OHMIC HEATING AND MELTING HEAT TRANSFER EFFECTS ON ELECTRICALLY CONDUCTING DISSIPATIVE MAGNETIC NANOFLUID

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Abstract— In this study, we investigated the Ohmic heating and melting heat transfer effects on Casson nanofluid past a stretching sheet in the presence of magnetic field and viscous dissipation and non-uniform heat source/sink parameters and we have presented dual solutions for the flow of Ag-water and Al_2O_3-water nanofluids. Further, the transformed governing equations are solved numerically using fourth order Runge-Kutta scheme with shooting technique. Obtained results are compared with the open literature and found a good accuracy. The influence of various non-dimensional governing parameters such as melting parameter, Casson parameter and viscous dissipation parameter on velocity and temperature profiles along with skin friction factor and local Nusselt number is thoroughly discussed and presented with the help of tables and graphs. It is found that Al_2O_3-water nanofluid is more significant in velocity profiles than Ag-water nanofluid. But, we observed a reverse action in temperature profiles. And the fluid flow decreases with Ec for both the fluids.

Index Terms— Heat transfer, Ohmic heatin , nanofluids.

I. INTRODUCTION

Heat transferred during the melting of a stretching sheet has many real life uses as making of semi-conductor materials, volcanic emission solidification, dissolving of permafrost and melting of frozen ground, etc. Cheng and Chung [1] presented the melting effect on the mixed convective heat transfer from the vertical plate in a liquid-saturated porous medium. Ishak et al. [2] studied the flow and heat transfer from a steady laminar liquid flow on a melting surface. The stagnation-point flow of steady boundary layer micropolar fluid in a horizontal linearly stretching/shrinking sheet has been analysed by Yacob et al. [3]. Heat transfer in a stretching/shrinking surface past a nanofluid with magnetic effect is examined by Sandeep et al. [18].

This Piece of this investigation is arranged to analyse the melting effect of a Casson fluid on horizontally stretching sheet with non-uniform source/sink. The governing systems of equation are comprehensively solved by using Runge-Kutta fourth order scheme. Melting heat transfer and Joule heating extended from the work of Hayat et al. [8]. The aim of this work is to provide an alternate numerical route for solving the problem of heat transfer with melting and Ohmic effect in presence non-uniform heat source/sink of a dissipative magnetic field. The non-linear coupled differential equations are subjected to the similarity solutions. The results are presented for the various dimensionless parameters on the velocity and temperature fields. And found that result admit the general behaviour of the fluid and well justified with the results of previously published works. To the best of our Knowledge, the present work has not been examined before.

II. MATHEMATICAL FORMULATION

Consider an incompressible flow of a Casson nano fluid over a stretching surface located at $x = 0$ and *y*-axis is taken normal to it and the flow is confined to $y \geq 0$. It is assumed that the velocity of the stretching sheet is $u_w(x) = ax$, where a is a positive constant. We have chosen $T_\infty > T_m$ where $T_m (= T_\infty - bx^2)$ the non-uniform temperature of the melting surface is and T_∞ is the ambient temperature. Also a uniform magnetic field of intensity B_0 acts in the *y*-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison with the applied magnetic field. We incorporate the Joule heating effects in the energy equations which govern such type of flow are written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\rho_{nf} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu_{nf} \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \sigma_{nf} B_o^2 u, \qquad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_{nf} B_0^2}{(\rho C_p)_{nf}} u^2 + q''' \qquad (3)$$

The subjected boundary conditions are,

$$u = u_w = ax, \quad v = 0, \quad T = T_m \ \ at \qquad y = 0 \tag{4}$$
$$u \to 0, \ T \to T_\infty \ \ as \qquad y \to \infty$$
$$and$$
$$k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho_{nf}[l + c_s(T_m - T_0)]v(x,0) \tag{5}$$

Where u and v are the velocity components along the *x*- and *y*- directions respectively, σ_{nf} is the electrical conductivity of the nanofluid, $l(=l_0 x^2)$ is the non-uniform latent heat of the fluid and c_s is the heat capacity of the solid surface. The boundary condition (5) shows that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid temperature $T_0(=T_m - cx^2)$ to its melting temperature T_m.

