DOMATIC EDGE DOMINATION IN BOOLEAN FUNCTION GRAPH B(G, L(G), NINC) OF A GRAPH

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***Abstract :** For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph B(G, L(G), NINC) of G is a graph with vertex set V(G) $\cup$ E(G) and two vertices in B(G, L(G), NINC) are adjacent if and only if they correspond to two adjacent vertices of G, two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_1(G)$. In this paper, Domatic edge domination numbers of Boolean Function Graphs of some standard graphs are obtained.*

***Keywords:** Boolean Function graph, Edge Domination Number.*

I. INTRODUCTION

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A subset D of V(G) is called a dominating set of G if every vertex not in D is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets of G. The open neighborhood N(v) of v in V is the set of vertices adjacent to v, and the set $N[v] = N(v)\cup\{v\}$ is the closed neighborhood of v. An edge e of a graph is said to be incident with the vertex v if v is an end vertex of e. In this case, we also say that v is incident with e.

A subset $F \subseteq E$ is called an edge dominating set of G if every edge not in F is adjacent to some edge in F. The edge domination number $\gamma'(G)$ of G is the minimum cardinality taken over all edge dominating sets of G. The maximum order of a partition of E into edge dominating sets of G is called the edge domatic number of G and is denoted by $d'(G)$. The concept of edge domination was introduced by Mitchell and Hedetniemi [8]. Jayaram [6] studied line (edge) dominating sets and obtained bounds for the line (edge) domination number and obtaind Nordhaus-Gaddum results for the line domination number. Arumugam and Velammal [1] have discussed edge domination number and edge domatic number. The complementary edge domination in graphs is studied by Kulli and Soner [7]. For graph theoretic notations and terminology, we follow Harary [2]. Janakiraman et al., introduced the concept of Boolean function graphs [3 - 5]. For a real x, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

II. PERIOR RESULTS

Observation 2.1 [3]

1. G and L(G) are induced subgraphs of $B_1(G)$.

2. Number of vertices in $B_1(G)$ is p + q and if $d_i = deg_G(v_i)$, $v_i \in V(G)$, then the number of edges in $B_1(G)$ is $q(p - 2) + ½\sum_{1\le i \le p} d_i^2$.

3. The degree of a vertex of G in $B_1(G)$ is q and the degree of a vertex of L(G) e′ in $B_1(G)$ is $deg_{L(G)}(e') + p - 2$. Also if $d^*(e')$ is the degree of a vertex e′ of L(G) in $B_1(G)$, then $0 \le d^*(e') \le p + q - 3$. The lower bound is attained, if $G \cong K_2$ and the upper bound is attained, if $G \cong K_{1,n}$, for $n \ge 2$.

Theorem 2.2. [9]. $\gamma' (B_1(P_n)) = n-1$.

Theorem 2.3. [9]. $\gamma' (B_1(C_n)) = n-1$.

Theorem 2.4. [9]. $\gamma' (B_1(K_{1,n})) = (n+4)/3$.

Theorem 2.5. [10]. $\gamma' (B_1(P_n^{+})) = \left\lceil \frac{3n}{2} \right\rceil$.

Theorem 2.6. [10]. $\gamma' (B_1(C_n^{+})) = \left\lceil \frac{3n}{2} \right\rceil = n-1$.

Theorem 2.7. [10]. $\gamma' (B_1(K_{1,n}^{+})) = n+2$.

Theorem 2.8. [3]. $\gamma'(B_1(G)) \le q+ \gamma'(G)$.

In this paper, Domatic edge domination number of Boolean Function Graph B(G, L(G), NINC) of some standard graphs are obtained.

III. MAIN RESULTS

In the following domatic edge domination number of $B_1(P_n)$, $B_1(C_n)$, $B_1(K_{1,n})$ and $B_1(W_n)$ are found.

Theorem 3.1. For the Path P_n on n ($n \ge 4$) vertices, $d'(B_1(P_n)) = n-1$.

