

TWO DIMENSIONAL STEADY INCOMPRESSIBLE LAMINAR BOUNDARY LAYER ALONG A POROUS WALL

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ABSTRACT : The point of separation for the two dimensional incompressible boundary layer with uniform Suction Along a porous wall in Tani's flow for which the potential flow velocity is given by $U_{(x)} = U_0 \left(1 - \frac{x^2}{a^2}\right)$ has been obtained. The momentum integral, the K.E. integral equation and wall compatibility conditions have been employed for numerical solution by Runge kutta method.

Key words: Runge-Kutta method, Momentum integral equation, K.E. integral equation, wall compatibility condition, divergent channel.

BOUNDARY LAYER ALONG POROUS WALL IN TANI'S FLOW

It is proposed to investigate the boundary layer with suction along porous wall in Tani's flow for which the potential flow velocity is given by $U_{(x)} = U_0 \left(1 - \frac{x^2}{a^2}\right)$.

This type of flow occurs when a straight channel with parallel walls is succeeded by divergent channel with adverse pressure gradient. U_0 is the velocity at the entrance where the diverging flow states and 'a' is reference length.

for this flow $\frac{U_{(x)}}{U_0} = 1 - \frac{x^2}{a^2}$

i.e. $\bar{U} = 1 - \bar{x}^2$

and $\Lambda = \frac{\theta^2}{v} \frac{dU}{dx} = t^* \frac{d\bar{U}}{d\bar{x}} = -2\bar{x}t^*$

The momentum integral equation

$$\frac{dt^*}{d\bar{x}} = \frac{2}{\bar{U}} [I - (H + 2)\Lambda + \lambda] \quad (1)$$

The K.E integral equation

$$\frac{dH_\epsilon}{d\bar{x}} = \frac{1}{\bar{U}t^*} [2D - H_\epsilon \{I - (H - 1)\Lambda + \lambda\} \lambda] \quad (2)$$

$$\frac{dH_\epsilon}{d\bar{x}} = g(\bar{x}, t^*, H_\epsilon) \quad (2a)$$

and the wall compatibility condition.

$$(K + 1) \left(\frac{\theta}{\delta}\right)^2 + \lambda \left(\frac{\theta}{\delta}\right)^2 \left\{1 + \left(1 - \frac{\pi}{\sigma}\right)K\right\} - \Lambda = 0 \quad (3)$$

becomes

$$\frac{dt^*}{\partial \bar{x}} = f(\bar{x}, t^*, H_\epsilon) \quad (4)$$

VALUES AT STARTING POINT

The entrance suction ($\bar{x} = 0$) of the diverging flow is taken as the starting point for the numerical step by step calculation.

At $\bar{x} = 0, t^* = 0$ and hence from equation

$$(\lambda'^2 + 15\lambda + 90)K_8 = -90Xt^* \left(\frac{\delta^2}{\theta^2}\right) - 100.17\lambda'$$

we have

$$K_8 = \frac{-100.17\lambda'}{\lambda'^2 + 15\lambda + 90} \quad (5)$$

$\lambda' = 0$ then $K_8 = 0$

and

$$\frac{\theta}{\delta} = 0.09997$$

$$H = 2.5900$$

$$I = 0.2225$$

$$H_e = 1.5699$$

$$D = 0.1685$$

SOLUTION OF THE MOMENTUM INTEGRAL AND KINETIC ENERGY INTEGRAL EQUATIONS

With the values at the starting point numerical solutions of the momentum integral equation (4) and K.E. Integral equation 2(a) has been obtained for $\lambda' = 0$ and similarly, we obtained $\lambda' = -0.5, -1.0, \dots$

Numerical integration has been carried out for constant values of $\lambda' = 0.0, -0.5$ and -1.0 by Runge-Kutta method.

DISCUSSION OF RESULT

The momentum integral and the K.E integral equation have been solved numerically by Runge-Kutta method with the aid of

(i) Eight degree velocity profile P_8 and

(ii) Schlichting profile for

$\lambda' = 0$ in case of P_8 and $\lambda = 0$ in case of schlichting profile.

COMPARISON TABLE:

Point of separation in Tani's flow for solid (wall)

Authors	Point of separation
\bar{x}	
Tani	0.2710 (Exact method)
Pohlhausen	0.3180
Schlichting Profile	0.3400
Eight degree velocity profile (P_8) (Present method)	0.2650

The point of separation with the aid of eight degree velocity profile (P_8) is $\bar{x} = 0.2650$ which is very close to the result of Tani which is $\bar{x} = 0.2710$. Hence, it is expected the results obtained for porous wall with the aid of eight degree velocity profile would be sufficiently accurate.

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