Decompositions of Ideal Topological Sets

Amala Priya. A
Research Scholar, Department of Mathematics, P. M. T College, Usilampatti.
Affiliated to Madurai Kamaraj University, Madurai, Tamil Nadu, India.

ABSTRACT: In this paper, some relationships of α-I-open sets, pre-I-open sets, semi-I-open sets, strongly β-I-open sets and b-I-open sets in ideal topological spaces are discussed. Furthermore, decompositions of ideal topological sets are established.

Keywords: α-open set, I-αlc-set, I-slc-set, λ-closed set.

1. Introduction

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

1. $A \in I$ and $B \subseteq A \Rightarrow B \in I$ and
2. $A \in I$ and $B \in I \Rightarrow A \cup B \in I$ [8].

Given a topological space (X, τ) with an ideal I on X and if ℘ (X) is the set of all subsets of X, a set operator $(.)*: \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I , is defined as follows: for $A \subseteq X$, $A^*(I, \tau)=\{x \in X \mid U \cap A \notin I)$, called the ⋆-topology, finer than τ is defined by $cl^*(A)=A \cup A^*(I, \tau)$ [7]. When there is no chance for confusion, we will simply write A* for A* (I, τ) and τ* for τ*(I, τ). int*(A) will denote the interior of A in (X, τ*) and (X, τ, I) is called an ideal topological space or an ideal space. A subset A of an ideal topological space (X, τ, I) is called *-closed if $A^* \subseteq A$. In this paper, we introduce new classes of sets by using λ-I-closed sets in ideal topological spaces and study their basic properties; and their connections with other types ofideal topological sets. Moreover, some new decompositions of ideal topological sets are obtained.

2. Preliminaries

Definition 2.1 A subset A of a space (X, τ) is called

1. α-open [12] if $A \subseteq int(cl(int(A)))$;

2. preopen [11] if $A \subseteq int(cl(A))$;
3. semi-open [9] if $A \subseteq cl(int(A))$;
4. β-open [1] if $A \subseteq cl(int(cl(A)))$;
5. b-open [2] if $A \subseteq int(cl(A)) \cup cl(int(A))$.

The family of all α-open (resp. preopen, semi-open, β-open, b-open) sets of X is denoted by $\alpha O(X)$ (resp. $PO(X)$, $SO(X)$, $\beta O(X)$, $BO(X)$).

Definition 2.2 A subset A of a space (X, τ) is called

1. a Λ-set if $A = A^{\wedge}$ where $A^{\wedge} = \cap \{G: A \subseteq G, G \in \tau\}$ [10].
2. a Λ_α-set if $A = \Lambda_\alpha(A)$ where $\Lambda_\alpha(A) = \cap\{G: A \subseteq G, G \in \alpha O(X)\}$ [5].
3. a Λ_s-set if $A = \Lambda_s(A)$ where $\Lambda_s(A) = \cap\{G: A \subseteq G, G \in SO(X)\}$ [4].
4. a Λ_p-set if $A = \Lambda_p(A)$ where $\Lambda_p(A) = \cap\{G: A \subseteq G, G \in PO(X)\}$ [4].
5. a Λ_β-set if $A = \Lambda_\beta(A)$ where $\Lambda_\beta(A) = \cap\{G: A \subseteq G, G \in \beta O(X)\}$ [13].
6. a Λ_b-set if $A = \Lambda_b(A)$ where $\Lambda_b(A) = \cap\{G: A \subseteq G, G \in BO(X)\}$ [6].

Definition 2.3 A subset A of a topological space (X, τ) is called λ-closed [3] if $A = L \cap F$, *where L is a Λ-set and F is closed.*

3. Characterizations of generalized I-closed sets

Definition 3.1 A subset A of an ideal topological space (X, τ, I) is called

1. λ-I-αg*-closed if $A = L \cap F$, where L is a Λ_α-set and F is *-closed.
2. λ-I-sg*-closed if $A = L \cap F$, where L is a Λ_s-set and F is *-closed.
3. λ-I-pg*-closed if $A = L \cap F$, where L is a Λ_p-set and F is *-closed.
4. λ-I-βg*-closed if $A = L \cap F$, where L is a Λ_β-set and F is *-closed.
5. λ-I-bg*-closed if $A = L \cap F$, where L is a Λ_b-set and F is *-closed.

Definition 3.2 A subset A of an ideal topological space (X, τ, I) is called

1. an I-αlc-set if $A = L \cap F$ where L is α-open and F is *-closed.
2. an I-slc-set if $A = L \cap F$ where L is semi-open and F is *-closed.

Definition 3.3 A subset A of an ideal topological space (X, τ, I) is called

1. $I_{\alpha g^*}$-closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is α-open.
2. I_{sg^*}-closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi-open.

Lemma 3.4 Every Λ_α -set (resp. Λ_s -set, Λ_p -set, Λ_β -set, Λ_b -set) is λ -I- α g*- closed (resp. λ-I-sg*-closed, λ-I-pg*-closed, λ-I- βg*-closed, λ-I-bg*-closed)but not conversely.

