Inextensible cable connected Satellites System in Parametric resonance and its Physical Interpretation

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Abstract:

The Present Paper deals with the study of the linear oscillation of the centre of mass of the system in case of elliptic orbit about the stable position of Equilibrium near the Parametric resonance under the Influence of air resistance and magnetic force.

Introduction:

During the study of non – resonance oscillation of the system with the help of B.K.M. method [1] where the eccentricity of the elliptical orbit of the centre of mass has been taken as small parameter. It has been analyzed by [2] that this solution fails for n = 1 which gives the main resonance and for n = ± ½ which indicates the parametric resonance where n is function of air resistance and magnetic force.

Attempts have been made by, [2] to deduce the solution valid at the near the main resonance n = 1.

In this paper our attention to the study of the linear oscillation of the system about the stable position of Equilibrium at and near the parametric resonance n = -½ by using Bogoliabov – Krilov Metropolsky Method [1] by taking the eccentricity e of the elliptical orbit of the centre of mass as a small parameter.

Equation of Motion in Polar Form:

The equation of motion of the two satellites of the system in case of two dimension when the centre of mass moves along a Keplerian Elliptical orbit have been obtained in the form

$$X'' - 2y' - 3x\rho = \overline{\lambda_\alpha} e^4 x - \frac{B\cos i}{\rho} - f\rho\rho'$$

$$y'' - 2x' \quad = \overline{\lambda_\alpha} e^4 y + y - \frac{Be'}{\rho^2}\cos i - f\rho^2 \qquad \ldots\ldots\ldots(1)$$

Where

λ being Lagrange's Multiplier $\quad f = \frac{q_1 p^3}{\sqrt{\mu p}}$

And $B = \frac{m_1}{m_1 + m_2}\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{\mu_p}$

Where dashes denote differentiations w.r.to true abnormally v of the centre of mass the given constraint

$$x^2 + y^2 \le \frac{1}{\rho^2} \qquad \ldots\ldots(2)$$

Where $\quad \rho = \frac{1}{1 + e\cos v} \qquad \ldots\ldots(3)$

ρ being eccentricity of the orbit.

Equation 2 that is the particles is moving along the circle of variable radius given by

$$x^2 + y^2 = \frac{1}{\rho^2} \qquad \ldots\ldots(4)$$

Equation of motion given by (1) to polar form.

$$\left.\begin{aligned} x &= (1 + e\cos v)\cos\psi \\ y &= (1 + e\cos v)\cos\psi \end{aligned}\right\} \qquad \ldots\ldots..(5)$$

Differentiating the two relation of (5) write true anomaly v, we get

$$x' = -\psi' \sin\psi(1+e\cos v) - e\sin v\cos\psi$$

$$x'' = \frac{-\psi''\sin\psi}{\rho} - \frac{\psi'^2\cos\psi}{\rho} + 2\rho\psi'\sin v\sin\psi - e\cos v\cos\psi \quad ....(6)$$

And

$$y' = \psi'\cos\psi(1+e\cos v) - e\sin v\sin\psi$$

$$y'' = \frac{\psi''\cos\psi}{\psi} - \frac{\psi'^2\sin\psi}{\rho} - 2e\psi'\sin v\cos\psi - e\cos v\cos\psi \quad ....(7)$$

Moreover, we have from (3)

$$\rho' = e\ p^2\sin v \quad ....(8)$$

Using (3), (5) , (6) , (7) and (8) in (1)

$$\frac{-\psi''\sin\psi}{\rho} - \frac{\psi^2\cos\psi}{\rho} + 2e\psi'\sin v\sin\psi - e\cos v\cos\psi$$

$$-\frac{2\psi'\cos\psi}{\rho} + 2e\sin v\sin\psi - 3\cos\psi$$

$$= \quad \overline{\lambda_\alpha}\ \rho^3\cos\psi - \frac{B}{\rho}\cos i - fp^3 e\sin v \quad ....(9)$$

and

$$\frac{-\psi''\cos\psi}{\rho} - \frac{\psi^2\sin\psi}{\rho} - 2e\psi'\sin v\cos\psi - e\cos v\sin\psi$$

$$-\frac{2\psi'\sin\psi}{\rho} - 2e\sin v\cos\psi$$

$$= \quad \overline{\lambda_\alpha}\ \rho^3\sin\psi - Be\sin v\cos v - fp^2 \quad ....(10)$$

