that the denominator should be positive so in the solution it is intentioned that no variable can take zero value.

Keywords : Fuzzy variables, Triangular fuzzy number,

1. INTRODUCTION Linear fractional programming has applications in many fields of operations research. It is concerned with the optimization of a linear fractional function while satisfying a set of linear equality and or inequality constraints or restrictions. In real world situation the available information in the system under consideration are not exact, therefore fuzzy linear fractional programming (FLFP) was introduced and studied by many researchers. Fuzzy set theory has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications. FLFP problems have an essential role in fuzzy modeling, which can formulate uncertainty in actual environment. Afterwards, many authors have considered various types of the FLP problems and proposed several approaches for solving these problems. Fuzzy mathematical programming (FMP) is derived through incorporation of fuzzy set theory within ordinary mathematical programming frameworks. The FMP methods contain two major categories: fuzzy flexible programming (FFP) and fuzzy possiblilistic programming (FPP) (Inuiguchi 1990).

In 2009 [2] K. Eshghi1 and J. Nematian discussed Special Classes of Fuzzy Integer Programming Models with All-Die rent Constraints. In 2008 [4] James Clarke jcl, Mirella Lapata discussed ,Global Inference for Sentence Compression an Integer Linear Programming Approach. In 2010 [5] ,B. Kheirfama and F. Hasani,gave the discussion about sensitivity analysis for fuzzy linear programming problems with fuzzy variables In 2008 [7] Pop B., and Stancu-Minasian, I.M.,gave a method of solving fully fuzzified linear fractional programming problems. In 2009 [9] Teng-San Shih Jin-Shieh Su and Jing-Shing Yao, discussed Fuzzy linear Programming based on interval-valued Fuzzy sets.

2. PRELIMINARIES

We need the following definitions of the basic arithmetic operators on fuzzy triangular numbers based on the function principle which can be found in [9] .

Definition 2.1. A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2, a_3 real numbers and its membership function are $\mu_{\tilde{a}}(x)$ is given below.

$$\mu_{\tilde{a}}(x) = \left\{ \begin{array}{l} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{array} \right\}.$$

Definition 2.2.

Let (a_1, a_2, a_3) and (b_1, b_2, b_3) be two triangular fuzzy numbers. Then

(i) $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii)$(a_1, a_2, a_3) \Theta (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

(iii) $k(a_1, a_2, a_3) = (\mathrm{k}a_1, ka_2, ka_3)$, for $\mathrm{k} \geq 0$

(iv)$k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$, for $\mathrm{k} < 0$

Let F(R) be the set of all real triangular fuzzy numbers then

Definition 2.3.

.Let $\tilde{A} = (a_1, a_2, a_3)$, and $\tilde{B} = (b_1, b_2, b_3)$, be in F (R).then

$(i)\ \tilde{A} = \tilde{B} \Leftrightarrow a_i = b_i$ for all i=1,2,3 and

(ii) $\tilde{A} \leq \tilde{B} \Leftrightarrow a_i \leq b_i$ for all i=1,2,3

Definition 2.4.

Let $\tilde{A} = (a_1, a_2, a_3)$ be in F (R).then

$(i)\ \tilde{A}$ is said to be positive if $a_i \geq 0$, for all i = 1,2,3

(ii) $\tilde{A}$ is said to be integer if $a_i \geq 0$, for all i = 1,2,3 are integers and

(iii) $\tilde{A}$ is said to be symmetric if $a_2 - a_1 = a_3 - a_2$

3. FUZZY INTEGER LINEAR PROGRAMMING MATHEMATICAL Consider the following integer linear fractional programming problem with fuzzy variables : (

Maximize $\tilde{Z} = \dfrac{c\tilde{x}}{d\tilde{x}}$ ....(1)

Subjected to $A\tilde{x} \leq \tilde{b}$ ....(2)

$\tilde{x} > 0$ and are integers..(3)

