Secondary Stage Creep Deformations and Stresses in Thick Spherical Vessels Considering Plain Strain

Sukhjinder Singh Sandhu[a], Tejeet Singh[b] and V.K.Gupta[c]

[a] *Assistant Professor, Department of Mechanical Engg, S. B. S. S. T. C., Ferozepur, 152004, INDIA*

[b] *Associate Professor, Department of Mechanical Engg, S. B. S. S. T. C., Ferozepur, 152004, INDIA*

[c] *Professor, Mech. Engg., UCoE, Punjabi Univ., Patiala-147002, INDIA*

Abstract

The secondary stage creep deformations and creep stress in a thick walled spherical vessels made of functionally graded composites has been obtained in the present study. The spherical pressure vessel chosen for the investigation is subjected to internal and external pressure under constant temperature field. The material of the vessel is incompressible and volume constancy condition was assumed. The creep behaviour of material is governed by threshold stress based creep law. The study reveals that for linear variation of reinforcement and assumed pressure ratio the compressive value of radial stress reduces with increase in pressure ratio from 2 to 5 over entire radial distance. However, the longitudinal stress remains compressive at inner radius, however, the value become tensile for pressure ratio 3, 4 and 5.

Key words: Spherical Vessel; Secondary Creep; Functionally Graded Material; Axial Stress.

1. Introduction

The use of functionally graded materials for applications such as spherical pressure vessels subjected to high pressure and temperature is an important area of investigation (Sultana and Mondal, 2012). It is seen that many application involving spherical pressure vessels are subjected to high temperatures for long duration and therefore material of the vessel undergoes creep stresses and thereby reducing its service life. Most of the components used under high pressure and temperature are designed for minimum strain rate in the secondary stage creep during their lifetime. Therefore, study of secondary stage creep is very important from structural design point of view. In applications like power plants and petrochemical industry the spherical vessels are also subjected to a radial thermal gradient of the order of 50°C along with pressure (Johnson and Khan, 1963; Sim, 1973; Durban and Baruch, 1974). Therefore, it requires a different analysis as compared to the vessels exposed to constant temperature conditions.

The general theory of creep in pressure vessels was developed by Bhatnagar and Arya (1973) and applied to the solution of a specific problem using Norton's creep law. Steady-state creep analysis of thick-walled spherical pressure vessels with varying creep properties has been presented by You and Ou (2008). Stresses in a spherical pressure vessels undergoing creep and dimensional changes has been presented by Miller (1995). Analytical and numerical analysis for the Functionally Graded thick sphere under combined pressure and temperature loading has been presented by Bayat et al (2012). In all these studies the strains are assumed to be infinitesimal and the deformation is referred with respect to original dimensions of the sphere.

The excellent properties of metal matrix composites (MMCs) like high specific strength and stiffness, and high temperature stability make them suitable choice for applications involving high pressure and temperature (Harris, 1999). Therefore, it was decided to investigate the secondary creep in a sphere made of Al- SiC composite and subjected to internal and external pressure and exposed to uniform temperature field. A mathematical model has been developed to describe the secondary stage creep behavior of the functionally graded sphere. The model developed is used to investigate the effect of different pressure ratios on the steady state creep in a thick-walled functionally graded spherical vessel.

The content of silicon carbide in Al matrix has been assumed to vary linearly, with maximum amount at the inner radius and minimum at the outer radius of spherical vessel. A mathematical model has been developed and used to analyze the effect of varying the radial distribution of SiC on the steady state creep response of sphere.

