Nanofluid with Heat Source for Free Convection Boundary Layer Flow

[1*]Dr. Jaweriya Sultana
[1*] Dept. of Mathematics, School of Engineering, Central University of Karnataka-585367, Karnataka, India,

***Abstract*— This paper presents an outline the derivation of conservation equations applicable to a nanofluid in the absence of a solid matrix. The modified equations to the case of a porous medium saturated by the nanofluid in presence of heat source. In this paper, we have employed a Darcy model for the momentum equation. We do not anticipate that the inclusion of a Brinkman term in that equation will have a major qualitative effect.**

***Index Terms*— Nanofluid, heat transfer, stability, instability.**

I. Introduction

Nanotechnology provides new area of research to process and produce materials with average crystallite sizes below 100nm called nanomaterials. The term “nanomaterials” encompasses a wide range of materials including nanocrystalline materials, nanocomposites, carbon nanotubes and quantum dots. The term “nanofluid” refers to a liquid containing a dispersion of submicronic solid particles (nanoparticles). The term was coined by choi[1]. The characteristic feature of nanofluid is thermal conductivity enhancement, a phenomenon observed by masuda et al [2]. This phenomenon suggests the possibility of using nanofluid in advanced nuclear systems [3]. Another recent application of nanofluid flow is nano-drug delivery [4].

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [5], who says that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on the further heat transfer enhancement observed in convective situations. Buongiorno notes that several authors have suggested that convective heat transfer enhancement could be due to the dispersion of the suspended nanoparticles but she argues that this effect is too small to explain the observed enhancement. Buongiorno also concludes that turbulence is not affected by the presence of the nanoparticles so this cannot explain the observed enhancement. Particle rotation has also been proposed as a cause of heat transfer enhancement, but Buongiorno calculates that this effect is too small to explain the effect. With dispersion, turbulence and particle rotation ruled out as significant agencies for heat transfer enhancement, Buongiorno proposed a new model based on the mechanics of the nanoparticle/ base-fluid relative velocity.

Buongiorno [5] noted that the nanoparticle absolute velocity can be viewed as the sum of the base-fluid velocity and a relative velocity (that he calls the slip velocity). He considered in turn seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity settling. He concluded that in the absence of turbulent effects it is the Brownian diffusion and the thermophoresis that will be important. Buongiorno proceeded to write down conservation equations based on these two effects.

The Bénard problem (the onset of convection in a horizontal layer uniformly heated from below) for a nanofluid was studied by Tzou [6], [7] on the basis of the transport equations of Buongiorno [5]. In the present project the corresponding problem for flow in a porous medium (the Horton–Rogers–Lapwood problem) is studied. We will assume that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents particles from agglomeration and deposition on the porous matrix.

For completeness, we mention that a substantially different treatment of the Bénard problem for a nanofluid has been given by Kim et al [8]–[10]. These authors simply modified three quantities that appear in the definition of the Rayleigh number namely the thermal expansion coefficient, the thermal diffusivity and the kinematic viscosity.

We are not aware of any publications on convection of nanofluids in porous media as such. (We are aware of the paper by Tsai and Chein [11] who modelled a microchannel heat sink, with a nanofluid, as a porous medium.) There have been studies done on convection in porous media with thermophoresis particle deposition (e.g.,[12]) but an essential feature of nanofluids is that with a special treatment particle deposition can be made negligible.

Likewise it appears that studies involving Brownian motion and porous media are confined to deposition phenomena and so are irrelevant to the present investigation. In the present work, we have extended the work of Nield and Kuznetsov[13].

The organization of the paper is as fallows. Brief introduction to the nanoparicles , nonofulid applications are discussed in the section I. Pertubation soluation for nonofluid flow is given in the section II.Results and discussion is given in the section III. Finally the conclusion is given in the section IV.

II. Constricting the Problem

Consider the superimpose perturbations on the basic solution is given by equation .1

$$V = V', p = p_b + p', T = T_b + T', \phi = \phi_b + \phi' \quad\text{................(1)}$$

The following equations are obtained by number of modification

$$\nabla \cdot V' = 0 \tag{2}$$

$$0 = -\nabla p' - V' + RaT'\hat{e}_z - Rn\phi'\hat{e}_z \tag{3}$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + \frac{N_B}{Le}\left(\frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right) - \frac{2N_A N_B}{Le}\frac{\partial T'}{\partial z} + \alpha(T_b + T') \tag{4}$$

$$\frac{1}{\sigma}\frac{\partial \phi'}{\partial t}+\frac{1}{\varepsilon}w'=\frac{1}{Le}\nabla^2\phi'+\frac{N_A}{Le}\nabla^2 T' \quad (5)$$

$$w'=0,\ T'=0,\ \phi'=0, \text{ at } z=0, \text{ and at } z=1. \quad (6)$$

It will be noted that the parameter Rm is not involved in these and subsequent equations. It is just a measure of the basic static pressure gradient.

