STAR RELATED REVERSE-GRAPHOIDAL MAGIC STRENGTH

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Abstract ***Let $G = (V, E)$ be a graph and let ψ be a graphoidal cover of G .The magic labelling f of G, there is a constant $c(f)$ such that $f(x) + f(y) + f(xy)$, , for every edge $xy \in E(G)$. The magic strength of G is defined as $m(G) = min\{c(f)$: if f is a magic labeling of $G\}$.In this paper we determine reverse process of graphoidal of magic strength called reverse- graphoidal magic strength and also proved reverse- graphoidal magic strength of $[P_n: S_2]$,Double Crowned star $K_{1,n} \odot 2K_1$, graph. $< K_{1,n} : n >$, graph $K_2 + mK_1$.***

Index Terms: Graphoidal Constant, Magic Graphoidal, Magic Srength, reverse- magic graphoidal, reverse- grahoidal magic strength.

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1. INTRODUCTION

Let P be a path $\{v_1, v_2, \ldots\ldots, v_n\}$ in ψ with $f^*(P) = f(v_1) + f(v_n) + \sum_{i=1}^{n-1} f(v_i v_{i+1}) = k$ is a constant, where f^* is the induced labeling on ψ. A graph G is said to be magic if there exist a bijection $f: V \cup E \to \{1,2,3 \ldots\ldots. m + n\}$; where '$n$' is the number of vertices and 'm' is the number of edges of a graph. Such that for all edges xy, $f(x) + f(y) + f(xy)$ is a constant. Such a bijection is called a magic labeling of G. . Then, we say that G admits ψ- magic graphoidal total labeling of G. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ- magic graphoidal total labelling of G.

B.D. Acharya and E. Sampath Kumar [1] defined graphoidal covering of graph. Selvam,Vasuki, Jeyanthi [9] introduced the concept of magic strength of a graph .

Here combination of graphoidal and magic strength we introduced a new concept (ie. Reverse) process of graphoidal of a magic strength is called ***reverse- graphoidal magic strength***.

Definition1.1

A complete bipartite graph $K_{1,n}$is called a ***star*** and it has $(n + 1)$ vertices and n edges

Definition 1.2

The ***Trivial graph*** K_1 or P_1 is the graph with one vertex and no edges

Definition1.3

Let $K_{1,n}\Theta 2K_1$ be the ***Double Crowned Star*** which is the graph obtained from a star $K_{1,n}$ by attaching double edge at each end vertex of $K_{1,n}$.

Definition 1.4

Let $S_2 = (v_1 v_0 v_1)$ be a star and let $[P_n : S_2]$ be the graph obtained from n copies of S_2 and the path $P_n = (u_1, u_2, u_3, \ldots\ldots\ldots\ldots., u_n)$ by joining u_j with the vertex v_0 of the j^{th} copy of S_2 by means of an edge, for $1 \le j \le n$

Definition 1.5

The graph $< K_{1,n} : n >$ is obtained by the subdivision of the edges of star $K_{1,n}$

II. MAIN RESULTS

Definition 2. 1

A reverse magic graphoidal labeling of a graph G is one-to-one map f from $V(G) \cup E(G) \to \{1,2,3, \ldots\ldots\ldots, m + n\}$where *'n'* is the number of vertices of a graph and *'m'* is the number of the edges of a graph, with the property that , there is an integer constant *'μ'* such that

$f^*(P) = \sum_{i=1}^{n-1} f(v_i\, v_{i+1}) - \{f(v_1) + f(v_n)\} = \mu_{rmgc}$, is a contant

Then the reverse methodology of magic graphoidal labeling is called reverse- magic graphoidal labeling (rmgl). Reverse process of magic graphoidal of a graph is called reverse - graphoidal magic graph.(rgmg).

Selvam and Vasuki [9] made a note, Let f be a magic labeling of G with constant $c(f)$.Then adding all the constant obtained at each edge. We have

$$\mathcal{E}\, c(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$$

From the above equation we introduce the concept of reverse process of graphoidal of a magic strength is called ***reverse - graphoidal magic strength*** and it is denoted as $rgms(G)$, is defined as the minimum of all μ_{rmgc} where the minimum is taken over all reverse magic graphoidal total labeling f of (G).