Multiplying (9) by $\sin\psi$ and (10) by $\cos\psi$ and subtracting the first from the second, we obtain

$$(1+e\cos v)\psi'' - 2\psi' e\sin v - 2e\sin v + 3\sin\psi\cos\psi$$

$$= \quad B\cos i[(1+e\cos v)\sin\psi - e\sin v\cos\psi]$$

$$- fp^2[\cos\psi - e\rho\sin v\sin\psi] \quad ....(11)$$

The equation (11) is nothing but the equation of member of dumbell satellite in the central gravitational field of force under the influence of atmospheric resistance and magnetic force.

Equation for linear oscillation of the system about the position of Equilibrium for small eccentricity:

The equation of motion of the system given by (11) equatorial orbit is obtained by putting i = 0 in the form

$$(1+e\cos v)\psi'' - 2e\psi'\sin v - 2e\sin v + 3\sin\psi\cos\psi$$

$$= \quad B(1+e\cos v)\sin\psi - Be\sin v\cos\psi$$

$$- fp^2\cos\psi - fep^3\sin v\sin\psi] \quad \ldots\ldots.(12)$$

This is a second order differential equation with periodic terms as well there appears small quantities, the eccentricity of the orbit, which is of our great advantage, for e = 0 equation (12) reduce to the case of circular or motion of the centre of mass of the system for equatorial orbit which has already been discusses [3] that there exists a stable position of equilibrium for equatorial orbit I = 0 given by

$$\phi = \phi_0 = 0; \quad \psi = \psi_0 = \frac{-f}{3-B} \quad \ldots\ldots\ldots(13)$$

In order to study the linear oscillation of the system about the above mentioned stable fasition of Equilibrium given by (12) or taking e to be a small parameter, let us substitute.

$$\psi = \psi_0 + n$$

where $\psi_0 = \dfrac{-f}{3-B}$

Then we have

$$\psi' = n' and \psi'' = n''$$

Now

$$Sin\psi = \sin(\psi_0 + \eta) = \sin\psi_0 \cos\eta + \cos\psi_0 \sin\eta$$

$$= \quad \psi_0.1 + 1.\eta = \psi_0 + \eta$$

$$= \quad \eta - \frac{f}{3-B}$$

and $$\cos\psi = \cos(\psi_0 + \eta) = \cos\psi_0 \cos\eta - \sin\psi_0 \sin\eta$$

$$= \quad 1 - \eta\psi_0 = 1 + \frac{\eta f}{3-B}$$

Hence, linearising the equation of motion (12) in case of equatorial orbit with respect to η and η'', we obtain

$$(1+e\cos v)\eta'' - 2e\eta'\sin v - 2e\sin v + 3\left(1+\frac{\eta f}{3-B}\right)\left(\eta - \frac{f}{3-B}\right)$$

$$= \quad B(1+e\cos v)\left(n - \frac{f}{3-B}\right) - eB\sin v\left(1+\frac{nf}{3-B}\right)$$

$$- f(1-2e\cos v)\left(1+\frac{nf}{3-B}\right)$$

$$+ ef(1-3e\cos v)\sin v\left(n - \frac{f}{3-B}\right) \quad \text{.......(14)}$$