Proof: Let $v_1, v_2, ..., v_n$ be the vertices and $e_{12}, e_{23}, …, e_{n-1,n}$ be the edges of P_n respectively. Then $v_1, v_2, ..., v_n, e_{1,2}, e_{2,3}, …, e_{n-1,n} \in V((B_1(P_n))$ where $e_{i,\ i+1} = (v_i, v_{i+1})$, i = 1, 2, …, n -1. $B_1(P_n)$ has 2n-1 vertices and n^2 - n- 1 edges. $\gamma '(B_1(P_n)) = n – 1$.

Case 1. n is odd

Maximum edge domatic partition of $B_1 (P_n)$ is given by the sets D_i, i = 1, 2, …, n-1, where

$D_i = \{(v_1,e_{i+1,i+2}), (v_2, e_{i+2,\ i+3}), … ,(v_{n - (i+1)}, e_{n-1, n}), (v_{n-i}, e_{1,2}), (v_{n-i+1}, e_{2,3}), …, (v_{n-1}, e_{i, i+1}), (v_n, e_{i+1, i+2})\}$, i = 1, 2, …, n-3, $|D_i| = n$.

$D_{n-2} = \{(v_{1}, e_{n-1, n})\}\cup (\cup_{i=1}^{\frac{n-3}{2}}\{(v_{2i},v_{2i+1})\}) \cup \{(v_{n-1}, v_n)\} \cup (\cup_{i=1}^{\frac{n-3}{2}}\{(e_{3i-2,\ 3i-1},\ e_{3i-1,\ 3i})\})$, $|D_{n-2}| = n-1$.

$D_{n-1} = \cup_{i=i}^{n-1/2}\{(v_{2i-1}, v_{2i})\} \cup (v_n, e_{12}) \cup \cup_{i=2}^{n-2}\{(e_{i,\ i+1},\ e_{i+1,\ i+2})\}$, $|D_{n-1}| =(3n- 5) / 2$.

where the suffices are integers modulo n. Therefore, $d'(B_1(P_n)) = n-1$.

Case 2. n is even

Edge domatic partition of $B_1(P_n)$ is given by the sets D_i, i = 1, 2, …, n-2, where

$D_i = \{(v_1, e_{i+1,i+2}), (v_2, e_{i+2, i+3}), \ldots, (v_{n-(i+1)}, e_{n-1, n}), (v_{n-i}\, e_{12}), (v_{n-i+1}\, e_{23}), \ldots, (v_{n-1}, e_{i, i+1}), (v_n, e_{i+1, i+2})\}$, i=1, 2, …, n-3. $|D_i| = n^2 - 3n$

$D_{n-2} = (v_1, e_{n-1, n}) \cup (\bigcup_{i=1}^{\frac{n-2}{2}}\{(v_{2i}, v_{2i+1})\}) \cup (\bigcup_{i=1}^{\frac{n-4}{2}}\{(e_{3i-2,\ 3i-1},\ e_{3i-1,3i})\}) \cup \{(e_{n-3, n-2}, e_{n-2, n-1})\}$, if $n \equiv 0 \pmod 4$. $|D_{n-2}| = n-1$

$D_{n-2} = \{(v_1, e_{n-1, n})\} \cup (\bigcup_{i=1}^{\frac{n-2}{2}}\{(v_{2i}, v_{2i+1})\}) \cup (\bigcup_{i=1}^{\frac{n-4}{2}}\{(e_{3i-2,\ 3i-1},\ e_{3i-1,3i})\}) \cup \{(e_{n-2, n-1}, e_{n-1,n})\}$, if $n \equiv 2 \pmod 4$. $|D_{n-2}| = n-1$

$D_{n-1} = \bigcup_{i=1}^{n/2}\{(v_{2i-1}, v_{2i})\} \cup (v_n, e_{12}) \cup \bigcup_{i=2}^{n-2}\{(e_{i,\ i+1},\ e_{i+1,\ i+2})\}$, $|D_{n-2}| = (3n-4)/2$

$\bigcup_{i=1}^{n-1} D_i = E(B_1(P_n))$. In Case 1 and Case 2, $D' = \{D_1, D_2, D_3, \ldots, D_n\}$ is a maximum edge domatic partition of $B_1(P_n)$. Hence, $d'(B_1(P_n)) = n-1$.

Theorem 3.2 For the cycle C_n on n $(n \geq 4)$ vertices, $d'(B_1(C_n)) = n$.

