Analysis of Arrival and Service Distribution of Finite Population Queuing Model by using Binomial Distribution and Uniform Distribution

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Abstract:

The aim of this paper is to study the application of binomial distribution and uniform distribution by using basic parameters average arrival rate , inter-arrival time , average service rate of server and service time of a customer to finite population queuing model. It gives some important information in the form of relation that total time required for the arrival of finite number of customers, total time required to complete the service of finite number of customers. It also gives the relation of probability of finite number of customers in the system. The study of arrival and service distribution of accepted and rejected customers by the server is discussed in this paper.

Keywords: Average arrival rate, inter-arrival time, average service rate, service time of a customer, Utilization factor.

1. *Introduction:*

As we know that in most of the queuing models population is consider to be infinite countable and for this Poisson distribution is use for arrival rate and service rate. For the inter-arrival time of customers and service time of customer both are continuous, exponential distribution is used. Here we discussed about the finite population queuing model which easily countable. Binomial distribution is more suitable distribution for arrival and service rate. In steady state condition time is uniformly distributed and therefore we apply here uniform distribution for the inter-arrival time of customers and service time of customer. There are two types of customers we consider here for the discussion, accepted customer and rejected customer by the server. Customers who are rejected by the server, it takes some time to check their status. This checking time we consider it as a service time of rejected customers.

2. Methodology:

From the data which we collect from the finite population queuing model having inter-arrival time of customer and service time of each customer, average arrival rate and average service rate can be calculated. Queuing model is working on (FCFS) queue discipline.

Let 'M' be the capacity of system and 'α'be the average arrival rate of customers. 'β_1' and 'β_2'be the average service rate of accepted customers and average service rate of rejected customers respectively. $\frac{1}{\alpha}$ average inter-arrival time, $\frac{1}{\beta_1}$ and $\frac{1}{\beta_2}$ average service time of accepted and rejected customers respectively.

Utilization factor= server is busy for service is $\rho = \frac{\alpha}{\beta}$. In the similar way we can find utilization factor for accepted and rejected customer.

3. Method to find β_1, β_2 and β :

Let 'm' numbers of customers are accepted by the server and 'n' be the number of customers rejected by the server. Let $t_1, t_2, t_3, \ldots\ldots\ldots, t_m$ be the service time recorded for 'm' number of accepted customers respectively and $s_1, s_2, s_3 \ldots\ldots s_n$ be the service time recorded for 'n' number of rejected customers respectively

$$\therefore \beta_1 = \frac{\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\ldots\ldots+\frac{1}{t_m}}{m} \quad \ldots\ldots (1) \qquad \text{and} \qquad \beta_2 = \frac{\frac{1}{s_1}+\frac{1}{s_2}+\frac{1}{s_3}+\ldots\ldots+\frac{1}{sn}}{n} \quad \ldots\ldots\ldots(2)$$

If β be the total average service rate of accepted and rejected customers then,

$$\therefore \beta = \frac{\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\ldots\ldots+\frac{1}{t_m}+\frac{1}{s_1}+\frac{1}{s_2}+\frac{1}{s_3}+\ldots\ldots+\frac{1}{s_n}}{m+n} = \frac{\beta_1 m+\beta_2 n}{m+n} \quad \ldots\ldots\ldots (3)$$

By binomial distribution, probability of 'x' customers arrived in the system is $P_x = {}^M C_x p^x q^{M-x}$

$$\therefore P_x = {}^M C_x \left(\frac{\alpha}{M}\right)^x \left(1-\frac{\alpha}{M}\right)^{M-x} \quad \text{where} \quad \alpha = Mp$$

$\therefore$ Probability of 'x' customers arrived in time 't' is $P_x(t) = {}^M C_x \left(\frac{\alpha t}{M}\right)^x \left(1-\frac{\alpha t}{M}\right)^{M-x}, t < \frac{M}{\alpha}$ ………(4)

From this relation it is clear that the time required for the arrival of 'M' number of customers is $o \le t < \frac{M}{\alpha}$.

If $t \ge \frac{M}{\alpha}$ probability of arriving customer is either zero or negative and negative probability is not possible.

Probability of 'x' customers (both accepted and rejected together) served by the server is

$$\therefore P_x = {}^M C_x \left(\frac{\beta}{M}\right)^x \left(1-\frac{\beta}{M}\right)^{M-x} \quad \text{where } \beta = Mp$$

$\therefore$ Probability of 'x' customers served by the server in time 't' is $P_x(t) = {}^M C_x \left(\frac{\beta t}{M}\right)^x \left(1-\frac{\beta t}{M}\right)^{M-x}, \ t < \frac{M}{\beta}$

……. (5)

From this relation it is clear that the service time required for 'M' customers is $o \le t < \frac{M}{\beta}$.

If $t \ge \frac{M}{\beta}$ probability of customer for service is either zero or negative and negative probability is not possible.