For the case of a regular fluid (not a nanofluid) the parameters Rn, N_A and N_B are zero, the second term in Eq. (5) is absent because $\partial\phi/\partial z=0$ and then Eq. (5) is satisfied trivially. The remaining equations are reduced to the familiar equations for the Horton–Roger–Lapwood problem.

The six unknowns u',v',w',p',T',ϕ' can be reduced to three by operating on Eq. (3) with $\hat{e}_z\cdot curlcurl$ and using the identity $curlcurl = graddiv-\nabla^2$ together with Eq. (2).

The result is

$$\nabla^2 w' = Ra\nabla_H^2 T' + Rn\nabla_H^2\phi' \quad (7)$$

Here ∇_H^2 is the two-dimensional Laplacian operator on the horizontal plane.

The differential Equations and the boundary conditions constitute a linear boundary-value problem that can be solved using the method of normal modes.

We write

$$(w',T'\phi') = [W(z),\Theta(z),\Phi(z)]\exp(st+ilx+imy) \quad (8)$$

and substitute into the differential equations to obtain

$$(D^2-\gamma^2)W + Ra\gamma^2\Theta - Rn\gamma^2\Phi = 0, \quad (9)$$

$$W+\left(D^2+\frac{N_B}{Le}D-\frac{2N_AN_B}{Le}D-\gamma^2-s+\alpha\right)\Theta-\frac{N_B}{Le}D\phi+\alpha(T_b-T')=0, \quad (10)$$

$$\frac{1}{\varepsilon}-\frac{N_A}{Le}(D^2-\gamma^2)\Theta-\left(\frac{1}{Le}(D^2-\gamma^2)-\frac{s}{\sigma}\right)\Phi=0, \quad (11)$$

$$W=0,\Theta=0,\Phi=0,z=0,z=1, \quad (12)$$

where

$$D=\frac{d}{dz},\gamma=(l^2+m^2)^{1/2}. \quad (13)$$

Thus γ is a dimensionless horizontal wave number.

For neutral stability the real part of s is zero. Hence we now write $s=i\omega$, where ω is real and is a dimensionless frequency.

We now employ a Galerkin-type weighted residuals method to obtain an approximate solution to the system of Eqs. (9)-(12). We choose as trial functions (satisfying the boundary conditions)

$$W_p=\Theta_p=\Phi_p=\sin p\pi z;\ p=1,2,3\ldots\ldots\ W=\sum_{p=1}^{N}A_pW_p,\Theta=\sum_{p=1}^{N}B_p\Theta_p,\Theta=\sum_{p=1}^{N}C_p\Theta_p \quad (14)$$

substitute into Eqs. (9)-(12), and make the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, thereby obtaining a system of $3N$ linear algebraic equations in the $3N$ unknowns $A_p,B_p,C_p;\ p=1,2\ldots\ldots N$. The vanishing of the determinant of coefficients produces the eigenvalue equation for the system. One can regard Ra as the eigenvalue. Thus Ra is found in terms of the other parameters.

III. Results and Discussions

A sketch of $Rn\left(N_A+\frac{Le}{\varepsilon}\right)$ versus Ra is given in Fig. 1. The sketch is made on the assumption that $(\varepsilon N_A+Le)/\sigma$ is greater than unity. If that inequality is reversed than the labels on the axes need to be swapped around. The stability diagram is qualitatively similar to Fig. 9.2 in [14] which pertains to the double-diffusive Horton–Rogers–Lapwood problem.

There appears to be a qualitative discrepancy between our results and Fig. 4(b) in Tzou [6],[7]. This figure indicates that the analysis in Tzou [6],[7] leads to the prediction that the critical Rayleigh number is reduced by a substantial amount in the bottom-heavy case, whereas our analysis leads to a predicted increase in the value of the critical Rayleigh number for non-oscillatory instability in this case. Tzou offers no physical explanation for the substantial reduction. Tzou [6],[7] uses the symbol Le to denote a Lewis number divided by the nanoparticle fraction decrement rather than a regular Lewis number. This means that his parameter Le tends to infinity as the nanoparticle fraction decrement tends to zero, i.e. in the limit as the nanofluid is replaced by a regular fluid. Accordingly, we hypothesize that it is possible that the solution obtained by Tzou [6],[7].