2. Creep Analysis and Mathematical Solution

For aluminium matrix composites undergoing secondary stage creep, the relation between effective strain rate $(\dot{\varepsilon}_e)$ and effective stress (σ_e) can be described by the well-known threshold stress (σ_o) based creep law (Singh and Gupta, 2011) and is given by,

$$\dot{\varepsilon}_e = \{M(r)(\sigma_e - \sigma_0(r))\}^n \qquad (1)$$

In the present study, the values of M and σ_0 have been obtained from following regression equations developed from experimental data of Pandey et al (1992). The developed regression equations are given below,

$$M(r) = 0.0287611 - \frac{0.00879}{P} - \frac{14.02666}{T} - \frac{0.032236}{V(r)} \quad (2)$$

$$\sigma_o(r) = -0.084P - 0.0232T + 1.1853\big(V(r)\big) + 22.207 \quad (3)$$

Where P, $V(r)$, T, $M(r)$ and $\sigma_o(r)$ are respectively the particle size, particle content, temperature, creep parameter and threshold stress at any radius (r) of the FGM spherical pressure vessels. In the present work particle size is assumed as 1.7 μm while operating temperature is kept as 623K. The reinforcement content i.e. silicon carbide particle (SiCp), in the sphere is assumed to vary linearly from the inner radius (a) to the outer radius (b). As a result, creep parameters will vary with the radius. The variation of SiCp is described by following equation (Singh and Gupta, 2011)

$$\text{Particle content } V(r) = V_{max} - \frac{(r-a)}{(b-a)}(V_{max} - V_{min}) \quad (4)$$

Where, V_{max} and V_{min} are respectively the maximum and minimum SiCp, at the inner and outer radii.

The average particle content (V_{avg}) in spherical vessel can be expressed as,

$$V_{avg} = \frac{\int_a^b 2\pi r l.V(r)dr}{\pi(b^2-a^2)l} \quad (5)$$

Substituting the value of particle content, $V(r)$, from Eqn. (4) into Eqn. (5) and integrating the resulting equation, we get,

$$V_{min} = \frac{3V_{avg}(1-\alpha^2)(1-\alpha) - V_{max}(1-3\alpha^2+2\alpha^3)}{(2-3\alpha+\alpha^3)} \quad (6)$$

Where, α is the ratio of inner to outer radius (*i.e.* a/b) of spherical vessel.

Thus for a given FG sphere containing particle gradient both the creep parameters will be function of radius. The value of M(r) and σ_o (r) at any radius could be estimated by substituting the reinforcement content V(r) from equation (4) in into equation (2) and (3).

Let us consider a thick-walled, spherical vessel made of functionally graded Al-SiCp composite. The vessel is assumed to have inner and outer radii as a and b respectively and is subjected to both internal pressure p and external pressure q.

The geometric relationships between radial and circumferential strain rates and radial displacement rate are

$$\dot{\varepsilon}_r = \frac{d\dot{x}_r}{dr} \tag{7}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{x}_r}{r} \tag{8}$$

Where, $\dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta$ are respectively radial and circumferential strain rates, $\dot{x}_r\left(=\frac{dx}{dt}\right)$ is the radial displacement rate and x is the radial displacement.

Eliminating $\dot{x}_r$, from Eqns. (7) and (8) the deformation compatibility equation is obtained as,

$$r\frac{d\dot{\varepsilon}_\theta}{dr} = (\dot{\varepsilon}_r - \dot{\varepsilon}_z) \tag{9}$$

Considering the equilibrium of forces on an element of spherical vessel along the radial direction, we get the equilibrium equation as below,

$$\frac{r}{2}\frac{d\sigma_r}{dr} = (\sigma_z - \sigma_r) \tag{10}$$

Where, σ_θ and σ_r are respectively the circumferential and radial stresses.

Since the material of the sphere is assumed to be incompressible, therefore,

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0 \tag{11}$$

Where, $\dot{\varepsilon}_z$ is the axial strain rate.

The steady state creep deformations in thick-walled spherical pressure vessels are spherically symmetric i.e. $\sigma_\theta = \sigma_z$, thus the constitutive equations (Singh and Gupta, 2011) for creep along the principal direction r, θ and z can be written as below,

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{\sigma_e}(\sigma_r - \sigma_\theta) \tag{12}$$

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{2\sigma_e}(\sigma_z - \sigma_r) \tag{13}$$

Where $\dot{\varepsilon}_e$ is the effective strain rate, σ_e is the effective stress respectively.

