A Mathematical Review on Common Coupled Fixed point theorems in Topology

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ABSTRACT

The main purpose of this paper is to study the common coupled fixed point and the coupled coincidence theorem of contractive type mappings involving rational expressions in the framework of partial metric spaces. Fixed point problems have also been considered in partially ordered probabilistic Metric spaces and in partially ordered Metric spaces. The study in this paper generalizes and extends the results of several previous known results in complex-valued metric spaces.

Keywords: Coupled Concidence point, Sets, Coupled Common fixed point, Mappings.

I. INTRODUCTION

The main fixed point theorem was given by Brouwer in 1912, however the credit of making idea valuable and prominent goes to the Polish mathematician Stephan Banach who demonstrated the renowned withdrawal mapping theorem in 1922 which expresses that: Let (X, d) be an entire metric space and let T : X → X be a constriction on X, that is, there exists a steady λ [0, 1) to such an extent that d(Tx, Ty) λd(x, y) for all x, y X. At that point T has a one of a kind fixed point in X. The Banach withdrawal standard is a standout amongst the most critical and valuable outcomes in the metric fixed point theorem. It ensures the presence and uniqueness of the fixed purpose of certain self-maps of metric spaces and gives a valuable technique to locate those fixed focuses. This guideline incorporates distinctive course in various spaces received by mathematicians; for instance, 2-metric spaces, halfway metric spaces, cone metric spaces have just been acquired. As of late, Azam et al presented another space called complex-esteemed metric space which is more broad than the notable metric space, and got sufficient conditions for the presence of basic fixed purposes of a couple of contractive sort mappings including normal articulation.

In this way, a few creators have considered the presence and uniqueness of the fixed point and regular fixed purposes of self-mappings in perspective of differentiating contractive conditions. Despite the fact that the complex-esteemed metric spaces frame an extraordinary class of cone metric spaces, yet this thought is expected to characterize objective articulations which are not significant in cone metric spaces and in this way many aftereffect of examination can't be summed up to cone metric spaces. Bhaskar and Lakhmikantham presented the idea of coupled fixed focuses for a given mostly requested set X.

Samet et al demonstrated that a large portion of the coupled fixed point hypotheses are contaminate prompt outcomes of the outstanding fixed point hypotheses in the writing. Recently, Kutbi et al demonstrated the presence and uniqueness of the normal coupled fixed point in entire complex-esteemed metric spaces in perspective of various contractive condition. The point of this paper is to set up a coupled fortuitous event point theorem for mappings on complex-valued metric spaces alongside summed up withdrawal including balanced articulation and a one of a kind basic coupled fixed point theorem utilizing the idea of w-good mappings. Our outcomes broaden and enhance a few existing fixed point brings about the writing.

The investigation of regular fixed purposes of mappings fulfilling certain contractive conditions has been inquired about widely by numerous mathematicians since fixed point theorem assumes a noteworthy part in arithmetic and connected sciences. For a review of fortuitous event point theorem in metric and cone metric spaces, we allude the peruser to Mustafa and Sims presented another thought of summed up metric space called a G-metric space. Mustafa, Sims and others contemplated fixed point theorem for mappings fulfilling diverse contractive conditions. Abbas and Rhoades acquired some normal fixed point hypotheses for non driving maps without coherence, fulfilling diverse contractive conditions in the setting of summed up metric spaces. While V. Lakshmikantham et al. in presented the idea of a coupled incident purpose of a mapping F from X × X into X and a mapping g from X into X, and concentrated fixed point hypotheses in different applications.

The traditional Banach's withdrawal guideline is a power device in nonlinear investigation and has been broadened and enhanced by many creators. In 2004, the presence of fixed focuses for withdrawal mappings in somewhat requested metric spaces has been examined by Ran and Reurings, Nietto and Lopez. Augmentations and uses of these works show up in numerous conditions.

MATHEMATICAL PRELIMINARIES

The concept of coupled fixed point theorem was introduced by Guo and Lakshmikantham . In 2006, Bhaskar and Lakshmikantham introduced the concept of the mixed monotone property as follows…..