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices and $e_{1,2}, e_{2,3}, \ldots, e_{n-1, n}$ be the edges in C_n, where $e_{i,i+1} = (v_i, v_{i+1})$, i = 1, 2, …, n – 1, $e_{n,1} = (v_n, v_1)$. Then $v_1, v_2, \ldots, v_n, e_{1,2}, e_{2,3}, \ldots, e_{n-1,n}, e_{n1} \in V(B_1(C_n))$. $B_1(C_n)$ has 2n vertices and n^2 edges. $\gamma'(B_1(C_n)) = n - 1$.

Edge domatic partition of $B_1(C_n)$ is given by the sets D_i, i = 1,2, …, n, where $D_i = \{(v_1, e_{i+1,\ i+2}), (v_2, e_{i+2,\ i+3}), \ldots, (v_{n-(i+1)}, e_{n-1, n}), (v_{n-i}\ e_{1n}), (v_{n-i+1}, e_{12}), \ldots, (v_{n-1}, e_{i+1, i+2}), (v_n, e_{i+2, i+3})\}$, i = 1, 2, …, n-2. $|D_i| = n$.

$D_{n-1} = \{(v_1, v_2), (v_3, v_4), \ldots, (v_{i+(n-4)}), (v_{i+(n-3)}), (e_{1, i+1}, e_{i+1, i+2}), (e_{i+2, i+3}), (e_{i+3, i+4}), \ldots, (e_{n-1, n}, e_{1n})\}$, i =1,2, …, (n-2) / 2. $|D_i| = n-1$.

$D_n = \{(v_1, v_n)\} \cup (\bigcup_{i=1}^{\frac{n-3}{2}}\{(v_{2i}, v_{2i+1})\}) \cup (\bigcup_{i=1}^{\frac{n-3}{2}}\{(e_{2i,\ 2i+1},\ e_{2i+1,\ 2i+2})\}) \cup \{(e_{n-1, n}, e_{1n})\} \cup \{(e_{1n}, e_{12})\}$, if n is odd. $|D_n| = n$.

$D_n = \{(v_1, v_n)\} \cup (\bigcup_{i=1}^{\frac{n-2}{2}}\{(v_{2i}, v_{2i+1})\}) \cup (\bigcup_{i=1}^{\frac{n-2}{2}}\{(e_{2i,\ 2i+1},\ e_{2i+1,\ 2i+2})\}) \cup (e_{1n}, e_{12})$, if n is even. $|D_n| = n$.

$\bigcup_{i=1}^{n} D_i = E(B_1(C_n))$ and $D' = \{D_1, D_2, D_3, \ldots, D_n\}$ is a maximum edge domatic partition of $B_1(C_n)$. Hence, $d'(B_1(C_n)) = n$.

Theorem 3. 3 For the star $K_{1,n}$ on (n + 1) vertices $d'(B_1(K_{1,n})) = n$, where $n \geq 2$.

Proof: Let $v_1, v_2, v_3, \ldots, v_{n+1}$ be the vertices with v_1 as the central vertex and $e_{1,2}, e_{1,3}, \ldots, e_{1,n+1}$ be the edges in $K_{1,n}$, where $e_{1, i+1} = (v_1, v_{i+1})$, i = 2, 3, …, n. Then $v_1, v_2, \ldots, v_n, v_{n+1}, e_{1,2}, e_{1,3} \ldots, e_{1, n+1} \in V(B_1(K_{1,n}))$. $B_1(K_{1,n})$ has 2n + 1 vertices and (n(3n - 1))/2 edges. $\gamma'(B_1(K_{1,n})) = (n + 4)/3$.

Case 1. n is odd

Edge domatic partition of $B_1(K_{1,n})$ is given by the sets D_i, i = 1,2, …, n, and are given by $D_{i-2} = \{(v_2, e_{1,i}), (v_3, e_{1, i+1}), \ldots, (v_n, e_{1,i+(n-2)}), (v_{n+1}, e_{1, i-1}), (e_{1, i-1}, e_{1i}), (e_{1,i-1}, e_{1, i+1}), \ldots, (e_{1, i+n-3/2})\}$, i = 3, 4,…, n+1. $|D_{i-2}| = n$. $D_n = \{(v_1, v_2), (v_1, v_3), \ldots, (v_1, v_{n+1}), (e_{1,n+1}, e_{1,2}), (e_{1, n+1}, e_{1,3}), \ldots, (e_{1,n+1}, e_{1, (n+1)/2})\}$, where the suffcies are integers modulo n and $e_{1,0} = e_{1,n}$.