From Eqs. (12) and (13), the relationship between the radial and axial strain rates can be obtained as,

$$\dot{\varepsilon}_r = -2\dot{\varepsilon}_z \tag{14}$$

Substituting Eqn. (14), into Eqn. (9), the deformation compatibility becomes,

$$\frac{d\dot{\varepsilon}_z}{\dot{\varepsilon}_z} = -3\frac{dr}{r} \tag{15}$$

The integration of Eqn. (15), gives the axial strain rate as,

$$\dot{\varepsilon}_z = \frac{A_1}{r^3} \tag{16}$$

Where, A_1 is constant of Integration.

The effective stress in thick-walled spherical vessels subjected to internal pressure is assumed to be expressed by von-Mises equation (Singh and Gupta, 2011),

$$\sigma_e = (\sigma_z - \sigma_r) \tag{17}$$

Using Eqn. (17) into Eqn. (13), we get,

$$\dot{\varepsilon}_e = 2\dot{\varepsilon}_z = \frac{2A_1}{r^3} \tag{18}$$

Using Eqn. (4) into above Eqn. (18), we get,

$$\sigma_e = \frac{(2A_1)^{\frac{1}{n}}}{r^{3/n}M(r)} + \sigma_o(r) \tag{19}$$

Using Eqn. (17) into above Eqn. (19), we get,

$$\sigma_z - \sigma_r = \frac{A_2(r)}{r^{\frac{3}{n}}} + \sigma_o(r) \tag{20}$$

Where $A_2(r) = \left[\frac{(2A_1)^{\frac{1}{n}}}{M(r)}\right]$

Substituting equilibrium Eqn. (10) into above equation and integrating the resulting equation between limits *a* to *r*, we get,

$$\sigma_r = 2\int_a^r \frac{A_2(r)}{r^{\frac{(n+3)}{n}}}dr + 2\int_a^r \frac{\sigma_o(r)}{r}dr + A_3 \tag{21}$$

Where, A_3 is constant of integration.

The following boundary conditions are assumed for the spherical vessel,

(i). $at\ r = a, \sigma_r = -p$ (22)

(ii). $at\ r = b, \sigma_r = -q$ (23)

Eqn. (21) may be solved between limits *a* and *b* and under the enforced boundary conditions given in Eqs. (22) and (23) to get the value of 'A_1' as

$$A_1 = \frac{1}{2}\left[\frac{p-q-2A_4}{2A_5}\right]^n \tag{24}$$

Where $A_4 = \int_a^b \frac{\sigma_0(r)}{r} dr$ and $A_5 = \int_a^b \frac{1}{r^{(n+3)/n} M(r)} dr$

Applying boundary conditions in Eqn. (21), the radial stress is obtained as,

$$\sigma_r = 2\int_a^r \frac{A_2(r)}{r^{\frac{(n+3)}{n}}} dr + 2\int_a^r \frac{\sigma_o(r)}{r} dr - p \tag{25}$$

A computer program has been developed to calculate the steady state creep response of the FG spherical vessel for various combinations of size and content of the reinforcement, and operating temperature. For the purpose of numerical computation, the inner and outer radii of the spherical vessel are taken 500 *mm* and 800 *mm* respectively, and the internal pressure is assumed to be 100 *MPa* and external pressure varies as 50 *MPa, 33.33 MPa, 25 MPa and 20 MPa* so that ratio of p/q = 2, 3, 4 and 5. The radial stress at different radial locations of the sphere is calculated respectively from Eqs. (25). The creep parameters *M(r)* and $\sigma_o(r)$, required during the computation process, are estimated respectively from Eqs. (2) and (3).