To proceed further, we make the following equation.

Note 1. Let f be a reverse magic graphoidal labeling of G with the constant μ_{rmgc}. Then ,adding all constant obtained at each edge, we get

$$rgms\,(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

Theorem 2.1

$rgms\,[P_n : S_2] = 3, \quad for\ n \geq 2$

Proof:

Let $\{v_1, v_2, \dots\dots, v_n, u_1, u_2, \dots\dots u_n, u_{11}, u_{12}, u_{22}, \dots\dots, u_{n1}, u_{n2}\}$ be the vertex set and $\{(v_1v_2), (v_2v_3), \dots\dots, (v_{n-1}v_n), (v_1u_1), (v_2u_2), \dots\dots, (v_nu_n), (u_1u_{11}), (u_1u_{12}), (u_2u_{21}), (u_2u_{22}), \dots\dots, (u_nu_{n1}), (u_nu_{n2})\}$ be the edge set of $[P_n : S_2]$

Here $m + n = 8n - 1$

Define $f: V \cup E \rightarrow \{1,2, \dots\dots, m+n\}$ by

$f(u_1) = 1, f(u_2) = 8n-1, f(u_3) = 8n-2, \dots\dots, f(u_n) = 7n+1$

$f(v_2) = 2, f(v_3) = 3\ f(v_4) = 4, \dots\dots, f(v_{n-1})) = n-1$

$f(u_{11}) = n, f(u_{21}) = n+1, f(u_{31}) = n+2, \dots\dots, f(u_{n1}) = 2n-1$

$f(v_{12}) = 7n, f(u_{22}) = 7n-1, f(u_{32}) = 7n-2, \dots\dots, f(u_{n2}) = 6n+1$

$f(u_1 v_1) = 2n$

$f(v_1v_2) = 2n+1$

$f(v_2u_2) = 4n+2$

$f(v_2v_3) = 4n, f(v_3v_4) = 4n-1, f(v_4v_5) = 4n-2, \dots\dots, f(v_{n-1}v_n) = 3n+3$

$f(v_3u_3) = 4n+3, f(v_4u_4) = 4n+4, f(v_5u_5) = 4n+5, \dots\dots, f(v_nu_n) = 5n$

$f(u_{11}u_1) = 6n, f(u_{21}u_2) = 6n-1, f(u_{31}u_3) = 6n-2, \dots\dots, f(u_{n1}u_{n2}) = 5n+1$

$f(u_1u_{12}) = 2n+3, f(u_2u_{22}) = 2n+4, f(u_3u_{32}) = 2n+5, \dots\dots, f(u_nu_{n2}) = 3n+2$

Let $\psi = \{P_1 = (u_1 v_1 v_2 u_2),$

$P_2 = (v_2v_3u_3), (v_3v_4u_4), \dots\dots (v_{n-1}v_nu_n),$

$P_3 = (u_{11}u_1u_{12}), (u_{21}u_2u_{22}), \dots\dots (u_{n1}u_nu_{n2})\}$

And we have the equation,

$$\mu_{rmgc}(f) = \sum_{e\in E} f(e) - \sum_{v\in V} d(v)f(v)$$

Then the equation becomes,

$\mu_{rmgc} f(P_1) = f(u_1 v_1) + f(v_1v_2) + f(v_2u_2) - \{1 \times f(u_1) + 1 \times f(u_2)\}$

$= 2n + 2n + 1 + 4n + 2 - \{1 \times 1 + 1 \times (8n-1)\}$

$= 8n + 3 - \{1 + 8n - 1\}$

$= 3$ ________________ (1)

$\mu_{rmgc} f(P_{2(1)}) = f(v_2v_3) + f(v_3u_3) - \{1 \times f(v_2) + 1 \times f(u_3)\}$

$= 4n + 4n + 3 - \{1 \times 2 + 1 \times (8n-2)\}$

$= 8n + 3 - \{2 + 8n - 2\}$

$= 3$ ________________ (2)

Continuing this process,

$\mu_{rmgc} f(P_{2(k)}) = f(v_{n-1}v_n) + f(v_nv_n) - \{1 \times f(v_{n-1}) + 1 \times f(u_n)\}$