Definition 1-

An element $(x, y) \in X^2$ is said to be a coupled fixed point of the mapping $F : X^2 \longrightarrow X$ if $F(x, y) = x$ and $F(y, x) = y$. Lakshmikantham and Ćirić extended the concept of mixed monotone property to mixed g-monotone property as follows.

Definition 2-

Let $(X,\preccurlyeq)$ be a partially ordered set and $F : X^2 \longrightarrow X$ be a mapping. Then a map F is said to have the mixed monotone property if $F(x, y)$ is monotone nondecreasing in x and is monotone non-increasing in y; that is, for any $x, y \in X$,

$x1,x2 \in X, x1 \preccurlyeq x2 \text{ implies } F(x1,y) \preccurlyeq F(x2,y)$

$y1,y2 \in X, y1 \preccurlyeq y2 \text{ implies } F(x,y1) \preccurlyeq F(x,y2)$.

Definition 3-

Let $(X,\preccurlyeq)$ be a partially ordered set and $F : X^2 \longrightarrow X$ and $g : X \longrightarrow X$.We say F is called the mixed g-monotone property iffor any $x, y \in X$,

$x1,x2 \in X, gx1 \preccurlyeq gx2 \text{ implies } F(x1,y) \preccurlyeq F(x2,y)$ $y1,y2 \in X, gy1 \preccurlyeq gy2 \text{ implies } F(x,y1) \preccurlyeq F(x,y2)$.

Definition 4-

An element $(x, y) \in X^2$ is said to be a coupled common fixed point of the mappings $F : X^2 \longrightarrow X$ and $g : X \longrightarrow X$ if $F(x, y) = gx = x$ and $F(y, x) = gy = y$.

Definition 5-

An element $(x, y) \in X^2$ is said to be a coupled coincidence point of the mappings $F : X^2 \longrightarrow X$ and $g : X \longrightarrow X$ if $F(x, y)=gx$and $F(y, x) = gy$.

Definition 6 -

Let X be a nonempty set and $F : X^2 \longrightarrow X$ and $g : X \longrightarrow X$. We say F and g are commutative if $gF(x, y) = F(gx, gy)$for all$x,y \in X$.

In 2010, Choudhury and Kundu introduced the notion of compatibility in the context of coupled coincidence point problems as follows.

Definition 7-

The mappings $F : X^2 \longrightarrow X$ and $g : X \longrightarrow X$ are said to be compatible if

$\lim_{n\to\infty} d(g(F(xn,yn)),F(gxn,gyn))=0$ and $\lim_{n\to\infty} d(g(F(yn,xn)),F(gyn,gxn))=0$ whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $\lim_{n\to\infty} F(x_n, y_n)= \lim_{n\to\infty} gx_n = x$ and $\lim_{n\to\infty} F(y_n,x_n) =\lim_{n\to\infty} gy_n = y$ with $x,y \in X$.

Definition 8-

Let Φ denote the class of functions $\phi : [0,\infty) \longrightarrow [0, \infty)$ which satisfies the following conditions:

- (ϕ_1) ϕ is lower semi-continuous and (strictly) increasing;
- $(\phi_2)\phi(t) < t$ for all $t > 0$;
- (ϕ_3) $\phi(t + s) \le \phi(t) + \phi(s)$ for all $t,s \in [0,\infty)$.
- Note that $\lim_{n\to\infty} \phi(t_n) = 0 \Leftrightarrow \lim_{n\to\infty} t_n = 0$for $t_n \in [0,\infty)$.
- Also, for $\phi \in \Phi$, $\Psi_\phi$denote all functions $\psi : [0,\infty) \longrightarrow [0,\infty)$which satisfy the following conditions: $(\Psi_1)\limsup_{n\to\infty} \Psi(t_n) < \phi(r)$ if $\lim_{n\to\infty} t_n = r > 0$;
- (Ψ_2) $\lim_{n\to\infty} \Psi(t_n) = 0$ if $\lim_{n\to\infty} t_n = 0$ for $t_n \in [0,\infty)$.