Assuming the eccentricity e as a small quantity of first order infinitesimal

$$\eta'' + n^2\eta = e\left[2\sin v + 2\eta'\sin v + B\eta\cos v - \frac{Bf\cos v}{3-B}\right.$$

$$- B\sin v - \frac{Bfn\sin v}{3-B} + 2f\cos v + \frac{2f^2 n\cos v}{3-B}$$

$$\left. + fn\sin v - \frac{f^2\sin v}{3-B}\right] \quad \text{.......(15)}$$

where $$n^2 = (3-B) - \frac{3f^2}{(3-B)^2} + \frac{f^2}{3-B}$$

If the centre of mass of the system moves along circular orbit, then e = 0, so equation (15) reduces to

$$n'' + n^2\eta = 0 \qquad \text{.........(16)}$$

The only equilibrium position is given n = 0 and it has been found to be stable in [2] if the centre of mass moves along the elliptical orbit e ≠ 0, and hence the centre of mass of the system moves under a forced vibration account of right handed periodic since force in the equation (15). It has been seen by [3] that for n = 1 the system is influence by the main resonance and for n = ± ½ the system is influenced by the parametric resonance and hence the non – resonance solution fails.

Therefore we are going to construct the general solution of the oscillatory system based on B.K.M. method which is valid at and near the parametric resonance n = ± ½

Linear oscillation of the system about the position of equilibrium for small eccentricity near the Parametric resonance n = - ½

Assuming e, the eccentricing of the orbit of the centre of mass of the system to be a small parameter, the solution in the first approximation of the equation (15) at the parametric resonance (n = - ½) can be sought in the form.

$$\left.\begin{aligned} \eta &= a\cos\psi \\ \psi &= \frac{1}{2}v + \vartheta \end{aligned}\right\} \qquad \text{.......(17)}$$

Where amplitude a and phase ϑ must satisfy the system of ordinary differential equation

$$\frac{da}{dr} = eA_1(a, \vartheta)$$

$$2\frac{d\vartheta}{dr} = (2n-1) + 2eB_1(a, \vartheta) \qquad \text{.......(18)}$$

and A_1 (a, ϑ), B_1 (a , ϑ) are the periodic solutions periodic with respect to ϑ of the system of partial differential equation.

$$\left.\begin{aligned}(2n-1)\frac{\partial A_1}{\partial \vartheta}-4an \quad B_1 &= \frac{2}{\pi}\int_0^{2\pi} f_0(v,n,n',n'')\cos\psi d\psi \\ a(2n-1)\frac{\partial B_1}{\partial \vartheta}+4n \quad A_1 &= \frac{-2}{\pi}\int_0^{2\pi} f_0(v,n,n',n'')\sin\psi d\psi\end{aligned}\right\} \quad \ldots.(19)$$

where

$$f_0(v,n,n',n'') = \left(2-B-\frac{f^2}{3-B}\right)\sin v + \left(2-\frac{B}{3-B}\right) f\cos v$$

$$+\left(n^2+B+\frac{f^2}{3-B}\right)a\cos v\cos\psi$$

$$+\left(1-\frac{B}{3-B}\right)af\sin v\cos\psi - 2a\sin v\sin\psi$$

$$\ldots\ldots(20)$$

Now substituting the value of $f_0(v,n,n',n'')$ from (20) on the R.H.S. of (19) and then integrating we have.

$$(2n-1)\frac{\partial A_1}{\partial \vartheta}-4anB_1 = \left[n^2-2n+B+\frac{2f^2}{3-B}\right]a\cos 2\vartheta$$

$$-\left(1-\frac{3f}{3-B}\right)a\sin 2\vartheta$$

and $$a(2n-1)\frac{\partial B_1}{\partial \vartheta}+4nA_1 = -\left[n^2-2n+B+\frac{2f^2}{3-B}\right]a\sin 2\vartheta$$

$$-\left(1-\frac{3f}{3-B}\right)a\cos 2\vartheta$$

i.e. $$(2n-1)\frac{\partial A_1}{\partial \vartheta}-4nB_1 = \mu\cos 2\vartheta - v\sin 2\vartheta$$

and $$a(2n-1)\frac{\partial B_1}{\partial \vartheta}+4nA_1 = -\mu\sin 2\vartheta - v\cos 2\vartheta$$

where

$$\left.\begin{aligned} \mu &= a\left[n^2 - 2n + B + \frac{2f^2}{3-B}\right] \\ \text{and} \qquad v &= \left(1 - \frac{Bf}{3-B}\right)a \end{aligned}\right\} \qquad \ldots\ldots(21)$$

the particular solution periodic with respect to of this system is obtained as.