Where the coefficient matrix A=$(\mathrm{a}_{ij})_{m \times n}$ is a nonnegative real matrix,

the cost vector c=$(\mathrm{c}_1, \mathrm{c}_2, ...\mathrm{c}_n)$ and d=$(d_1, d_2, ...d_n)$

is nonnegative vector and $\tilde{x} = (\tilde{x}_j)_{n \times 1}$ and $\tilde{b} = (\tilde{b}_i)_{m \times 1}$ are nonnegative real fuzzy vectors

such that $\tilde{x}_j, \tilde{b}_i \in F(R)$ for all $1 \leq \mathrm{j} \leq \mathrm{n}$

and $1 \leq i \leq m.$

Definition 3.1. A fuzzy vector $\tilde{x}$ is said to be a feasible solution of the problem (P) if $\tilde{x}$ satisfies (2) and (3).

Definition 3.2. A feasible solution $\tilde{x}$ of the problem (P) is said to be an optimal solution of the problem (P) if there exists no feasible $\tilde{u} = (\tilde{u}_j)_{n \times 1}$ of (P) such that $\dfrac{c\tilde{u}}{d\tilde{u}} > \dfrac{c\tilde{x}}{d\tilde{x}}$.

Using the arithmetic operations of fuzzy numbers, we can obtain the following result.

Theorem 3.1.A fuzzy vector $\overset{\sim 0}{x} = (x_1^0, x_2^0, x_3^0)$ is an optimal solution of the problem (P) iff x_2^0, x_1^0 and x_3^0 are optimal solutions of following integer linear fractional programming

$(\mathrm{P}_2), (\mathrm{P}_1)$ and (P_3) respectivly where

(P_2) Maximize $\mathrm{Z}_2 = \dfrac{cx_2}{dx_2}$

subjected to $\mathrm{A}x_2 \le b_2, x_2 > 0$ are integers

(P_1) Maximize $\mathrm{Z}_1 = \dfrac{cx_1}{dx_1}$

subjected to $\mathrm{A}x_1 \le b_1, x_1 > 0\ x_1 \le \mathrm{x}_2^0$ are integers

(P_3) Maximize $\mathrm{Z}_3 = \dfrac{cx_3}{dx_3}$

subjected to $\mathrm{A}x_3 \le b_3, x_3 > 0, x_3 \ge \mathrm{x}_2^0$ are integers

Proof. Suppose that $\overset{\sim 0}{x} = (x_1^0, x_2^0, x_3^0)$ is an optimal solution of (P).This implies that

$$Z_1 \le Z_1^0, Ax_1 \le b_1, \text{ where } \mathrm{Z}_1 = \frac{cx_1}{dx_1}, Z_1^0 = \frac{cx_1^0}{dx_1^0}$$

$$Z_2 \le Z_2^0, Ax_2 \le b_2, \text{where } \mathrm{Z}_2 = \frac{cx_2}{dx_2}, Z_2^0 = \frac{cx_2^0}{dx_2^0} \text{]}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4)$$

$$Z_3 \le Z_3^0, Ax_3 \le b_3 \text{ where } \mathrm{Z}_3 = \frac{cx_3}{dx_3}, Z_3^0 = \frac{cx_3^0}{dx_3^0}$$

From (4) we have,

$$\text{Max. } \mathrm{Z}_1 = \frac{cx_1^0}{dx_1^0}, \text{ Max. } \mathrm{Z}_2 = \frac{cx_2^0}{dx_2^0}, \text{ Max. } \mathrm{Z}_3 = \frac{cx_3^0}{dx_3^0}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Now from (4) and (5) , We can conclude that $x_2^0, x_1^0,$ and x_3^0 are optimal solutions of the integer linear fractional programming $(\mathrm{P}_2), (\mathrm{P}_1)$ and (P_3) respectivly.

This implies that $\overset{\sim 0}{x} = (x_1^0, x_2^0, x_3^0)$ is an optimal solution of the problem (P) with optimal value $\overset{\sim 0}{Z} = (Z_1^0, Z_2^0, Z_3^0)$. Hence the Theorem.