$= 3n + 3 + 5n - \{1 \times (n-1) + 1 \times (7n+1)\}$

$= 8n + 3 - \{n - 1 + 7n + 1\}$

$= 3$ ________________ (3)

$\mu_{rmgc} f(P_{3(1)}) = f(u_{11}u_1) + f(u_1u_{12}) - \{1 \times f(u_{11}) + 1 \times f(u_{12})\}$

$= 6n + 2n + 3 - \{1 \times n + 1 \times 7n\}$

$= 8n + 3 - \{n + 7n\}$

$= 3$ ________________ (4)

Continuing this process,

$\mu_{rmgc}(f(P_{3(k)}) = f(u_{n1}u_n) + f(u_nu_{n2}) - \{1 \times f(u_{n1}) + 1 \times f(u_{n2})$

$= 5n - 1 + 3n + 2 - \{1 \times (2n-1) + 1 \times (6n-1)\}$

$= 8n + 3 - \{8n\}$

$= 3$ ________________ (5)

from (1), (2), (3), (4), and (5), we conclude that

$$\mu_{rmgc}[P_n : S_2] = 3$$

$$\therefore \quad rgms\,[P_n : S_2] = 3$$

Theorem 2.2

$rgms\,(K_{1,n}\theta\, 2K_1) = 0 \quad for\ n \geq 2$

Proof:

Let $\{u, u_1, u_2, \dots\dots u_n, u_{11}, u_{12}, u_{21}, \dots\dots, u_{n1}, u_{n2}\}$ be the vertex set and $\{uu_1, uu_2, \dots\dots uu_n, u_1u_{11}, u_1u_{12}, u_2u_{21}, u_2u_{22}, \dots\dots, u_nu_{n1}, u_nu_{n2}\}$ be the edge set of of $(K_{1,n}\theta\, 2K_1)$.

Here, $m + n = 6n + 1$

Define $f: V \cup E \rightarrow \{1,2, \dots\dots, m+n\}$ by

When n is even :

$f(u_1) = 1, f(u_2) = 6n+1, f(u_3) = 2, f(u_3) = 6n, \ldots, f(u_{n-1}) = \frac{n}{2}, f(u_n) = \frac{11n}{2} + 2$

$f(u_{11}) = \frac{n}{2}+1, f(u_{21}) = \frac{n}{2}+2, f(u_{32}) = \frac{n}{2}+3, \ldots\ldots\ldots\ldots\ldots f(u_{n1}) = \frac{n}{2}+n$

$f(u_{12}) = \frac{11n}{2}+1, f(u_{22}) = \frac{11n}{2}, f(u_{32}) = \frac{11n}{2}-1, \ldots\ldots\ldots\ldots\ldots f(u_{n2}) = \frac{9n}{2}+2$

$f(uu_1) = \frac{3n}{2}+1, f(uu_2) = \frac{9n}{2}+1, f(uu_3) = \frac{3n}{2}+2, \ldots f(uu_{n-1}) = 2n, f(uu_n) = 4n+2$

$f(u_1u_{11}) = 2n+1, f(u_2u_{21}) = 2n+2, f(u_3u_{31}) = 2n+3, \ldots\ldots\ldots\ldots\ldots, f(u_nu_{n1}) = 3n$

$f(u_1u_{12}) = 4n+1, f(u_2u_{22}) = 4n, f(u_3u_{32}) = 4n-3, \ldots\ldots\ldots\ldots\ldots, f(u_nu_{n2}) = 3n+2$

Let $\psi = \{ P_1 = (u_1uu_2), (u_3uu_4), \ldots\ldots\ldots\ldots\ldots\ldots\ldots, (u_{n-1}uu_n)$

$P_2 = (u_{11}u_1u_{12}), (u_{21}u_2u_{22}), \ldots\ldots\ldots\ldots\ldots\ldots\ldots, (u_{n1}u_nu_{n2})\}$

And we have the equation,

$$\mu_{rmgc}(f) = \sum_{e\in E} f(e) - \sum_{v\in V} d(v)f(v)$$

Then the equation becomes,

$\mu_{rmgc}f(P_{1(1)}) = f(u_1u) + f(uu_2) - \{1\times f(u_1) + 1\times f(u_2)\}$

