DOUBLE LAYERED FUZZY PLANAR GRAPH WITH POINTS OF INTERSECTION BETWEEN THE EDGES

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Abstract: Fuzzy planar graph is an important subclasses of fuzzy graph. In this paper, we define a new fuzzy graph double layered fuzzy planar graph (DLFPG) and weak double layered fuzzy planar graph.

Keywords: Fuzzy graphs, Fuzzy planar graph, double layered fuzzy planar graph, weak double layered fuzzy planar graph, fuzzy faces.

Introduction

Fuzzy set theory was introduced by Zadeh in 1965 [8], Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [1]. Though introduced recently, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang [2] have also introduced various concepts in connectedness with fuzzy graph. Nagoorgani and Malarvizhi [3] have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs Abdul – Jabbar and Naoom [5] introduced the concept of fuzzy planar graph. Also, Nirmala and Dharabal [6] defined special fuzzy planar graph A.Pal, S.Samanta and M.Pal [4] have defined fuzzy planar graph in a different concept where crossing of edge are allowed. Pathinathan and JesinthaRosline [5] have introduced double layered fuzzygraph. The intuitionistic double layered fuzzy graph is given by JesinthaRoseline and Pathinathan [7]. In this paper we define double layered fuzzy planar graph (DLFPG) and we discuss some properties. We introduce a new fuzzy graph called double layered fuzzy planar graph, the theoretical concepts of double layered fuzzy planar graph and finally we give conclusions on DLPFG.

Definition 4.1

Let ψ =(V,σ,E) be a fuzzy multi-graph with the underlying crisp multi-graph ψ^*=(V,σ^*,E^*). The vertex set of DL(ψ) be $\sigma^* \cup E^*$.

The geomentrical representation DLP_1, DLP_2,…DLP_n be the points of intersections between the edges DL(ψ)=(V,σ_{DL},E_{DL})is said to be double layered fuzzy planar graph(DLFPG) with double layered fuzzy planarity value f_{DL}, where

$$f_{DL} = \frac{1}{1+\{I_{DLP_1} + ...I_{DLP_N}\}}$$

It is obvious that f_{DL} is bounded and the range of f_{DL} is $0<f_{DL}\leq 1$.

Algorithm For Double Layered Fuzzy Planar Graph

Step: 1

We consider the fuzzy planar graph

Step:2

We choose the edges of the fuzzy planar graph to consider new vertex of the fuzzy planar graph. We get new vertex set of the double layered fuzzy planar graph (V+E).

Step:3

In the given graph, draw edgesfor the adjacent vertex and also for the adjacent edges.

Step:4

Here we will get a new graph which has many intersecting edges. Plot the intersecting point as DLP_1,……DLP_N.

Step:5

Evaluate the value for the intersecting point using the formula

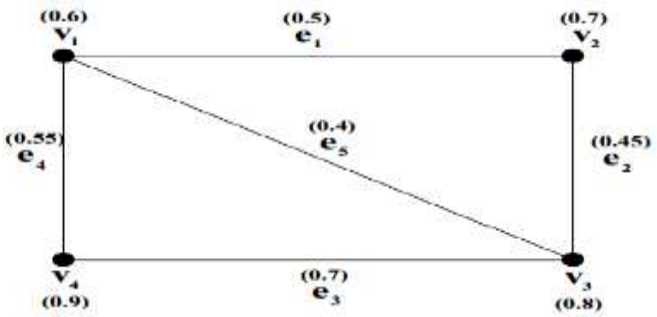
$$DLI_{(a,b)} = \frac{(a,b)\mu^i}{(\sigma(a)\Lambda\sigma(b))} \text{ and } I_{DLP_K} = \frac{DLI_{(a,b)} + DLI_{(c,d)}}{2} \text{ ,where k=1,2,….N}$$

Step:6

If the double layered planarity value is $0<f_{DL}\leq 1$. Then the graph is double layered fuzzy planar graph.