The space and temperature dependent heat generation/absorption (non-uniform heat source/sink) is given by,

$$q''' = \frac{k_f u_w(x)}{x\upsilon_f}\left[A^*(T_\infty - T_m)f' + B^*(T - T_m)\right] \tag{6}$$

The effective dynamic viscosity of nanofluid μ_{nf} given by

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{7}$$

Where ϕ is the nanoparticle volume fraction. The effective nanofluid density ρ_{nf}, heat capacity $(\rho C_p)_{nf}$ and thermal diffusivity α_{nf} are taken as follows:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{8}$$
$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \tag{9}$$
$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \tag{10}$$

The effective thermal conductivity and electrical conductivity of nanofluid k_{nf} and σ_{nf} written as,

$$k_{nf}^* = \left\{\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}\right\}, \qquad \sigma_{nf}^* = 1 + \frac{3(\sigma - 1)\phi}{(\sigma+2) - (\sigma-1)\phi}, \tag{11}$$

where, $k_{nf}^* = \dfrac{k_{nf}}{k_f}$, $\sigma_{nf}^* = \dfrac{\sigma_{nf}}{\sigma_f}, \sigma = \dfrac{\sigma_s}{\sigma_f}$

We now introduce the following similarity transformations,

$$u = axf'(\eta), \quad v = -(\upsilon_f a)^{0.5} f(\eta), \ \eta = (\frac{a}{\upsilon_f})^{0.5} y, \ \theta(\eta) = \frac{T - T_m}{T_\infty - T_m} \tag{12}$$

Substituting eqn. (11) into eqns. (1) - (3) yields the following non-dimensional ordinary differential equations.

$$A_1(1 + \frac{1}{\beta}) f''' - MA_1(1-\phi)^{2.5}\frac{\sigma_{nf}}{\sigma_f} f' - f'^2 + ff'' = 0 \tag{13}$$

$$\frac{1}{\Pr}\frac{k_{nf}}{k_f}(1-\phi)^{2.5}\theta'' + \frac{1}{\Pr}(1-\phi)^{2.5}(A^* f' + B^*\theta) + Ec(1+\frac{1}{\beta}) f''^2 + EcM(1-\phi)^{2.5}\frac{\sigma_{nf}}{\sigma_f} f'^2$$
$$+ \frac{1}{A_2}(f\theta' - 2\theta f' + 2f') = 0 \tag{14}$$

where prime indicates the differentiation with respect to η, M is the Hartman number, Pr is the Prandtl number, Ec is the Eckert number. Which are given by,

$$M = \frac{\sigma_f B_0^2}{a\rho_f}, \quad \Pr = \frac{\nu_f}{\alpha_f}, \quad Ec = \frac{\rho_f a^2}{b(\rho C_p)_f} \tag{15}$$

The relevant boundary conditions of eqn. (4) and (5) are,

$$f'(\eta) = 1, \ \Pr f(\eta) + \frac{k_{nf}}{k_f}\delta\theta'(\eta) = 0, \ \theta(\eta) = 0, at \qquad y = 0$$
$$f'(\eta) \to 0, \ \theta(\eta) \to 1, \qquad at \qquad y = \infty \tag{16}$$

Where δ is the dimensionless melting parameter

$$\delta = \frac{c_f(T_\infty - T_m)}{l + c_s(T_m - T_0)} = \frac{c_f b}{l_0 + c_s c} \tag{17}$$

Which is the combination of the Stefan numbers $\frac{c_f(T_\infty - T_m)}{l}$ and $\frac{c_s(T_m - T_0)}{l}$ for the liquid and solid phases, respectively. When $\phi = 0$ we obtain the governing equations for a viscous fluid. In absence of melting i.e., for δ =0 eq. (17) reduces to the classical equation,

$$A_1 = \frac{1}{(1-\phi)^{2.5}(1-\phi+\frac{\rho_s}{\rho_f}\phi)}, \ A_2 = \frac{1}{(1-\phi)^{2.5}(1-\phi+\frac{(\rho C_p)_s}{(\rho C_p)_f}\phi)},$$
$$A_3 = 1-\phi+\frac{\rho_s}{\rho_f}\phi \tag{18}$$

Local skin friction coefficient C_f and the Nusselt number Nu_x are given by,

$$C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{q_w x}{k_{nf}(T_\infty - T_m)} \tag{19}$$

where the surface shear stress τ_w and wall heat flux q_w are given by,

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{20}$$

Using equations above equations we obtain,

$$C_f(\mathrm{Re}_x)^{1/2} = \frac{1}{(1-\varphi)^{2.5}}(1+\frac{1}{\beta})f''(0), \ Nu_x(\mathrm{Re}_x)^{-0.5} = -\frac{k_{nf}}{k_f}\theta'(0) \tag{21}$$

here $\mathrm{Re}_x = x(a/\nu_f)$ is the local Reynolds number.