Now, we have the following coupled fixed point theorem as the main conclusion.

Theorem 1 -

Let $(X, \le)$ be a partially ordered set and there is a metric d on X such that (X, d) is a complete metric space. Suppose that $F : X^2 \longrightarrow X$ is a mapping having the mixed monotone property on X. Assume there exists $\phi \in \Phi$ and $\Psi \in \Psi_\phi$ such that

$\varphi(\{d(F(x,y),F(u,v))+d(F(y,x),F(u,v))\}\times 2-1)\leq\psi(\{d(x,u)+d(y,u)\}\times 2-1)$ all $x, y, u, v \in X$ with $x \geq u$ and $y \leq v$.

Suppose that either

a. F is continuous or;
b. X has the following properties:
1. if a non-decreasing sequence $\{x_n\} \to x$, then $x_n \leq x$ for all n,
2. if a non-increasing sequence $\{y_n\} \to y$, then $y \leq y_n$ for all n.

If there exist two elements $x_0, y_0 \in X$ with $x_0 \leq F(x_0,y_0)$ and $y_0 \geq F(y_0,x_0)$. Then there exist $x, y \in X$ such that $x = F(x,y)$, $y = F(y, x)$,that is, F has a coupled fixed point in X.

The fixed point theorem using the context of metric spaces endowed with a graph was initiated by Jachymski . Other results for single valued and multivalued operators in such metric spaces were given by Beg et al .

Let (X, d) be a metric space, Δ be a diagonal of X^2, and G be a directed graph with no parallel edges such that the set V (G) of its vertices coincides with X and $\Delta \subseteq E(G)$, where E (G) is the set of the edges of the graph. That is, G is determined by (V (G), E (G)). We will use this notation of G throughout this work.

In this section, we study the existence of solution of the nonlinear integral equations, as an application of the fixed point theorem proved in Main Results.

Nonlinear Integral Equation as an Application of Fixed Point Theorem

Consider the following nonlinear integral equation:

$x(t)=q(t)+\int T0A(t,s)h(s,x(s),y(s))ds, y(t)=q(t)+\int T0A(t,s)h(s,y(s),x(s))ds$, where $t \in I = [0, T]$ with $T > 0$.

We considered the space $X := C(I, \mathbb{R}^n)$. Let $\|x\| = \max_{t\in I} |x(t)|$, for $x \in X$.Consider the graph G with partial order relation by

$x,y\in X, x\leq y \Leftrightarrow x(t)\leq y(t)$ foranyt$\in I$.

Then $(X, \|.\|)$ is a complete metric space endowed with a directed graph G.

Let $E(G) = \{(x, y) \in X^2 : x \leq y\}$. Thus E (G) satisfies the transitivity property, and $(X, \|.\|, G)$ has property A. We consider the following conditions:

1. $h : I \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ and $q : I \to \mathbb{R}^n$ are continuous
2. there exists a continuous $0 \leq \alpha < 1$ such that $|h(s,x,y)-h(s,u,v)|\leq\alpha(|x-u|+|y-v|)$ for all $x, y, u, v \in \mathbb{R}^n$ and for all $s \in I$;
3. for all $t, s \in I$, there exists a continuous $A : I \times \mathbb{R} \to \mathbb{R}$ such that there exists $(x_0, y_0) \in X^2$ such that $x0(t)\leq q(t)+\int T0A(t,s)h(s,x0(s),y0(s))ds, y0(t)\leq q(t)+\int T0A(t,s)h(s,y0(s),x0(s))ds$, where $t \in I$.

CONCLUSION

We studied about the coupled coincidence and coupled common fixed point theorems for satisfying nonlinear contractions in partially ordered metric spaces. Presented theorems are generalizations of the numerous past results of Coupled fixed point hypotheses in summed up metric spaces. Then again, there has been late interest for building up fixed point hypotheses in partially ordered complete metric spaces with a contractivity condition which holds for all focuses that are connected by partial ordering.

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