3. Results and Discussions

On the basis of mathematical analysis, numerical calculations have been carried out to obtain the secondary stage creep behaviour of functionally graded spherical pressure vessels. The results have been obtained for different pressure ratios in FG spheres. The internal pressure is taken as 100 MPa, however external pressure is varied to obtain pressure ratio as 2, 3, 4 and 5.

3.1 : Variation of Particle Content and Creep Parameters.

The distribution of SiC particles in spherical vessel is linear with maximum particle content is 30 vol% at inner radius. The content of reinforcement is assumed to decrease linearly along the radial distance. The average particle content is kept as 20 vol%.

The threshold stress will reduce linearly (Fig.1) with maximum value at in a radius and minimum value at outer radius of spherical vessel. The threshold stress is higher at locations have greater density of silicon carbide particles. The variation of σ_0 become stepper with increase in particle gradient in functionally graded spherical vessel.

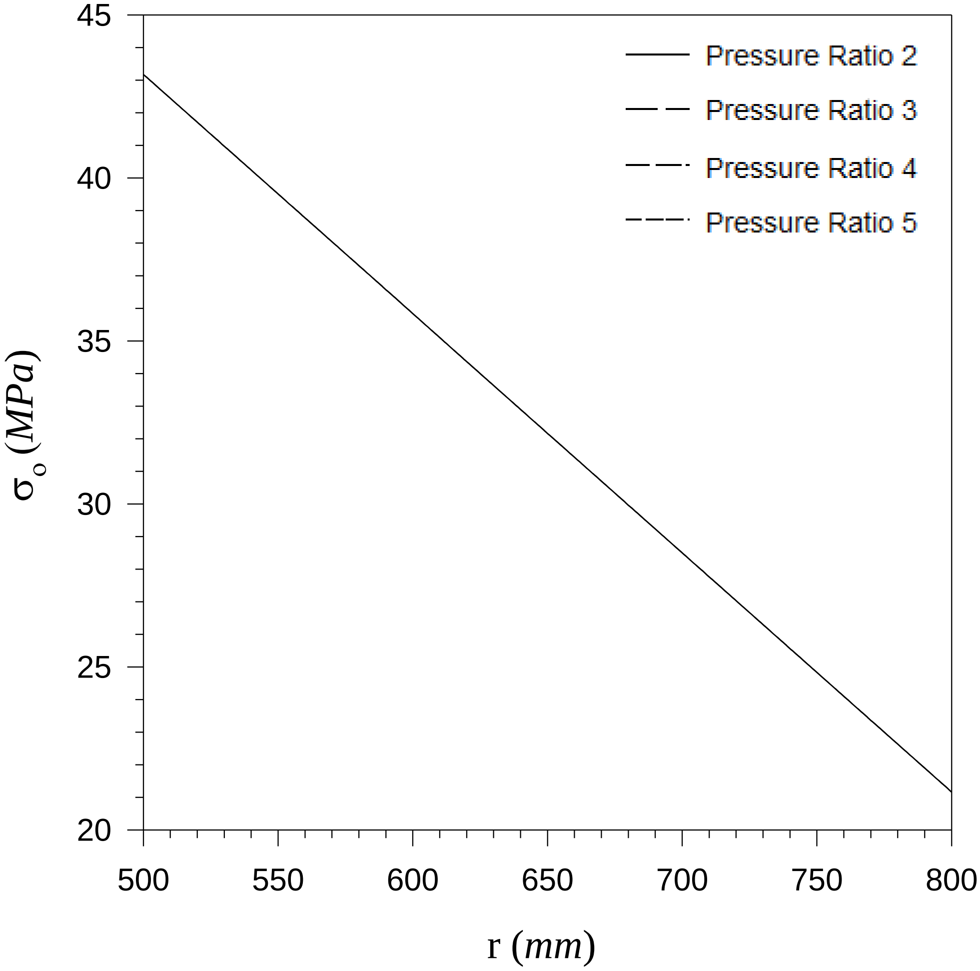

Fig. 1: Variation of Threshold stress.