Similarly, for the accepted and rejected customers the service time required for 'M' customers are $o \le t < \frac{M}{\beta_1}$ and $o \le t < \frac{M}{\beta_2}$ respectively.

4. Time Distribution for the Arrival and Service Provider of Finite Population Queuing Model by using Uniform Distribution:

As we apply binomial distribution for arrival rate of customer and service rate of server, it has been observed that the time limit is finite so we apply here uniform distribution for arrival time and service time.

By uniform distribution probability density function for continuous random variable't' is $f(t) = \dfrac{1}{b-a}$, $a \le t \le b$

$= 0$, $otherwise$ ……. (6)

The cumulative distribution function of uniform distribution is

$$F(t) = \int_{-\infty}^{t} f(t)dt = \int_{a}^{t} \frac{1}{b-a} dt = \frac{1}{b-a}[t-a] \qquad \text{…….. (7)}$$

For arrival time distribution, average inter-arrival time of customer is $\dfrac{1}{\alpha}$ and mean of uniform distribution is $\dfrac{a+b}{2}$

$$\therefore \frac{1}{\alpha} = \frac{a+b}{2} \Rightarrow \alpha = \frac{2}{a+b} \Rightarrow b = \frac{2}{\alpha} - a$$

Here time starts from $t = 0$ which gives $a = 0 \Rightarrow b = \dfrac{2}{\alpha}$

$\therefore$ Probability density function is $f(t) = \dfrac{1}{\frac{2}{\alpha} - 0} = \dfrac{\alpha}{2}$, $0 \le t \le \dfrac{2}{\alpha}$ …….. (8)

And cumulative distribution function is

$$F(t) = \frac{1}{b-a}[t-a] = \frac{1}{\frac{2}{\alpha} - 0}[t-0] = \frac{\alpha t}{2} \ , \ 0 \le t \le \frac{2}{\alpha} \ . \qquad \text{……… (9)}$$

Which gives the probability of next customer arrive in time't' if a customer already arrived.

The time limit indicates that the 100% arrival of next customer in time $\dfrac{2}{\alpha}$.

For service time distribution of customers (both accepted and rejected together), average service time between two successive customers is $\dfrac{1}{\beta}$

$\therefore$ Probability density function is $f(t) = \dfrac{1}{\frac{2}{\beta} - 0} = \dfrac{\beta}{2}$, $0 \le t \le \dfrac{2}{\beta}$ ………… (10)

And cumulative distribution function is

$$\therefore F(t) = \frac{\beta t}{2} \ , \ 0 \le t \le \frac{2}{\beta} \qquad \text{………… (11)}$$

Which gives the probability of next customer served in time't' if a customer already served.

The time limit indicates that the 100% served the next customer in time $\dfrac{2}{\beta}$.

Similarly, **f**or service time distribution of accepted customers and rejected customers are calculated.

5. Relation of probability for number of customers in the system:

System will have probability of containing 'x' number of customers at time $(t + \omega t)$ is

$$P_x(t+\omega t) = P_x(t)\{\Pr ob(zero \ \ arrival \ \& \ zero \ departure)\} + P_{x+1}(t)\{\Pr ob(zero \ \ arrival \ \& \ one \ departure)\} + P_{x-1}(t)\{\Pr ob(one \ \ arrival \ \& \ zero \ departure)\}$$

$$= P_x(t)\left\{\left(1-\frac{\alpha\omega t}{M}\right)^M\left(1-\frac{\beta\ \omega t}{M}\right)^M\right\} + P_{x+1}(t)\left\{\left(1-\frac{\alpha\omega t}{M}\right)^M M\left(\frac{\beta\ \omega t}{M}\right)\left(1-\frac{\beta\ \omega t}{M}\right)^{M-1}\right\}$$

$$+ P_{x-1}(t)\left\{M\left(\frac{\alpha\omega t}{M}\right)\left(1-\frac{\alpha\omega t}{M}\right)^{M-1}\left(1-\frac{\beta\ \omega t}{M}\right)^M\right\}$$

………. (from equations (4) & (5))

$$= P_x(t)\left\{\left(1-M\frac{\alpha\omega t}{M}\right)\left(1-M\frac{\beta\ \omega t}{M}\right)\right\} + P_{x+1}(t)\left\{\left(1-M\frac{\alpha\omega t}{M}\right)M\left(\frac{\beta\ \omega t}{M}\right)\left(1-M\frac{\beta\ \omega t}{M}\right)\left(1+\frac{\beta\ \omega t}{M}\right)\right\}$$

$$+ P_{x-1}(t)\left\{M\left(\frac{\alpha\omega t}{M}\right)\left(1-M\frac{\alpha\omega t}{M}\right)\left(1+\frac{\alpha\omega t}{M}\right)\left(1-M\frac{\beta\ \omega t}{M}\right)\right\}$$

…… (As ωt is very small so neglecting higher power terms greater than or equal to 2 of ωt)

$$= P_x(t)\{1-(\alpha+\beta\ )\omega t\} + P_{x+1}(t)\left\{(1-(\alpha+\beta\ )\omega t)(\beta\ \omega t)\left(1+\frac{\beta\ \omega t}{M}\right)\right\} + P_{x-1}(t)\left\{(\alpha\omega t)(1-(\alpha+\beta\ )\omega t)\left(1+\frac{\alpha\omega t}{M}\right)\right\}$$ …..