Example 3.5 Let X = {a, b, c}, τ = {θ, X, {c}, {a, c}, {b, c}} and I = {θ}. Then {a} is λ-I-αg*-closed but not a Λ_α-set.

Example 3.6 In Example 3.5, {a} is λ-I-sg*-closed but not a Λ_s-set.

Example 3.7 In Example 3.5, {a} is λ -I-pg*-closed but not a Λ_p-set.

Example 3.8 In Example 3.5, {a} is λ-I- βg*-closed but not a Λ_β-set.

Example 3.9 In Example 3.5, {a} is λ-I-bg*-closed but not a Λ_b -set.

Lemma 3.10 1. A subset $A \subseteq (X, \tau, I)$ is Iαg*-closed if and only if $cl^*(A) \subseteq \Lambda\alpha$ (A).

2. A subset $A \subseteq (X, \tau, I)$ is I sg* -closed if and only if $cl^*(A) \subseteq \Lambda s$ (A).

Lemma 3.11 For a subset A of an ideal topological space (X, τ, I), the following conditions are equivalent.

1. A is λ -I- α g*-closed.
2. A = L $\cap$ cl*(A) where L is a $\Lambda\alpha$ -set.
3. A = $\Lambda\alpha$ (A) $\cap$ cl*(A).

Theorem 3.12 For a subset A of an ideal topological space (X, τ, I), the followingconditions are equivalent.

1. (i) A is *-closed.

   (ii) A is Iαg*-closed and an I-αlc-set.

   (iii) A is Iαg*-closed and λ -I-αg*-closed.

2. (i) A is *-closed.

   (ii) A is Isg*-closed and an I-slc-set.

   (iii) A is Isg*-closed and λ -I-sg*-closed.

Remark 3.13 The following examples show that the concepts of

1. $I_{\alpha g}$*-closed sets and I-αlc-sets are independent of each other.
2. $I_{\alpha g}$*-closed sets and λ-I- α g*-closed sets are independent of each other.
3. I_{sg}*-closed sets and I-slc-sets are independent of each other.
4. I_{sg}*-closed sets and λ -I-sg*-closed sets are independent of each other.

Example 3.14 Let X = {a, b, c, d}, τ = { θ , X, {a}, {b, c}, {a, b, c}} and I = { θ }. Then

1. {b, d} is $I_{\alpha g}$* -closed but not an I-αlc-set.
2. {a} is an I-α lc-set but not Iαg*-closed.

Example 3.15 1. In Example 3.14, {b, d} is $I_{\alpha g}$*-closed but not λ-I-αg*- closed.

2. In Example 3.14, {a} is λ-I- α g*-closed but not $I_{\alpha g}$*-closed.

Example 3.16 1. In Example 3.14, {b, d} is I_{sg}*-closed but not an I-slc-set.

2. In Example 3.14, {a} is an I-slc-set but not I_{sg}*-closed.

Example 3.17 1. In Example 3.14, {b, d} is I_{sg}*-closed but not λ-I-sg*-closed.

2. In Example 3.14, {a} is λ-I-sg*-closed but not I_{sg}*-closed.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assuit Univ. 12(1983), 77-90.

[2] D. Andrijevic, On b-open sets, Mat. Vesnik, 48(1996), 59-64.

[3] F. G. Arenas, J. Dontchev and M. Ganster, On λ -sets and the dual of generalizedcontinuity, Questions Answers Gen. Topology, 15(1) (1997), 3-13.

[4] M. Caldas and J. Dontchev, G.Λ_s -sets and G.$\vee_s$ -sets, Mem. Fac. Sci. Kochi. Univer. (Math). 21(2000), 21-30.

[5] M. Caldas, D. N. Georgiou and S. Jafari, Study of (Λ, α)-closed sets and the related notions in topological spaces, Bull. Malay. Math. Sci. Soc. (2), 30(1)(2007),23-36.

[6] M. Caldas, S. Jafari and T. Noiri, On Λ_b -sets and the associated topology $\tau^{\Lambda b}$,Acta Math. Hungar. 110(4)(2006), 337-345.

[7] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97(4) (1990), 295-310.

[8] K. Kuratowski, Topology, Vol. I, Academic Press, New York, 1966.

[9] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[10] H. Maki, Generalized Λ-sets and the associated closure operator in special issuein commemoration of Prof. Kazusada IKEDA's Retirement, (1986), 139-146.

[11] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuousand weak pre continuous mappings, Proc. Math. and Phys. Soc. Egypt, 53(1982), 47-53.

[12] O. Njastad, On some classes of nearly open sets, Pacific J. Math. 15(1965), 961-970.

[13] T. Noiri and E. Hatir, Λ_{sp}-sets and some weak separation axioms, Acta Math. Hungar. 103(3) (2004), 225-232.