# Spectral Estimation Using Complex Morlet Transform

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*Abstract*— **Spectral estimation is very useful in the estimation of the power and frequency of the signal. The goal of *spectral estimation* is to describe the distribution of the power contained in a signal, based on a finite set of data. Estimation of power spectra is useful in a variety of applications, including the detection of signals buried in wide-band noise. The *power spectrum* of a stationary random process is mathematically related to the correlation sequence by the discrete-time Fourier transform. Frequency estimation of a sinusoid in white noise using the maximum entropy power spectral method has been shown to be very sensitive to the initial sinusoid. Previously work has been done which showed the dependence of the frequency estimate of the sinusoid in white noise on the phase of the sinusoid, and the Hilbert transform has been used to obtain the analytic signal. This paper used the wavelet method to obtain the analytic signal and then employed the maximum entropy power spectral method for the frequency estimation. The use of Morlet wavelet for the generation of the analytic signal is reducing the percentage change in frequency deviation. The analytic signal is generated using the complex Morlet wavelet.The dependence of the frequency estimate of a sinusoid in white noise on the phase of the sinusoid can be significantly reduced by employing an analytic signal approach.**

**Keywords**: Spectral Estimation, Frequency Estimation, Power Spectrum, maximum entropy power spectral method, Haar wavelet.

## I. Introduction

Frequency estimation of a sinusoid in white noise using the maximum entropy power spectral method has been shown to be very sensitive to the initial sinusoid. Kay [1] showed the dependence of the frequency estimate of the sinusoid in white noise on the phase of the sinusoid. Kay [1] used the Hilbert transform to obtain the analytic signal. This thesis used the wavelet method to obtain the analytic signal and then employed the maximum entropy power spectral method for the frequency estimation.

### ANALYTIC SIGNALS

A signal which has no negative frequency components is called as analytic signal. In continuous time every analytic signal $z(t)$ can be represented as

$$z(t) = \frac{1}{2\pi}\int_{0}^{\infty} Z(\omega)e^{j\omega t}d\omega \tag{1.1}$$

Where $Z(\omega)$ is the complex coefficient of the positive frequency complex sinusoid $e^{j\omega t}$ at frequency $\omega$.Any real sinusoid $A\cos(\omega t+\phi)$ may be converted to a positive frequency complex sinusoid $A\exp[j(\omega t+\phi)]$ by simply generating a phase quadrature.
Component $A\sin(\omega t+\phi)$ is working as the imaginary part

$$A\exp[j(\omega t+\phi)] = A\cos(\omega t+\phi) + jA\sin(\omega t+\phi) \tag{1.2}$$

The phase-quadrature component can be generated from the in-phase component by a simple quarter-cycle time shift.

## II. ANALYTIC SIGNALS USING HILBERT TRANSFORM

Complicated signals which are expressible as a sum of many sinusoids, a filter can be constructed which shifts each sinusoidal component by a quarter cycle. This is called a Hilbert transform filter. Let $H_t\{x\}$ denote the output at time $t$ of the Hilbert-transform filter applied to the signal $x$ . Ideally, this filter has magnitude $1$ at all frequencies and introduces a phase shift of $-\pi/2$ at each positive frequency and at each negative $+\pi/2$ frequency. When a real signal $x(t)$ and its Hilbert transform $y(t) = H_t\{x\}$ are used to form a new complex signal $z(t) = x(t) + jy(t)$ , the signal $z(t)$ is the (complex) analytic signal corresponding to the real signal $x(t)$ .

For any real signal $x(t)$ the corresponding analytic signal $z(t) = x(t) + jH_t\{x\}$ has the property that all negative frequencies of $x(t)$ have been filtered out. The phase shifts can be impressed on a complex sinusoid by multiplying it by $\exp(\pm j\pi/2) = \pm j$ . Consider the positive and negative frequency components at the particular frequency $\omega_0$ :

$$x_{+}(t) \square\ e^{j\omega_0 t}$$
$$x_{-}(t) \square\ e^{-j\omega_0 t} \qquad (2.1)$$

Now by applying a -90 degrees phase shift to the positive-frequency component, and a +90 degrees phase shift to the negative-frequency component:

$$y_{+}(t) = e^{-j\pi/2} e^{j\omega_0 t} = -je^{j\omega_0 t}$$
$$y_{-}(t) = e^{j\pi/2} e^{-j\omega_0 t} = je^{-j\omega_0 t} \qquad (2.2)$$

Adding them together gives

$$z_{+}(t) = x_{+}(t) + jy_{+}(t) = e^{j\omega_0 t} - j^2 e^{j\omega_0 t} = 2e^{j\omega_0 t}$$
$$z_{-}(t) = x_{-}(t) + jy_{-}(t) = e^{-j\omega_0 t} + j^2 e^{-j\omega_0 t} = 0 \qquad (2.3)$$

and the negative frequency component is filtered out. For example, let's start with the real sinusoid

$$x(t) = 2Cos(\omega_0 t) = e^{j\omega_0 t} + e^{-j\omega_0 t} \qquad (2.4)$$

Applying the ideal phase shifts, the Hilbert transform is

$$y(t) = e^{j\omega_0 t - j\pi/2} + e^{-j\omega_0 t + j\pi/2}$$
$$= -je^{j\omega_0 t} + je^{-j\omega_0 t} \qquad (2.5)$$
$$= 2Sin(\omega_0 t)$$

The analytic signal is then

$$z(t) = x(t) + jy(t) = 2Cos(\omega_0 t) + j2Sin(\omega_0 t) = 2e^{j\omega_0 t}$$
(2.6)

by Euler's identity. Thus, in the sum $x(t) + jy(t)$, the negative-frequency components of $x(t)$ and $jy(t)$ cancel out, leaving only the positive-frequency component. This is true for any real signal.