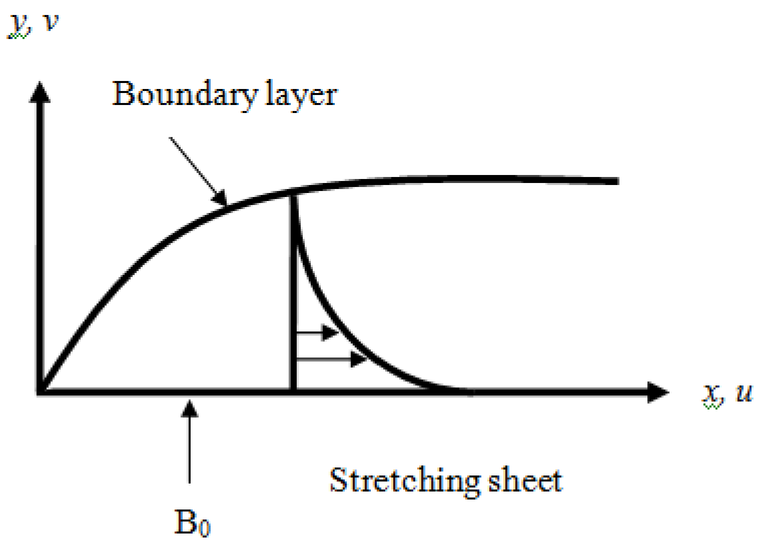


Figure 1. Sketch of Physical Flow

III. Discussions with Results

Under the influence of boundary conditions (16) the non-linear ordinary differential equations (13 and 14) are solved using Runge-Kutta fourth order method with Shooting technique. Consider the parameters at fixed values of $\delta = 0.5$, $Ec = 0.01$, $A^* = 0.1$, $B^* = 0.1$, $M = 1$, $\beta = 0.5$, $\phi = 0.1$. These values are fixed for the entire discussion except the changes in values are pictured in figures and tables. The non-dimensional velocities $f'(\eta)$ and temperature $\theta(\eta)$ is observed for two nanofluids, obtained by mixing Silver nanoparticle and Aluminium oxide nanoparticle in base fluid as water, represented by continuous lines and dashed lines respectively. The study is concentrated on the influence of governing parameters like Magnetic effect (M), volume fraction (ϕ), Casson fluid parameter (β), melting parameter (δ), Eckert number (Ec) and space dependent parameter A^* and temperature dependent parameter B^*.

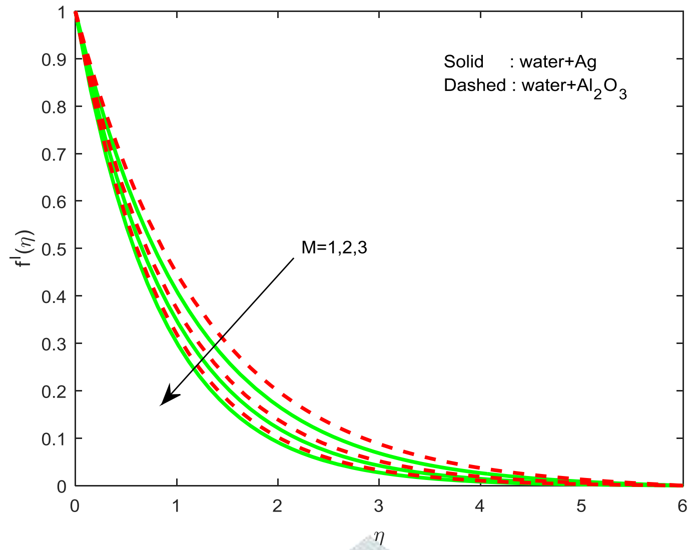
Table 1. Thermo physical properties of the nano particles with base fluid

Physical properties	Fluid phase (water)	Ag	Al_2O_3
$c_p(J/KgK)$	4179	235	765
$\rho(Kg/m^3)$	997.1	10500	3970
$k(W/mK)$	0.613	429	40
$\sigma(S/m)$	0.005	6.30x10^7	3.5x10^7

