E-CORDIAL LABELING OF SOME PATH UNION GRAPHS

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***Abstract:** Path union of graph G i.e.$P_m(G)$ is obtained by fusing a copy of graph G at each vertex of a path P_m. Vertex of fusion is same and fixed for all graphs. We discuss e-cordial labeling of $P_m(G)$ for G =claw, paw, kite and show that $P_m(G)$ is e-cordial.*

***Key words:** E-cordial, path union, fusion, edge, vertex.*
***Subject Classification:** 05C78*

Introduction:

In 1997 Yilmaz and Cahit [4] introduced a weaker version of edge graceful labeling called E-cordial. The word cordial was used first time in this paper. Let G be a graph with vertex set V and edge set E. Let f be a function that maps E into {0,1} .Define f on V by $f(v) = \sum\{f(uv)/(uv)\in E\}$(mod 2).The function f is called as E cordial labeling if $|e_f(0)-e_f(1)|\leq 1$ and $|v_f(0)-v_f(1)|\leq 1$. Where $e_f(i)$ is the number of edges labeled with i = 0,1 and $v_f(i)$ is the number of vertices labeled with i = 0,1. We also use $v_f(0,1) = (a,b)$ to denote the number of vertices labeled with 0 are a in number and that with 1 are b in number. Similarly $e_f(0,1)=(x, y)$ to denote number of edges labeled with 0 are x in number and that labeled with 1 are y in number. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of papers on cordial labeling. A graph that admits E-cordial labeling is called as E- cordial graph. Yilmaz and Cahit has shown that Trees T_n with n vertices and Complete graphs K_n on n vertices are E – cordial iff n is not congruent to 2 (modulo 4). Friendship graph $C_3^{(n)}$ for all n and fans F_n for n not congruent to 1 (mod 4).They observe that a graph with n vertices is not e-cordial if n≡ 2(mod 4). One may refer A Dynamic survey of graph labeling for more details on completed work.

Preliminaries:

Fusion of vertex. Let G be a (p,q) graph. let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.[].If $u\epsilon G_1$ and $v\epsilon G_2$, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with v.

Path union of G i.e. $P_m(G)$ is obtained by taking a path P_m and m copies of graph G. Fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges, where G is a (p, q) graph. If we change the vertex on G that is fused with vertex of Pm then we generally get a path union non isomorphic to earlier structure. In this paper we define a e-cordial function f that does not depends on which vertex of given graph G is used to obtain path union. This allows us to obtain path union in which the same graph G is fused with vertices of Pm at different vertices of G, as our choice and the same function f is applicable to all such structures that are possible on $P_m(G)$.

Main Results:

Theorem 4.1 Path union of claw is e-cordial.

Proof: There are two possible structures of $P_m(G)$.From figure 4.1 it follows that one can take path union at vertex 'u' giving structure 1 and the one point union at point v will give structure 2.The two structures are non-isomorphic. Take a path $P_m =(v_1, v_2, ..v_m)$. Each edge on Pm has label '0'. At each vertex fuse a copy of Type A labeling at vertex u. The resultant graph has label distribution given by $v_f(0,1) = (2m,2m)$, $e_f(0,1) = (2m-1,2m)$. If we form path union at vertex v of claw then also the above type A label will work and label distribution also same.

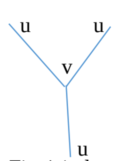


Fig 4.1 claw with vertevx u and v

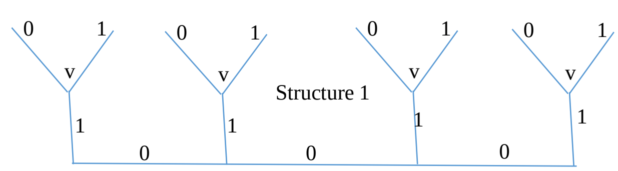
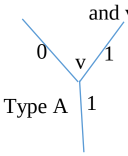


Fig 4.2 : P_4(claw) : $v_f(0,1) = (4,4)$, $e_f(0,1)= (7,8)$



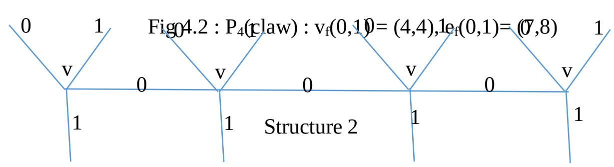


Fig 4.3 claw : $v_f(0,1) = (2,2)$, $e_f(0,1)= (1,2)$

Fig 4.4 : P_4(claw) : $v_f(0,1) = (4,4)$, $e_f(0,1)= (7,8)$

Theorem 4.2 Path union of paw is e-cordial for all m.