$= \frac{3n}{2}+1+\frac{9n}{2}+1-\{1\times 1+1\times(6n+1)\}$

$= \frac{12n}{2}+2-\{1+6n+1\}$

$= 0$ ________________ (1)

Continuing this process,

$\mu_{rmgc} f(P_{1(k)}) = f(u_{n-1}u) + f(uu_n) - \{1\times f(u_{n-1}) + 1\times f(u_n)$

$= 2n+4n+2-\{(1\times\frac{n}{2}) + 1\times(\frac{11n}{2}+2)\}$

$= 0$ ________________ (2)

$\mu_{rmgc}(f(P_{2(1)}) = f(u_{11}u_1) + f(u_1u_{12}) - \{1\times f(u_{11}) + 1\times f(u_{12})\}$

$= 2n+1+4n+1-\{\frac{n}{2}+1+\frac{11n}{2}+1\}$

$= 6n+2-\{\frac{12n}{2}+2\}$

$= 0$ ________________ (3)

Continuing this process,

$\mu_{rmgc}(f(P_{2(k)}) = f(u_{n1}u_n) + f(u_nu_{n2}) - \{1\times f(u_{n1}) + 1\times f(u_{n2});$

$= 3n+3n+2-\{1\times(\frac{n}{2}+n)+1\times(\frac{9}{2}n+2)\}$

$= 6n+2-\{\frac{n}{2}+n+\frac{9}{2}n+2\}$

$= 0$ ________________ (4)

From(1), (2), (3)and (4), we conclude that

$$\mu_{rmgc}(K_{1,n}\theta 2K_1) = 0$$

$$\therefore \ rgms\,(K_{1,n}\theta 2K_1) = 0$$

When n is odd :

$f(u) = 1, f(u_1) = 6n$

$f(u_2) = 2, f(u_4) = 3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, f(u_{n-1}) = \frac{n+1}{2}$

$f(u_3) = 6n-1, f(u_5) = 6n-2, \ldots\ldots\ldots\ldots\ldots\ldots, f(u_n) = \frac{11n+1}{2}$

$f(u_{11}) = \frac{n+3}{2}, f(u_{21}) = \frac{n+5}{2}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, f(u_{n1}) = \frac{3n+1}{2}$

$f(u_{12}) = \frac{11n-1}{2}, f(u_{22}) = \frac{11n-3}{2}, \ldots\ldots\ldots\ldots\ldots\ldots, f(u_{n2}) = \frac{9n+1}{2}$

$f(uu_1) = 6n+1, f(uu_2) = \frac{3n+3}{2}, f(uu_3) = \frac{9n-1}{2}, \ldots\ldots, f(uu_{n-1}) = 2n, f(uu_n) = 4n+1$

$f(u_1u_{11}) = 2n+1, f(u_2u_{21}) = 2n+2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, f(u_nu_{n1}) = 3n$

$f(u_1u_{12}) = 4n, f(u_2u_{22}) = 4n-1, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, f(u_nu_{n2}) = 3n+1$

Let $\psi = \{P_1 = (uu_1),$

$P_2 = (u_2uu_3), (u_4uu_5), \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (u_{n1}uu_n)$

$P_3 = (u_{11}u_1u_{12}), (u_{21}u_2u_{22}), \ldots\ldots (u_{n1}u_nu_{n2})]\}$

We have the equation,

$$\mu_{rmgc}(f) = \sum_{e\in E} f(e) - \sum_{v\in V} d(v)f(v)$$

Then the equation becomes,

$\mu_{rmgc} f(P_1) = f(uu_1) - \{1 \times f(u) + 1 \times f(u_1)\}$
$= 6n + 1 - \{1 \times 1 + 1 \times 6n\}$
$= (6n+1) - (6n+1)$
$= 0$ ______________________________ (1)

$\mu_{rmgc}(f(P_{2(1)}) = f(u_2 u) + f(uu_3) - \{1 \times f(u_2) + 1 \times f(u_3)\}$
$= \frac{3n+3}{2} + \frac{9n-1}{2} - \{1 \times 2 + 1 \times (6n-1))\}$
$= \frac{12n+2}{2} - \{2 + 6n - 1\}$
$= 0$ ______________________________ (2)