Example

Consider the fuzzy planar graph φ, whose crisp graph φ^* is a cycle with n=4 vertices



A fuzzy planar graph ψ =(V,σ,E)

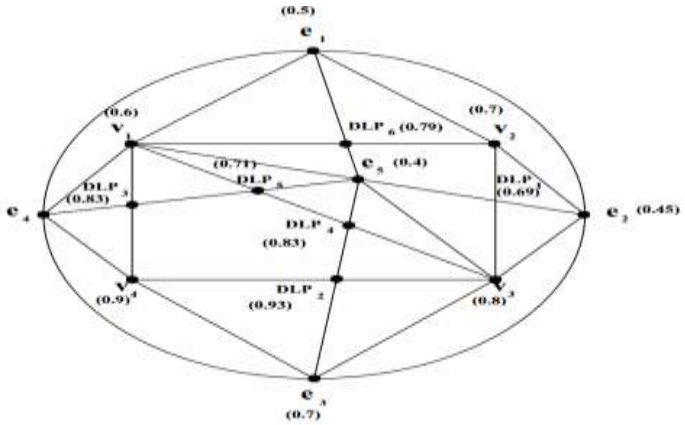


Fig 4.1 (a) Double layered fuzzy planar graph

Here we calculate the intersecting value at the intersecting point between two edges.
Two edges (b,c) and (e_2,e_5) are intersected where σ(a)=(0.7), σ(b)=(0.8), E(e_2)=(0.45), E(e_5)=(0.4), E(b,c)=0.5, E(e_2)∧E(e_5)=0.3. strength of the edge (b,c) is $\frac{0.45}{0.7}=0.57$, i.e., $DLI_{(b,c)}$=0.64 and that of (e_2,e_5) is $\frac{0.3}{0.4}=0.75$ i.e., $DLI_{(e_2,e_5)}=0.75.$ thus the intersecting value at the point is $\frac{0.65+0.75}{2}=0.695.$

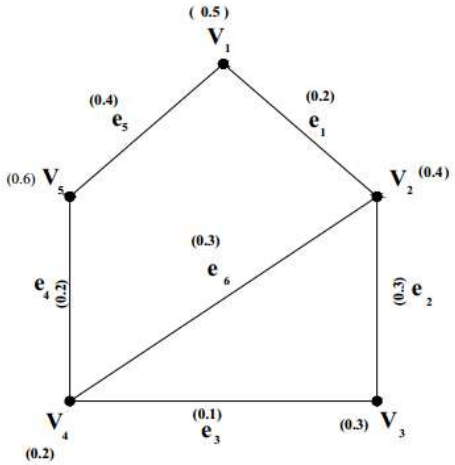
Therefore,

DLP_1=0.695

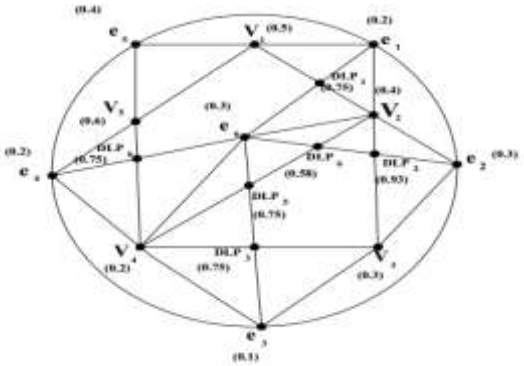
Similarly we can find,

DLP_2 = 0.93
DLP_3 = 0.83
DLP_4 = 0.83
DLP_5 = 0.17
DLP_6 = 0.79

Consider the fuzzy planar graph with n=5 vertices



A fuzzy planar graph ψ =(V,σ,E)



4.1 (b) Double layered fuzzy planar graph DL(ψ)=(V,σ_{DL},E_{DL})

Definition 4.3

A double layered fuzzy planar graph DL(φ) is called weak double layered fuzzy planar graph, if the double layered fuzzy planarity value of the graph is less than or equal to 0.5.

Definition 4.4

Let DL(ψ)=(V,σ_{DL},E_{DL}) be a double layered fuzzy planar graph and E_{DL}={((a,b),(a,b)μ^i),i=1,2,…P_{ab}/(a,b)∈V×V} P_{ab}=max{i/(a,b)μ^i≠0}.

A fuzzy face of DL(φ) is a region bounded by the set of fuzzy edges $E^1_{DL} \subset E_{DL}$ of a geometric representation of DL(φ). The membership value of the fuzzy face is

$$\min\left\{\frac{(a,b)\mu^i}{\sigma(a)\Lambda\sigma(b)}, i=1,2,....P_{ab}/(a,b)\in E^1{}_{DL}\right\}$$

A fuzzy face is called weak fuzzy face if its membership value is <0.5 and otherwise strong face. Every double layered fuzzy planar graph has an infinite region which is called outer fuzzy face. otherwise face called inner fuzzy faces.

THEORETICAL CONCEPTS

Theorem:4.1

Let DL(ψ) be a weak double layered fuzzy planar graph .The number of point of intersection between strong edges in DLP_1,……DLP_N.