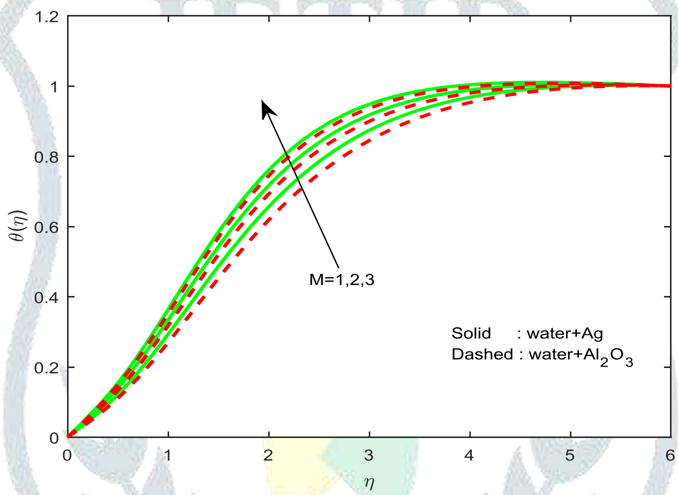
Table 1 shows the thermo physical properties of the nano particle and base fluid.

The variation of velocity and temperature profiles with the variation of magnetic field (M) is presented in the Fig.1 and 2 respectively. The non-dimensional velocity $f'(\eta)$ decreases with increasing magnetic field parameter (M). But the reverse result has been observed for temperature with an increase in the magnetic field parameter. This is due to the fact that the magnetic fields introduce the retarding body force known as the Lorentz force. The Lorentz force is a resistive force which opposes the fluid motion so heat is produced and as a result, the thermal boundary layer thickness becomes thicker for stronger magnetic field.

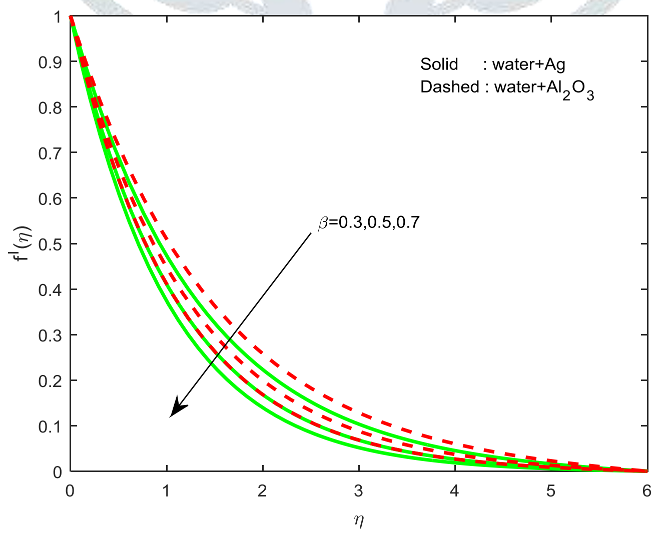
Fig. 3 and 4 shows changes in velocity and temperature due to the Casson parameter (β). It is observed that the rate of transport are considerably reduces with the increase in β. The temperature field increases with increasing values of β for both the nano-fluids. Since the increasing values of the Casson parameter decreases yield stress (the fluid behaves as a Newtonian fluid as Casson parameter becomes large). Hence Casson parameter suppresses the velocity. The thickness of the thermal boundary layer occurs due to increase in the elastic stress parameter.



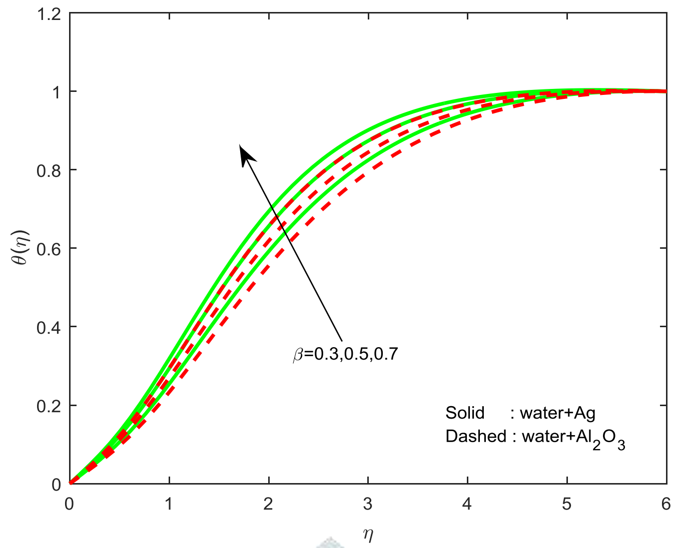
Graph 1: Fluid flow with respect to the Magnetic Field (M)