Continuing this process,

$\mu_{rmgc}(f(P_{2(k)}) = f(u_{n-1} u) + f(uu_n) - \{1 \times f(u_{n-1}) + 1 \times f(u_n)$
$= 2n + 4n + 1 - \{1 \times \frac{(n+1)}{2} + 1 \times \frac{(11n+1)}{2}\}$
$= 6n + 1 - \{\frac{12n+2}{2}\}$
$= 0$ ______________________________ (3)

$\mu_{rmgc} f(P_{3(1)}) = f(u_{11} u_1) + f(u_1 u_{12}) - \{1 \times f(u_{11}) + 1 \times f(u_{12})\}$
$= 2n + 1 + 4n - \{\frac{1 \times (n+3)}{2} + \frac{(11n-1)}{2}\}$
$= 0$ ______________________________ (4)

Continuing this process,

$\mu_{rmgc} f(P_{3(k)}) = f(u_{n1} u_n) + f(u_n u_{n2}) - \{1 \times f(u_{n1}) + 1 \times f(u_{n2})\}$
$= 3n + 3n + 1 - \{1x \frac{3n+1}{2} + \frac{(9n+1)}{2}\}$
$= 6n + 1 - \{\frac{12n+2}{2}\}$
$= 0$ ______________________________ (5)

From (1), (2), (3), (4) and (5), we conclude that

$$\mu_{rmgc}(K_{1,n} \theta\, 2K_1) = 0$$
$$\therefore \; rgms\,(K_{1,n} \theta\, 2K_1) = 0$$

Theorem 5.4

$rgms\,(K_2 + mK_1) = 0$

Proof:

Let $V(G) = \{v, u, w_1, w_2 \ldots \ldots \ldots \ldots \ldots, w_m\}$
and $E(G) = \{vu, vw_1, vw_2, \ldots \ldots \ldots \ldots \ldots, vw_m, uw_1, uw_2 \ldots \ldots \ldots \ldots \ldots \ldots, uw_m\}$

$f(v) = 1$
$f(u) = 3m + 2$
$f(vu) = 3m + 3$
$f(vw_1) = 2, f(vw_2) = 3, f(vw_2) = 4, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots, f(vw_m) = 1 + m$
$f(uw_1) = 3m + 1, f(uw_2) = 3m, f(uw_3) = 3m - 1, \ldots \ldots \ldots, f(uw_m) = 2m + 2$

Let $\psi = \{P_1 = \{(uv\}$
$P_{2=}\{(vw_1 u), (vw_2 u), (vw_3 u), \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots, (vw_m u)\}$

We have the equation,

$$\mu_{rmgc}(f) = \sum_e f(e) - \left[\sum_v d(v).f(v)\right]$$

Then the equation becomes,

$\mu_{rmgc} f(P_1) = f(uv) - \{1 \times f(u) + 1 \times f(v)\}$
$= 3m + 3 - \{1 \times 1 + 1 \times (3m+2)\}$
$= 0$ ______________________ (1)

$\mu_{rmgc} f(P_{2(1)}) = f(vw_1) + f(w_1 u) - \{1\, x\, f(v) + 1 \times f(u)\}$
$= 1 + 1 + 3m + 2 - 1 - \{1 \times (3m+2) + 1 \times 1\}$
$= 0$ ______________________ (2)

$\mu_{rmgc} f(P_{2(2)}) = f(vw_2) + f(w_2 u) - \{1\, x\, f(u) + 1 \times f(v)\}$
$= 1 + 2 + 3m + 2 - 2 - \{1 \times 1 + 1 \times (3m+2)\}$
$= 0$ ______________________ (3)

Continuing this process,

$\mu_{rmgc} f(P_{2(k)}) = f(vw_m) + f(w_m u) - \{1 \times f(u) + 1 \times f(v)\}$
$= 1 + m + 2m + 2 - \{1 \times 1 + 1 \times (3m+2)\}$
$= 0$ ______________________ (4)

From (1), (2), (3), and (4) we conclude that

$$\mu_{rmgc}(K_2 + mK_1) = 0$$
$$\therefore rgms(K_2 + mK_1)) = 0$$

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