On the other hand the value of creep parameter '*M*' will increase with increase in radial distance (Fig.2). The increase observed in the value of '*M*' may be due to decrease in particle content at outer regions.

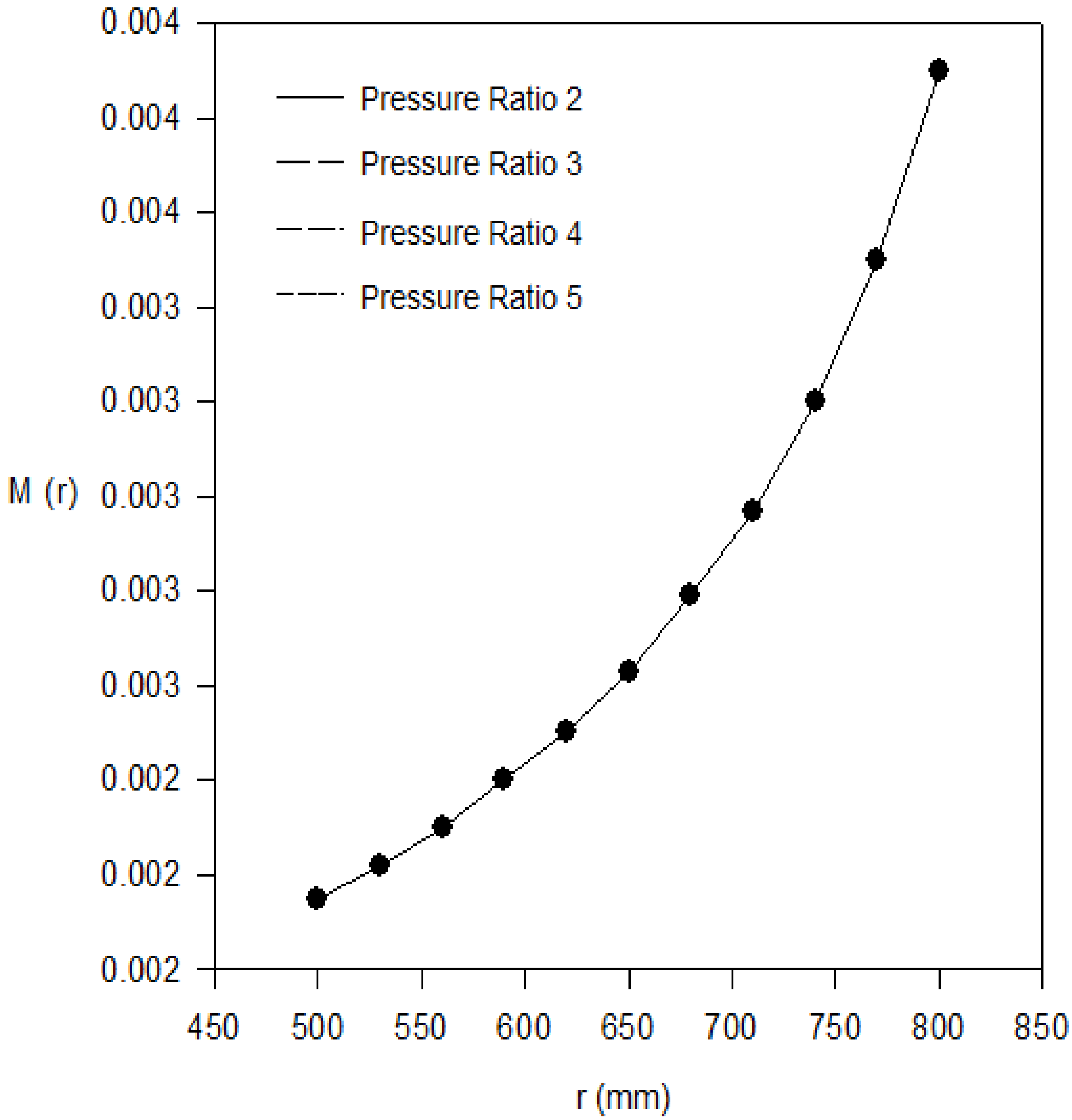

Fig. 2: Variation of Creep parameter '*M*'.