$\bigcup_{i=1}^{n} D_i = E(B_1(K_{1,n}))$ and $D' = \{D_1, D_2, D_3, \ldots, D_n\}$ is a maximum edge domatic partition of $B_1(K_{1,n})$. Hence, $d'(B_1(K_{1,n})) = (n+1-2) + 1 = n$.

Case 2. n is even

Edge domatic partition of $B_1(K_{1,n})$ is given by the sets D_i, i = 1,2, …, n and are given by

$D_{i-2} = \{(v_2, e_{1i}), (v_3, e_{1, i+1}), \ldots, (v_n, e_{i+(n-2)}), (v_{n+1}, e_{1, i-1}), (e_{1, i-1}, e_{1i}), (e_{1,i-1}, e_{1, i+1}), \ldots, (e_{1, i+n-2/2})\}$, i =3, 4,…, (n+4)/2. $|D_{i-2}| = n$.

$D_{i-1} = \{(v_2, e_{1i}), (v_3, e_{1, i+1}), \ldots, (v_n, e_{i+(n-2)}), (v_{n+1}, e_{1,i-1}), (e_{1,i-1}, e_{1i}), (e_{1,i-1}, e_{1,i+1}), \ldots, (e_{1, i-1,\ i+(n-4)/2})\}$, i =(n+6)/2, …, n+1. $|D_{i-1}| = n$.

$D_n = \{(v_1, v_2), (v_1, v_3), \ldots, (v_1, v_{n+1}), (e_{1,n+1}, e_{1,2}), (e_{1, n+1}, e_{1,3}), \ldots, (e_{1,n+1}, e_{1, n/2})\}$, $|D_n| = n-1$.

$\bigcup_{i=1}^{n} D_i = E(B_1(K_{1,n}))$ and $D' = \{D_1, D_2, D_3, \ldots, D_n\}$ is a maximum edge domatic partition of $B_1(K_{1n})$. Hence, $d'(B_1(K_{1n})) = ((n+4)/2) - 2 + ((n-2)/2) + 1 = (2n-2+2)/2 = n$.

Theorem 3.4 For the Wheel W_n on n $(n \geq 5)$ vertices, $d'(B_1(W_n)) = n$.

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices with v_1 as the central vertex and $e_{1,2}, e_{1,3}, \ldots, e_{1,n}, e_{2,3}, \ldots, e_{n-1, n}, e_{n,2}$ be the edges in W_n, where $e_{i, i+1} = (v_i, v_{i+1})$, and $e_{1, i+1} = (v_1, v_{i+1})$, i = 2, 3, ..., n-1, $e_{n2} = (v_n, v_2)$. Then $v_1, v_2, \ldots, v_n, e_{1,2}, e_{1,3}, \ldots, e_{1,n} \in V(B_1(W_n))$. $B_1(W_n)$ has 2n – 1 vertices and ((n – 1) (3n - 4))/2 edges. $\gamma'(B_1(W_n)) = n - 1$. Edge domatic partition of $B_1(W_n)$ is given by the sets D_i, i = 1,2, …, n, where

$D_{i-1} = \{(v_1, e_{i,\ i+1}), (v_2, e_{i+1,\ i+2}), \ldots, (v_{n-1}, e_{1,n}), (v_n\ e_{1,2}), (v_3, e_{1,4}), (e_{1, n-2}, e_{1,n}), (e_{1, i+1}, e_{1, i+n-3}), (e_{1,i+2}, e_{i+2, i+3})\}$, i= 2, 3, …, n-1. $|D_{i-1}| = n$.