Proof: Define a function f:E(G)→{0,1}.It gives us following two types of labeled copies of paw.

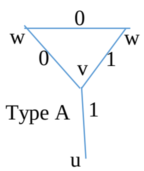


Fig 4.5 paw : $v_f(0,1) = (2,2)$, $e_f(0,1)= (2,2)$

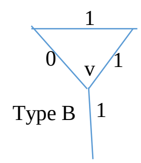


Fig 4.6 paw : $v_f(0,1) = (2,2)$, $e_f(0,1)= (1,3)$

There are three possible structures of $P_m(G)$.From figure 4.3 it follows that one can take path union at vertex 'u' giving structure 1 and the path union at point v will give structure 2 and path union at w giving structure 3. All structures are pairwise non-isomorphic. Take a path $P_m =(v_1, v_2, ..v_m)$. Each edge on P_m has label '0'. At each vertex v_i fuse a copy of Type A labeling (at vertex u) for all i ≡ 1 (mod 2) and copy of Type B if i≡ 0(mod 2). The resultant graph has label distribution given by $v_f(0,1) = (2m,2m)$, $e_f(0,1) = (2x+1,2x+1)$ for odd number m = 2x-1 ; x= 0, 1, 2. $v_f(0,1) = (2m,2m)$, $e_f(0,1) = (4+5(x-1),5x)$ for even number m = 2x= 1, 2. If we form path union at vertex 'v' or at 'w' of claw then also the above type A label will work and label distribution is also same. Thus the function f is independent of the vertex of paw used to form pathunion.

Theorem 4.3 Path union (P_m(house)) of house is e-cordial for m is not congruent to 2(mod 4)

Proof: Define a function f:E(G)→{0,1}.It gives us following three types of labeled copies of house graph. Take a path Pm = =$(v_1, v_2, ..v_m)$. Each edge on P_m has label '0'. At v_1 fuse a copy of type A label, at v_2 fuse a copy of type B label, at v_3 fuse type C label. For i>3 at each vertex v_i fuse a copy of Type A labeling for all i ≡ 0 (mod 4) and copy of Type B if i≡ 1,3(mod 4) copy of Type C if i≡ 2(mod 4). The resultant graph

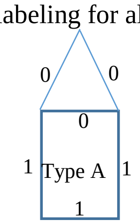


Fig 4.7 : $v_f(0,1) = (3,2)$, $e_f(0,1)= (3,3)$

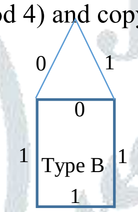


Fig 4.8 : $v_f(0,1) = (3,2)$, $e_f(0,1)= (2,4)$

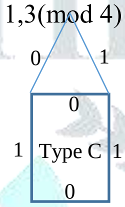


Fig 4.9 : $v_f(0,1) = (1,4)$, $e_f(0,1)= (3,3)$

has label distribution given by $v_f(0,1) = (3,2)$, $e_f(0,1) = (3,3)$ for m=1, for m = 2 is given by $v_f(0,1) = (6,4)$, $e_f(0,1) = (6,7)$. For all m of type 4x, x= 1, 2.. we have label distribution given by $v_f(0,1) = (10x,10x)$, $e_f(0,1) = (14x,14x-1)$.For all m of type 4x+1, x= 0, 1, 2.. we have label distribution given by $v_f(0,1) = (13+10x,12+10x)$, $e_f(0,1) = (14x+17,14x+17)$. For all m of type 4x+2, x= 0, 1, 2.. we have label distribution given by $v_f(0,1) = (14+10x,16+10x)$, $e_f(0,1) = (14x+21,14x+20)$. For all m of type 4x+3, x= 0, 1, 2.. we have label distribution given by $v_f(0,1) = (7+10x,8+10x)$, $e_f(0,1) = (14x+10,14x+10)$.