3.2 Variation of Radial and Axial Stresses

To observe the effect of pressure ratio on radial and axial stresses in a thick walled spherical vessel the pressure ratio is kept at 2, 3, 4 and 5. The compressive value of radial stress decreases parabolically from inner radius to outer radius due to imposed boundary conditions, Fig. 3. The compressive value of σ_r will reduce as pressure ratio increases from 2 to 5.

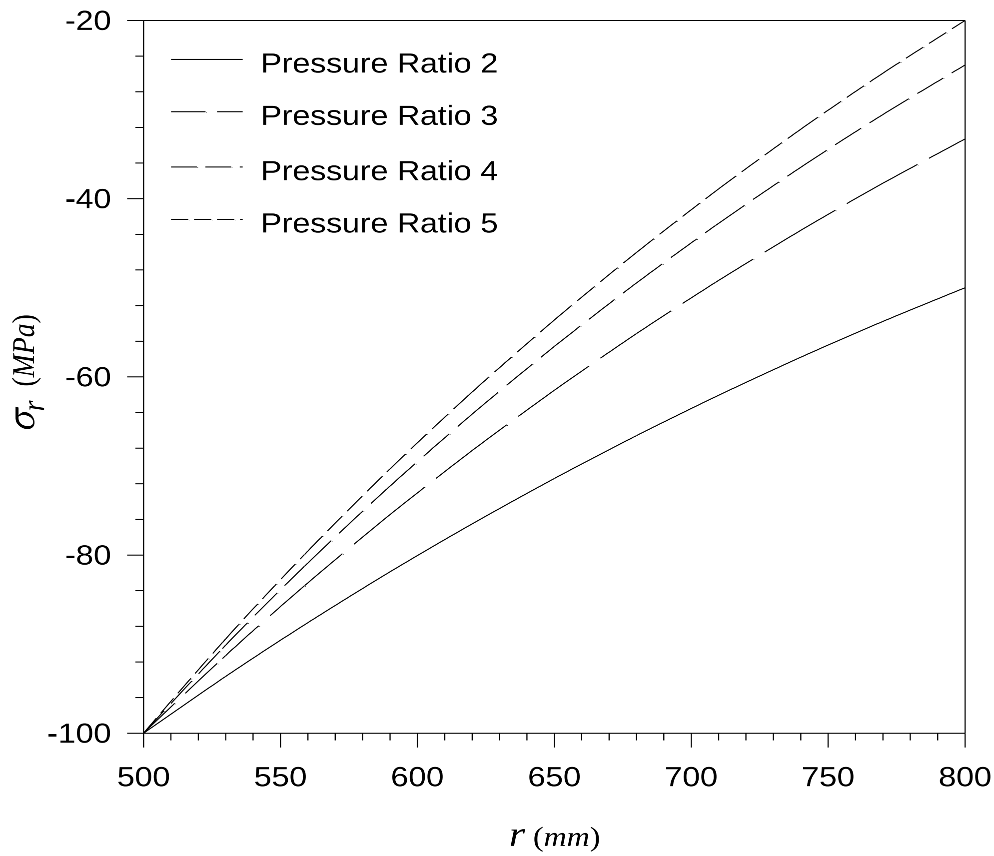

Fig. 3: Variation of Radial Stress.