Case 1. n is odd

$D_{n-1} = \{(v_1, v_{n-2})\} \cup (\bigcup_{i=1}^{\frac{n-1}{2}}\{(v_{2i}, v_{2i+1})\}) \cup \{(v_n, v_1), (e_{1,2}, e_{2n})\} \cup (\bigcup_{i=1}^{\frac{n-1}{2}}\{(e_{1,\ i+2},\ e_{i+1,\ i+2})\}) \cup \{(e_{1, n-1}, e_{n-2, n-1})\}$. $|D_{n-1}| = n+2$.

$D_n = \{(v_1, v_{n-1})\} \cup \{(v_2, v_n)\} \cup (\bigcup_{i=1}^{\frac{n-3}{2}}\{(v_{2i+1}, v_{2i+2})\}) \cup \{(e_{1n}, e_{2n})\} \cup (\bigcup_{i=1}^{\frac{n-1}{2}}\{(e_{i+1,\ i+2},\ e_{i+2,\ i+3})\}) \cup \{(e_{n-3,\ n}, e_{n-1,\ n})\} \cup \{(e_{1n}, e_{n-1,\ n})\}$. $|D_n| = n$.

Case 2. n is even

$D_{n-1} = \{(v_1, v_{n-2})\} \cup \bigcup_{i=1}^{\frac{n-2}{2}}\{(v_{2i}, v_{2i+1})\} \cup \{(v_n, v_1), (e_{12}, e_{2n})\} \cup \bigcup_{i=1}^{\frac{n-2}{2}}\{(e_{i,\ i+2},\ e_{i+1,\ i+2})\} \cup \{(e_{2,3}, e_{2,n})\} \cup \bigcup_{i=1}^{\frac{n-4}{2}}\{(e_{1,i+4},\ e_{i+3,i+4})\}$. $|D_n| = 3n/2$.

$D_n = \{(v_1, v_{n-1})\} \cup \{(v_2, v_n)\} \cup \bigcup_{i=1}^{\frac{n-4}{2}}\{(v_{2i+1}, v_{2i+2})\} \cup \{(e_{1n}, e_{2n})\} \cup \bigcup_{i=1}^{\frac{n-2}{2}}\{(e_{i+1,\ i+2},\ e_{i+2,\ i+3})\} \cup \{(e_{2n}, e_{n-1, n})\} \cup \{(e_{1n}, e_{n-1, n})\}$. $|D_n| = n+2$.

$\bigcup_{i=1}^{n} D_i = E(B_1(W_n))$ and $D' = \{D_1, D_2, D_3, \ldots, D_n\}$is a maximum edge domatic partition of $B_1(W_n)$. Hence, $d'(B_1(W_n)) = n$.

Theorem: 3.5 For any graph (p,q) graph G, $d'(B_1(G)) \geq \left\lceil \frac{2q(p-2)}{p+2q} \right\rceil$.

Proof: Number of edges in $B_1(G) = q(p-2) + \frac{1}{2}\Sigma d_i^2$. But $d_i \geq 0$.

Therefore, $E(B_1(G)) \geq q(p-2)$. Also, by Theorem [2.8]

$\gamma'(B_1(G)) \leq q + \gamma'(G) \leq q + \left\lfloor \frac{p}{2} \right\rfloor$

If P is even, then $\frac{E(B1(G))}{\gamma'(B1(G))} \geq \frac{q(p-2)}{(q+p/2)} = \frac{2q(p-2)}{(p+2q)}$ and hence of $B_1(G)$,

Number of edge dominating sets of $B_1(G) \geq \frac{2q(p-2)}{(p+2q)}$

Therefore, $d'(B_1(G)) \geq$ number of edge dominating sets of $B_1(G) \geq \left\lceil \frac{2q(p-2)}{p+2q} \right\rceil$.

If P is odd, then $\left\lfloor \frac{p}{2} \right\rfloor = \frac{(p-1)}{2}$.

$$\frac{E(B1(G))}{\gamma'(B1(G))} \geq \frac{q(p-2)}{(q+(p-1)/2)} = \frac{2q(p-2)}{(p+2q-1)} \geq \left\lceil \frac{2q(p-2)}{p+2q} \right\rceil .$$

Remark: $\gamma'(B_1(G)) \geq p$.

IV. CONCLUSION

In this paper, domatic edge domination numbers of Boolean Function Graph B(G, L(G), NINC) of path, cycle, stars and wheel are obtained.

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