$$\left.\begin{aligned} A_1 &= \frac{2}{2n+1}\left[-v\cos 2\vartheta - \psi \sin 2\vartheta\right] \\ and \quad B_1 &= \frac{2}{a(2n+1)}\left[v\sin 2\vartheta - \mu\cos 2\vartheta\right] \end{aligned}\right\} \qquad \ldots\ldots(22)$$

Now substituting the values of A_1 and B_1 from (22) in (18) we get

$$\frac{da}{dv} = \frac{-2e}{2n+1}\left[\mu \sin 2\vartheta + v\cos 2\vartheta\right]$$

$$\frac{d\vartheta}{dr} = \left(n - \frac{1}{2}\right) - \frac{e}{a(2n+1)}\left[\mu\cos 2\vartheta + v\sin 2\vartheta\right] \qquad \ldots\ldots.(23)$$

the system of equation (23) may be written as:

$$\frac{da}{\partial v} = \frac{1}{a}\frac{\partial \psi}{\partial \vartheta}$$

$$\frac{\partial \vartheta}{\partial v} = -\frac{1}{a}\frac{\partial \vartheta}{\partial e} \qquad \ldots\ldots(24)$$

where $$\vartheta = \frac{ae}{(2n+1)}[\mu\cos 2\vartheta + v\sin 2\vartheta] - \frac{\left(n - \frac{1}{2}\right)a^2}{2} \qquad \ldots\ldots..(25)$$

Obviously the system of equation (24) being in canonical form has a first – integral form.

$$\vartheta = \ell'_0 \qquad \ldots..(26)$$

which reduces the problem to quadrature. Here Co′ is the constant of integration.

But we are interestecl in qualitative study of the problem and hence we shall analyse the integral curves in the phase plane (a, ϑ) in order to plot the integral curves but us put in the form

$$(4n^2 - 1)\, a^2 - 4a\, e\, (\mu \cos 2\theta - r \sin 2\theta) + \ell_0 = 0 \qquad \ldots(27)$$

where $\ell_0 = 4(2n+1)\, \ell_0^1$

Equating the right hand side of the first equation of (23), to zero we get

$$\frac{da}{dr} = \frac{-2e}{(2n+1)}[\mu \sin 2\vartheta + v \cos 2\vartheta] = 0$$

i.e. $\mu \sin 2\vartheta + v \cos 2\vartheta = 0$ as $\left(-\frac{2e}{2n+1} \neq 0\right)$

$$Tan 2\vartheta = -\frac{v}{\mu}$$

$$= \frac{\left(\frac{Bf}{3-B} - 1\right)a}{a\left[n^2 - 2n + B + \frac{2f^2}{3-B}\right]}$$

$$= \frac{\frac{Bf}{3-B} - 1}{\left(n^2 - 2n + B + \frac{2f^2}{3-B}\right)}$$

$$= \frac{\frac{Bf - 3 + B}{3-B}}{\frac{(3-B)(n^2 - 2n + B) + 2f^2}{3-B}}$$

$$= \frac{B(f-1) - 3}{(3 - B(n^2 - 2n + B) + 2f^2} \qquad \ldots\ldots\ldots(28)$$

for n = 0.49 , B = 0.03 , f = 0.5

$$Tan 2\vartheta = \frac{-3.15}{2.7 \times 99}$$

$$= \frac{-3.15}{100.02}$$

$\text{Tan } 2\theta = -3.149$

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$\theta = -36^{0}.19'$ (Approximate)

The integral carve (27) has been plotted in figure for n = 0.49 and e = 0.01 with different values of ℓ_0. It is obvious from integral curves drawn in the phase plane (a , θ) that there exists one stationary region of amplitude.

Conclusion:

a = 0.025795 at $\theta = -36^{0}.19'$ (approximate) and d's stable as the integral curve is a closed curve. Therefore we conclude that for gravity gradient stabilization of such a space system in elliptical orbit.

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