The tangential stress σ_θ and axial stress σ_z remains equal due to spherical symmetry and observed to increase with radius. Further value axial stress remains compressive at inner and outer radius for pressure ratio 2. On the other hand, the stress become tensile at middle portion as pressure ratio is increased from 2 to 3. Further increase in pressure ratio from 4 to 5) leads to shift in tensile axial stress near inner region of sphere. Further the distribution of axial stress remain parabolic in nature Fig.4.

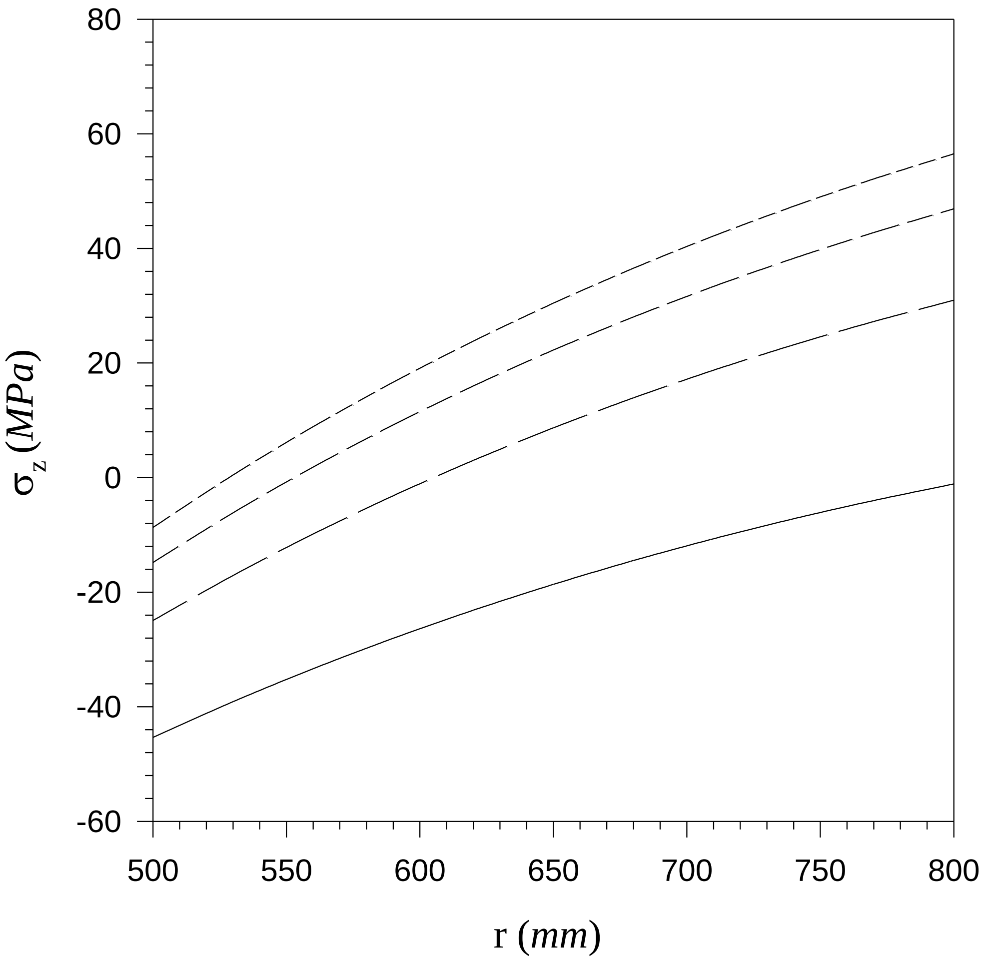

Fig. 4: Variation of Axial Stress.

4. Conclusions

The present study has led to following conclusions,

1) The radial stress in spherical pressure vessel decreases throughout with the increase in pressure ratio. The radial stress remains zero at the inner radius due to imposed boundary condition.
2) The axial stress is compressive near the inner and outer radius of spherical pressure vessel for pressure ratio 2.
3) The axial stress becomes tensile with increase in pressure ratio 3, 4 and 5.

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