(Neglecting higher powers of ωt)

$$= P_x(t)\{1-(\alpha+\beta\ )\omega t\} + P_{x+1}(t)\{(\beta\ \omega t)\} + P_{x-1}(t)\{(\alpha\omega t)\}$$ ….. (Neglecting higher powers of ωt)

$$= P_x(t) - P_x(t)(\alpha+\beta\ )\omega t + P_{x+1}(t)\{(\beta\ \omega t)\} + P_{x-1}(t)\{(\alpha\omega t)\}$$ ….. (Neglecting higher powers of ωt)

$$\therefore P_x(t+\omega t) - P_x(t) = -P_x(t)(\alpha+\beta\ )\omega t + P_{x+1}(t)\{(\beta\ \omega t)\} + P_{x-1}(t)\{(\alpha\omega t)\}$$

Dividing both sides by ωt and takes $\lim \omega t \to 0$

$$\lim_{\omega t\to 0}\frac{P_x(t+\omega t)-P_x(t)}{\omega t} = \alpha P_{x-1}(t) + \beta\ P_{x+1}(t) - (\alpha+\beta\ )P_x(t)$$

$$\therefore P_x'(t) = \alpha P_{x-1}(t) + \beta\ P_{x+1}(t) - (\alpha+\beta\ )P_x(t)$$ ……… (12)

Consider the system having no customer then $x=0$ and for this value no service gives $\beta\ =0$

$$\therefore P_0'(t) = \beta\ P_1(t) - \alpha P_0(t)$$ ……… (13)

We know for the steady state condition as $t\to\infty$, $P_x(t)=P_x$ and $P_x'=0, P_0'=0$

$\therefore$ Equation (12) becomes $0=\alpha P_{x-1}+\beta\ P_{x+1}-(\alpha+\beta\ )P_x$ ……….(14)

And equation (13) becomes $0=\beta\ P_1-\alpha P_0$

$$\therefore P_1 = \frac{\alpha}{\beta}P_0$$ ………. (15)

Put $x=1$ in (14), $0=\alpha P_0+\beta\ P_2-(\alpha+\beta\ )P_1$

$\therefore 0=\alpha P_0+\beta\ P_2-(\alpha+\beta\ )\dfrac{\alpha}{\beta}P_0$ from (15), which gives $P_2=\left(\dfrac{\alpha}{\beta}\right)^2 P_0$

$\therefore$ In general we get relation of probability which gives probability of 'x' customer in the system is $P_x=\left(\dfrac{\alpha}{\beta}\right)^x P_0$

……….(16)

We have $\sum_{x=0}^{M} P_x = 1 \Rightarrow \sum_{x=0}^{M}\left(\frac{\alpha}{\beta}\right)^x P_0 = 1$ from (16)

$$\therefore P_0 = \frac{1}{\sum_{x=0}^{M}\left(\frac{\alpha}{\beta}\right)^x} = \frac{1}{\frac{1-\left(\frac{\alpha}{\beta}\right)^{M+1}}{1-\frac{\alpha}{\beta}}} = \frac{1-\frac{\alpha}{\beta}}{1-\left(\frac{\alpha}{\beta}\right)^{M+1}} = \frac{1-\rho}{1-\rho^{M+1}} \quad \ldots\ldots\ldots (17)$$

$\therefore$ Equation (16) becomes probability of $'x'$ customer in the system is

$$P_x = \left(\frac{\alpha}{\beta}\right)^x \left(\frac{1-\frac{\alpha}{\beta}}{1-\left(\frac{\alpha}{\beta}\right)^{M+1}}\right) = (\rho)^x\left(\frac{1-\rho}{1-\rho^{M+1}}\right), x = 0,1,2,\ldots\ldots.M\ , \rho \neq 1, \alpha \neq \beta \quad \ldots\ldots\ldots (18)$$

$$= \frac{1}{M+1},\ \rho = 1, \alpha = \beta \quad \ldots\ldots\ldots(19)$$

Similar results can be obtained for $'x'$ accepted and rejected customers by the server in the system.

6. Conclusion: From the above results (4) and (5) it has been observed that the application of binomial distribution on finite queuing model gives the particular fixed time on which finite number of customers arrived and service time required to serve finite number of customers. Also the application of uniform distribution result (9) and (11) indicates the time of 100% arrival and served the next customer in the system. The waiting cost of rejected customer and cost of service mechanism during checking their status can be analyze and minimize it. Overall the application of binomial distribution and uniform distribution on finite population queuing model gives more accurate results as compared to the Poisson distribution and exponential distribution.

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