4.NUMERICAL EXAMPLE. The proposed method is illustrated by the following example.

Example 1. Consider the following Fuzzy integer(Non zero) linear fractional programming in which constraints parameters are fuzzy and the objective function is crisp.

$$\text{Maximize } \tilde{Z} = \frac{4\tilde{x}_1 + 3\tilde{x}_2}{2\tilde{x}_1 + \tilde{x}_2}$$

$$3\tilde{x}_1 + \tilde{x}_2 \le (9,15,21)$$

$$\tilde{x}_1 + 3\tilde{x}_2 \le (6,9,12)$$

$$\tilde{x}_1, \tilde{x}_2 > 0 \text{ and integer}$$

$$\tilde{Z} = (Z_1, Z_2, Z_3), \tilde{x}_1 = (y_1, x_1, t_1), \tilde{x}_2 = (y_2, x_2, t_2)$$

let now the problem (P_2) is given below

(P_2) Maximize

$$\text{Maximize } Z_2 = \frac{4x_1 + 3x_2}{2x_1 + x_2}$$

$$3x_1 + x_2 \le 15$$

$$x_1 + 3x_2 \le 9$$

$$x_1, x_2 > 0 \text{ and integer}$$

Now using Grid algorithm the solution of this fractional programming is (1,2) i.e.

$x_1 = 1,\ \ x_2 = 2,\ \ \ Z_2 = 2.5$

now the problem (P_1) is given below

(P_1) Maximize

$$\text{Maximize } Z_1 = \frac{4y_1 + 3y_2}{2y_1 + y_2}$$

$$3y_1 + y_2 \le 9$$

$$y_1 + 3y_2 \le 6$$

$$y_1, y_2 > 0 \text{ and integer}$$

Now using Grid algorithm the solution of this fractional programming is (1,1) i.e.

$y_1 = 1,\ \ y_2 = 1,\ \ \ Z_1 = 7/3 = 2.333$

now the problem (P_3) is given below

(P_3) Maximize

$$\text{Maximize } Z_3 = \frac{4t_1 + 3t_2}{2t_1 + t_2}$$

$$3t_1 + t_2 \le 21$$

$$t_1 + 3t_2 \le 12$$

$$t_1, t_2 > 0 \text{ and integer}$$

Now using Grid algorithm the solution of this fractional programming is (1,3) i.e.

$t_1 = 1,\ \ t_2 = 3,\ \ \ Z_3 = 13/5 = 2.6$

Therefore the solution of given fuzzy integer (Non zero) linear fractional programming is $\tilde{x}_1 = (y_1, x_1, t_1) = (1,1,1)$,

$\tilde{x}_2 = (y_2, x_2, t_2) = (1,2,3)$ and $\tilde{Z} = (2.333, 2.5, 2.6)$

Example 2. Consider the following Fuzzy integer(Non zero) linear fractional programming in which constraints parameters are fuzzy and the objective function is crisp.

$$\text{Maximize } \tilde{Z} = \frac{10\tilde{x}_1 + 20\tilde{x}_2}{5\tilde{x}_1 + 3\tilde{x}_2}$$

$$2\tilde{x}_1 + 5\tilde{x}_2 \le (15, 20, 30)$$

$$3\tilde{x}_1 + 4\tilde{x}_2 \le (20, 24, 30)$$

$$\tilde{x}_1, \tilde{x}_2 > 0 \text{ and integer}$$

$$\tilde{Z} = (Z_1, Z_2, Z_3), \tilde{x}_1 = (y_1, x_1, t_1), \tilde{x}_2 = (y_2, x_2, t_2)$$

Let now the problem (P_2) is given below

(P_2) Maximize

$$\text{Maximize } Z_2 = \frac{10x_1 + 20x_2}{5x_1 + 3x_2}$$

$$2x_1 + 5x_2 \le 20$$

$$3x_1 + 4x_2 \le 24$$

$$x_1, x_2 > 0 \text{ and integer}$$

Now using Grid algorithm the solution of this fractional programming is (1,3) i.e.