Graph 2: Fluid Temperature with respect to the Magnetic Field (M)



Graph 3: Fluid flow with respect to Casson parameter (β)



Graph 6: Fluid Temperature with respect to Casson Parameter (β)

Table no. 2, shows the Variation of $f''(0)$ (skin friction) and $-\theta'(0)$ (Nusselt number) for a Ag-water nanofluid (Silver nanoparticle)

Table.2 . Variation of $f''(0)$ (skin friction) and $-\theta'(0)$ (Nusselt number) for a Ag-water nanofluid (Silver nanoparticle)

M	ϕ	β	δ	Ec	A^*	B^*	$f''(0)$	$-\theta'(0)$
1							-0.890211	-0.223487
2							-1.055380	-0.263575
3							-1.198403	-0.295932
	0.1						-0.890211	-0.223487
	0.2						-0.886640	-0.231448
	0.3						-0.827645	-0.226223
		0.3					-0.746925	-0.200900
		0.5					-0.890211	-0.223487
		0.7					-0.983804	-0.237649
			1				-0.873900	-0.179662
			2				-0.853062	-0.134196
			3				-0.839077	-0.109751
				0.2			-0.838117	-0.666939
				0.4			-0.796088	-1.047104
				0.6			-0.763446	-1.358437
					1		-0.787557	-1.127028
					2		-0.724079	-1.755558
					3		-0.679426	-2.239555
						0.1	-0.880604	-0.303176
						0.3	-0.844788	-0.608530
						0.5	-0.756860	-1.417524

Table no. 3, Variation of $f''(0)$ (skin friction) and $-\theta'(0)$ (Nusselt number) for a Al_2O_3-water mixture nanofluid (Alumina Nanoparticle)

M	ϕ	β	δ	Ec	A^*	B^*	$f''(0)$	$-\theta'(0)$
1							-0.804446	-0.205618
2							-0.987186	-0.250562
3							-1.141413	-0.286106
	0.1						-0.804446	-0.205618
	0.2						-0.743305	-0.203378
	0.3						-0.685525	-0.198721
		0.3					-0.673638	-0.184359
		0.5					-0.804446	-0.205618
		0.7					-0.890421	-0.219359

			1				-0.793926	-0.167886
			2				-0.780107	-0.127368
			3				-0.770650	-0.105034
				0.2			-0.770445	-0.632843
				0.4			-0.742092	-1.004662
				0.6			-0.719442	-1.313042
					1		-0.733159	-1.125018
					2		-0.688684	-1.749894
					3		-0.656681	-2.229398
						0.1	-0.798992	-0.272878
						0.3	-0.779233	-0.520556
						0.5	-0.739554	-1.038695

Table 2 and 3 illustrates the influence of dimensionless parameter on the friction factor and Nusselt number. It is edvident that both friction factor and local Nusselt number reduces by enhancing magnetic field and Casson parameter. But the reverse action we saw in the influence of dissipation and melting parameter.

IV. Conclusion

The study presents a numerical solutions for the magneto-hydrodynamic flow of a Casson fluid over a stretching sheet in presence of the melting heat transfer and Ohmic heating. The effects of non-dimenssional governing parameters on velocity and temperature profiles of the flow are discussed with the help of graphs. The skin friction and Nusselt number are also calculated for variation of governing parameter and discussed. By the analysis the overall observation is listed out as:

- Fluid flow declines with rise in the magnetic field parameter but the reverse result is observed for temperature fields.
- The volume fraction of the nanoparticles increases the strength of the flow.
- By the variation of Casson parameter the rate of transport is considerably reduced and the rate of heat transfer is considerably enhances.
- The velocity is increased and the temperature is decreased with the variation of melying parameter δ
- The fluid flow decreases with Ec. The temperature is increasing initially and after some values of η the behaviour of graph is reversed.
- The temperature of the fluid increases by the influence of the non-uniform heat source/sink parameters
- The influence of the magnetic field declines the skin friction and Nusselt number.

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