Proof:

Let DL(ψ)=(V,σ_{DL},E_{DL}) be a weak double layered fuzzy planar graph .

If possible DL(ψ) has one point ofintersection between two strong edges in

DL(ψ).

For any strong edge $((v_1,v_2),(v_1,v_2)\mu^i)$

$(v_1,v_2)\mu^i > \frac{1}{2}(\sigma(v_1),\sigma(v_2))$

So $DLI_{(V1,V2)} \geq 0.5$

Thus for two intersection strong edges $((v_1,v_2),(v_1,v_2)\mu^i)$ and $((v_3,v_4),(v_3,v_4)\mu^i)$

$\frac{DLI_{(v_1,v_2)}+DLI_{(v_3,v_4)}}{2} \geq 0.5$, thatis $I_{DLP1} \geq 0.5$

Then $1+0.5 \geq 1.5$

Therefore $f_{DL}=\frac{1}{1+I_{DLP1}}>0.5$

The fact that the fuzzy graph is a weak double layered fuzzy planar graph.

So number of points of intersection between strong edges cannot be one.

It is clear that if the number of point of intersection of strong edges in one then

The double layered fuzzy planarity value $f_{DL} \geq 0.5$.

Thus we conclude that the number of point of intersection between strong edges DL(ψ) is DLP_1,……DLP_N.

Theorem 4.2

Let DL(ψ) be a double layered fuzzy planar graph .Then DL(ψ) is not strong double layered fuzzy planar graph

Proof

Let DL(ψ)=(V,σ_{DL},E_{DL}) be a weak double layered fuzzy planar graph. If possible DL(ψ) has at least two intersecting between point $DLP_1$and DLP_2.

For any strong edge $((v_1,v_2),(v_1,v_2)\mu^i)$

$(v_1,v_2)\mu^i > \frac{1}{2}(\sigma(v_1),\sigma(v_2))$

So $DLI_{(,v_1,v_2)} \geq 0.5$

Thus for two intersection strong edges $((v_1,v_2),(v_1,v_2)\mu^i)$ and $((v_3,v_4),(v_3,v_4)\mu^i)$

$\frac{DLI_{(v_1,v_2)}+DLI_{(v_3,v_4)}}{2} \geq 0.5$, That is $I_{DLP1} \geq 0.5$

Similarly $I_{DLP2} \geq 0.5$

Then $1+I_{DLP1}+I_{DLP2} \geq 2$

There for $f_{DL}=\frac{1}{1+IDLP1+IDLP2} \leq 0.5$.

It is clearly that double layered fuzzy planar is not strong double layered fuzzy planar graph.

Theorem 4.3

Let DL(ψ) be a double layered fuzzy planar graph with double layered planarity value $0< f_{DL} \leq 1$ .Then DL(ψ) has strong and weak fuzzy face.

Proof

Let DL(ψ) be a double layered fuzzy planar graph with double.

Let $F_1$and F_2 be two fuzzy faces, F_1 is bounded by the edges $((v_1,v_2),(v_1,v_2)\mu^i)$, $((v_3,v_4),(v_3,v_4)\mu^j)$ and $((v_4,v_5),(v_4,v_5)\mu^k$.

Then,

$$\min\left\{\frac{(v_1,v_2)\mu^i}{\sigma(v_1)\Lambda\sigma(v_2)},\frac{(v_3,v_4)\mu^j}{\sigma(v_3)\Lambda\sigma(v_4)},\frac{(v_5,v_6)\mu^k}{\sigma(v_5)\Lambda\sigma(v_6)}\right\}>0.5$$

i.e., its membership value is 0.5

similarly, F_2 is fuzzy bounded face with membership value 0.33.

Every strong fuzzy face has membership value is greater than 0.5.

So, every edge of a strong fuzzy face is a strong fuzzy edge, and every weak fuzzy face has membership value is less than 0.5. So, DL(φ) has strong and weak fuzzy faces.

Conclusion

This study describes the double layered fuzzy planar graph and its properties, we have defined a new term called double layered fuzzy planarity value of a fuzzy graph. If the double layered fuzzy planarity value of a fuzzy graph is one then no edge crosses other edge. This is a very important concept of fuzzy graph theory. Double layered fuzzy planarity value is less than 0.5 because this graph contain large number of intersecting points, so double layered fuzzy planar graph is defined as weak double layered fuzzy planar graph.

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