The function f is independent of the vertex of house used to form pathunion.

Theorem 4.4 Path union (G =P_m(bull) of bull is e-cordial for m is not congruent to 2(mod 4)

Proof: Define a function f:E(G)→{0,1}.It gives us following two types of labeled copies of bull graph.

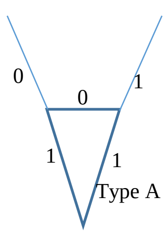


Fig 4.10 : $v_f(0,1) = (3,2)$, $e_f(0,1)= (2,3)$

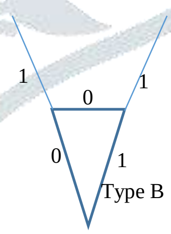


Fig 4.11 : $v_f(0,1) = (1,4)$, $e_f(0,1)= (2,3)$

Take a path Pm = =$(v_1, v_2, ..v_m)$. Each edge on P_m has label '0'. At v_1 fuse a copy of type A label, at v_2 fuse a copy of type A label, At v_3 fuse a copy of type B label, at v_4 fuse a copy of type A label, at v_5 fuse type A label. For i>5 at each vertex v_i fuse a copy of Type B labeling for all i ≡ 2 (mod 4) and copy of Type A if i≡ 0, 1, 3(mod 4). The resultant graph has label distribution given by $v_f(0,1) = (3,2)$, $e_f(0,1) = (2,3)$ for m=1, for m = 2 is given by $v_f(0,1) = (6,4)$, $e_f(0,1) = (5,6)$. For all m of type 4x, x= 1, 2.. we have label distribution given by $v_f(0,1) = (10x,10x)$, $e_f(0,1) = (12x-1,12x)$.For all m of type 4x+1, x= 1, 2.. we have label distribution given by $v_f(0,1) = (13+10(x-1),12+10(x-1))$, $e_f(0,1) = (12(x-1)+14,12(x-1)+15)$. For all m of type 4x+2, x= 1, 2.. we have label distribution given by $v_f(0,1) = (14+10(x-1),16+10(x-1))$, $e_f(0,1) = (12x+5,12x+6)$. For all m of type 4x+3, x= 0, 1, 2.. We have label distribution given by $v_f(0,1) = (7+10x,8+10x)$, $e_f(0,1) = (12x+8,12x+9)$.

The function f is independent of the vertex of bull used to form path union. The graph is e-cordial.

Theorem 4.5 Path union (G =P_m(bowtie)) of bull is e-cordial for m is not congruent to 2(mod 4) .

Proof: Define a function f:E(G)→{0,1}.It gives us following three types of labeled copies of bowtie graph. Take a path Pm = =$(v_1, v_2, ..v_m)$. Each edge on P_m has label '0'. At v_1 fuse a copy of type A label, at v_2 fuse a copy of type A label, At v_3 fuse a copy of type B label, at v_4 fuse a copy of type C label, at v_5 fuse type A label. For i>5 at each vertex v_i fuse a copy of Type C labeling for all i ≡ 1 (mod 4), Type B labeling for all i ≡ 2 (mod 4) and copy of Type A if i≡ 0, 3(mod 4).

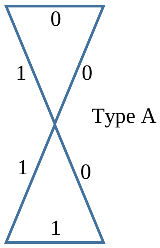


Fig 4.12 : $v_f(0,1)$ = (3,2), $e_f(0,1)$= (2,3)

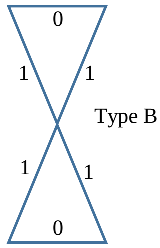


Fig 4.13 : $v_f(0,1)$ = (1,4), $e_f(0,1)$= (2,4)

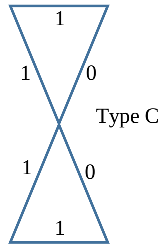


Fig 4.14 : $v_f(0,1)$ = (3,2), $e_f(0,1)$= (2,4)

The resultant graph has label distribution given by $v_f(0,1)$ = (3,2), $e_f(0,1)$ = (3,3) for m=1, for m = 2 is given by $v_f(0,1)$ = (6,4), $e_f(0,1)$ = (7,6). For all m of type 4x, x= 2, 3, .. We have label distribution given by $v_f(0,1)$ = (10x,10x), $e_f(0,1)$ = (14x,14x-1).