$x_1 = 1, \quad x_2 = 3, \quad Z_2 = 5$

now the problem (P_1) is given below

(P_1) Maximize

$$\text{Maximize } Z_1 = \frac{10y_1 + 20y_2}{5y_1 + 3y_2}$$

$$2y_1 + 5y_2 \le 15$$

$$3y_1 + 4y_2 \le 20$$

$$y_1, y_2 > 0 \text{ and integer}$$

Now using Grid algorithm the solution of this fractional programming is (1,2) i.e.

$y_1 = 1, \quad y_2 = 2, \quad Z_1 = 50/11 = 4.5454$

now the problem (P_3) is given below

(P_3) Maximize

Now using Grid algorithm the solution of this fractional programming is (1,5) i.e.

$t_1 = 1, \quad t_2 = 5, \quad Z_3 = 110/20 = 5.5$

Therefore the solution of given fuzzy integer (Non zero) linear fractional programming is $\tilde{x}_1 = (y_1, x_1, t_1) = (1,1,1)$,

$\tilde{x}_2 = (y_2, x_2, t_2) = (2,3,5)$ and $\tilde{Z} = (4.5454,\ 5,\ 5.5)$

5. CONCLUSION

The method provides an optimal solution to FILFP(Fuzzy Integer Linear Fractional Programming) problems applying classical integer linear fractional programming. This method can serve managers by providing the best solution to a variety of integer linear fractional programming problems with fuzzy variables in a simple and effective manner.

REFERENCES

1.Belén Melián, Dpto. Estadística, José Luis Verdegay(2005)"Fuzzy Optimization Problems in Wavelength Division Multiplexing (WDM) Networks" 'EUSFLAT - LFA

2.K. Eshghi1 and J. Nematian (2000)"Special Classes of Fuzzy Integer Programming Models with All-Die rent Constraints ", Transaction E: Industrial Engineering, Vol. 16, No. 1, pp. 10.

3.F. Hosseinzadeh Lotfi (2007)"A New Approach for Efficiency Measures by Fuzzy Linear Programming and Application in Insurance Organization",Applied Mathematical Sciences, Vol. 1, , no. 14, 647 – 663

4.James Clarke jcl, Mirella Lapata (2008)"Global Inference for Sentence Compression an Integer Linear Programming Approach", Journal of Artificial Intelligence Research ,31 399-429.

5.B. Kheirfama † F. Hasani (2010)"Sensitivity analysis for fuzzy linear programming problems with fuzzy variables", AMO – Advanced Modeling and Optimization, Volume 12, Number 2,

6.Mahdieh Allahviranloo (2010)"Mixed Integer Programming for Port Anzali Development Plan" ,International Journal of Humanities and Social Sciences ,4:1 A

7." Pop, B., and Stancu-Minasian, I.M(2008)., "A method of solving fully fuzzified linear fractional programming problems", J. Appl. Math. Comp., 27 , 227-242

8. P.A. Thakre, D.S. Shelar, S.P. Thakre (2009)"Solving Fuzzy Linear Programming Problem as Multi Objective Linear Programming Problem" Proceedings of the World Congress on Engineering ,Vol II WCE , London, U.K.

9.Teng-San Shih, Jin-Shieh Su and Jing-Shing Yao(2009) "Fuzzy linear Pprogramming based on interval-valued Fuzzy sets". , International Journal of Innovative Computing, Information and Control ICIC International °c 2009 ISSN 1349- Volume 5, Number 8,pp. 208

10. Mohanaselvi S & Ganesan K(2012) Fully Fuzzy linear programs with triangular fuzzy number International journal of Mathematical Archieve 3(5) 1838-1845.

11. Das S K, Mandal T.(2017)" A MOLFP Method for Solving Linear Fractional Programming under Fuzzy Environment" IJRIE ; 6(3): 202– 213