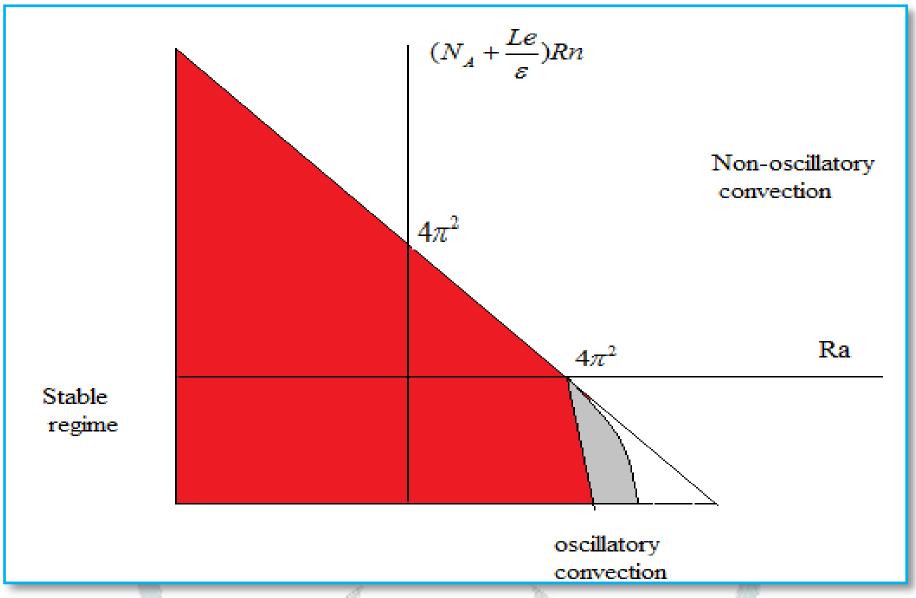

Fig. 1. Sketch of the stability and instability domains.

IV. Conclusion

In this paper, we have employed a Darcy model for the momentum equation. We do not anticipate that the inclusion of a Brinkman term in that equation will have a major qualitative effect. Rather, the expected result would be that the value 40 is replaced by a larger value Ra_0 that depends on the hydrodynamic boundary conditions and increases with increase of the Darcy number. A consequence of the increase in Ra_0 is that the change in the value of Ra, for a given value of Rn, decreases as a percentage of the value of Ra_0. Thus, for example, a change from free–free boundary conditions to the more restrictive rigid–rigid boundary conditions, something that increases the value of Ra_0, leads to a decrease in the sensitivity of Ra to a given change in Rn.

References

[1] S. Choi, Enhancing thermal conductivity of fluids with nanoparticle, in: D.A. Siginer, H.P. Wang (Eds.), Developments and Applications of Non-Newtonian Flows, ASME FED, vol. 231/ MD-vol. 66, 1995, pp. 99–105.

[2] H. Masuda, A. Ebata, K. Teramae, N. Hishinuma, Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles, Netsu Bussei 7 (1993) 227–233.

[3] J. Buongiorno, W. Hu, Nanofluid coolants for advanced nuclear power plants, Paper No. 5705, in: Proceedings of ICAPP'05, Seoul, May 15–19, 2005.

[4] C. Kleinstreuer, J. Li, J. Koo, Microfluidics of nano-drug delivery, Int. J. Heat Mass Transfer 51 (2008) 5590–5597.

[5] J. Buongiorno, Convective transport in nanofluids, ASME J. Heat Transfer 128 (2006) 240–250.

[6] D.Y. Tzou, Instability of nanofluids in natural convection, ASME J. Heat Transfer 130 (2008) 072401.

[7] D.Y. Tzou, Thermal instability of nanofluids in natural convection, Int. J. Heat Mass Transfer 51 (2008) 2967–2979.

[8] J. Kim, Y.T. Kang, C.K. Choi, Analysis of convective instability and heat transfer characteristics of nanofluids, Phys. Fluids 16 (2004) 2395–2401.

[9] J. Kim, C.K. Choi, Y.T. Kang, M.G. Kim, Effects of thermodiffusion and nanoparticles on convective instabilities in binary nanofluids, Nanoscale Microscale Thermophys. Eng. 10 (2006) 29–39.

[10] J. Kim, Y.T. Kang, C.K. Choi, Analysis of convective instability and heat transfer characteristics of nanofluids, Int. J. Refrig. 30 (2007) 323–328.

[11] T.H. Tsai, R. Chien, Performance analysis of nanofluid-cooled microchannel heat sinks, Int. J. Heat Fluid Flow 28 (2007) 1013–1026.

[12] A.J. Chamkha, I. Pop, Effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium, Int. Commun. Heat Mass Transfer 31 (2004) 421–430.

[13] D.A. Nield, A.V. Kuznetsov, The onset of convection in a nanofluid layer, ASME J. Heat Transfer, submitted for publication.

[14] D.A. Nield, A. Bejan, Convection in Porous Media, third ed., Springer, New York, 2006.

[15] A.V. Kuznetsov, A.A. Avramenko, Effect of small particles on the stability of bioconvection in a suspension of gyrotactic microorganisms in a layer of finite length, Int. Commun. Heat Mass Transfer 31 (2004) 1–10.