For m =4 we have $v_f(0,1)$ = (10,10), $e_f(0,1)$ = (13,14).

For all m of type 4x+1, x= 1, 2.. we have label distribution given by $v_f(0,1)$ = (13+10(x-1),12+10(x-1)), $e_f(0,1)$ = (17+14(x-1),17+14(x-1)).

For all m of type 4x+2, x= 1, 2.. we have label distribution given by $v_f(0,1)$ = (14+10(x-1),16+10(x-1)), $e_f(0,1)$ = (20+14(x-1),21+14(x-1).

For all m of type 4x+3, x= 0, 1, 2.. We have label distribution given by $v_f(0,1)$ = (7+10x,8+10x), $e_f(0,1)$ = (14x+10,14x+10).

The function f is independent of the vertex of bowtie used to form path union. The graph is e-cordial.

Theorem 4.6 Path union (G =P_m(dart) of dart is e-cordial for m is not congruent to 2(mod 4).

Proof: Define a function f:E(G)→{0,1}.It gives us following three types of labeled copies of dart graph.

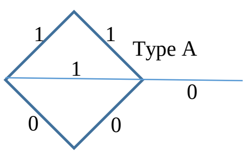


Fig 4.15 : $v_f(0,1)$ = (3,2), $e_f(0,1)$= (3,3)

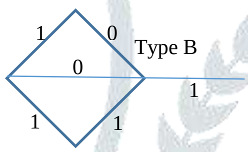


Fig 4.16 : $v_f(0,1)$ = (3,2), $e_f(0,1)$= (2,4)

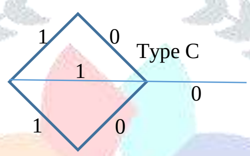


Fig 4.17 : $v_f(0,1)$ = (1,4), $e_f(0,1)$= (3,4)

Take a path Pm = =(v_1, v_2, ..v_m). Each edge on P_m has label '0'. At v_1 fuse a copy of type A label, at v_2 fuse a copy of type B label, at v_3 fuse type C label. For i>3 at each vertex v_i fuse a copy of Type A labeling for all i ≡ 0 (mod 4) and copy of Type B if i≡ 1,3(mod 4) copy of Type C if i≡ 2(mod 4). The resultant graph has label distribution given by $v_f(0,1)$ = (3,2), $e_f(0,1)$ = (3,3) for m=1, for m = 2 is given by $v_f(0,1)$ = (6,4), $e_f(0,1)$ = (6,7). For all m of type 4x, x= 1, 2.. we have label distribution given by $v_f(0,1)$ = (10x,10x), $e_f(0,1)$ = (14x,14x-1).For all m of type 4x+1, x= 0, 1, 2.. we have label distribution given by $v_f(0,1)$ = (13+10x,12+10x), $e_f(0,1)$ = (14x+17,14x+17). For all m of type 4x+2, x= 0, 1, 2.. we have label distribution given by $v_f(0,1)$ = (14+10x,16+10x), $e_f(0,1)$ = (14x+21,14x+20). For all m of type 4x+3, x= 0, 1, 2.. we have label distribution given by $v_f(0,1)$ = (7+10x,8+10x), $e_f(0,1)$ = (14x+10,14x+10).

The function f is independent of the vertex of house used to form pathunion.

Conclusions:

In this paper we have discussed and shown that path union P_m(G) is e- cordial for certain choice of G. The e-cordial function f we defined is such that irrespective of the vertex on G used to fuse with path vertex to obtain P_m(G), the graph is e-cordial.

We have shown that 1) Path union of claw is e-cordial for all m.2) Path union of paw is e-cordial for all m.3) Path union (P_m(house)) of house is e-cordial for m is not congruent to 2(mod 4). 4)Path union (G =P_m(bull) of bull is e-cordial for m is not congruent to 2(mod 4) 5) Path union (G =P_m(bowtie)) of is e-cordial for m is not congruent to 2(mod 4). 6) Path union (G =P_m(dart)) of dart is e-cordial for m is not congruent to 2(mod 4).Our work confirms one observation by Cahit that a graph with n vertices is not e-cordial if n≡ 2(mod 4).

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