## III. ANALYTIC SIGNALS USING WAVELET TRANSFROM

By considering an analytic wavelet function $g(t)$ and its Fourier transform $g(\omega)$ satisfying

$$g(t) \in L^1(R, dt) \cap L^2(R, dt) \qquad (3.1)$$
$$g(\omega) \in L^1(R \setminus \{0\}, d\omega / |\omega|) \cap L^2(R \setminus \{0\}, d\omega / |\omega|) \qquad (3.2)$$

For a given signal $s(t) \in L^2(R, dt)$ the wavelet transform of $s(t)$ with respect to wavelet $g(t)$ is defined as

$$S(b,a) = \frac{1}{a} \int_{-\infty}^{\infty} s(t) \overline{g}\left(\frac{t-b}{a}\right) dt \qquad (3.3)$$

Where

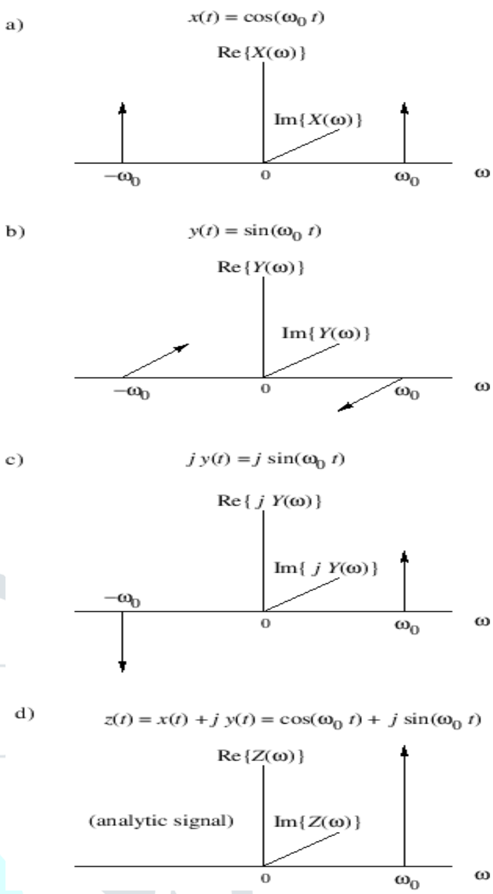


Fig. (1) Generation of Analytic Signal

the real number is set and $a > 0, \overline{g}(t)$ is complex conjugate of $g(t)$.

If $s(t)$ is a signal with finite energy and $g(t)$ is an analytic wavelet function, then $S(b,a)$, the wavelet transform $s(t)$, with respect to $g(t)$ is a complex function with respect to real valued variable $b$ and the scale factor $a(a > 0)$.For a fixed value of $a$,the imaginary part of this complex function is the Hilbert transform of the real part i.e., $S(b,a)$ is an analytic function with respect to $b$.

## IV. SPECTRAL ESTIMATION USING ANALYTIC SIGNAL

Kay[1] used maximum entropy power spectral estimations using Burg's estimate for the reflection coefficients is used for the frequency estimation. The location of the peak of the spectrum is very sensitive to the initial phase of the sinusoid. The dependence of the frequency estimate on phase is attributed to the interaction between the positive and negative frequency spectra in analogy with known results from Fourier power spectral estimation theory. Use of the discrete analytic signal for which the power spectrum is zero for $-(r / A_0) < \omega < 0$, where $A_0$ is the sampling interval. The analytic noise is now nonwhite and by decreasing the sampling rate by two, the resultant complex noise would be white. The whiteness of the noise is a desirable property in that the peak of the estimated power spectral density is then an asymptotically unbiased estimate of frequency**.** By forming the analytic signal and decreasing the sampling rate by 2, the data will consist of a complex sinusoid, in complex white noise, i.e., if *X;* is the real discrete signal, then

$$X_t^{'} = A\sin(\omega_0 + \phi) + W_t^{'} \tag{4.1}$$

Where $W_t^{'}$ is white noise with $E\left(W_t^{'}\right) = 0$ and

$$R_{\omega^{'}}(k) = E\left(W_t^{'}W_{t+k}^{'}\right) = \sigma_k^2\delta(k) \tag{4.2}$$

Then the analytic signal is

$$Z_t^{'} = X_t^{'} + jX_t^{'}$$
$$= -jAe^{j(\omega_0+\phi)} + W_t^{'} + jW_t^{'} \tag{4.3}$$

Where $X_t^{'}$ denotes the Hilbert transform of $X_t^{'}$ and the resultant signal after down sampling is

$$Z_t = Z_{2t}^{'} = -jAe^{j(2\omega_0 t+\phi)} + W_t^{c} \tag{4.4}$$

The real and complex maximum entropy power spectral estimators are

$$P_r(\omega) = \frac{P_{pr}\Delta_0}{\left|1 + a_1^r e^{-j\omega} + \ldots + a_{pr}^r e^{-j\omega p_r}\right|^2} \tag{4.5}$$

$$P_c(\omega) = \frac{P_{pc}\Delta}{\left|1 + a_1^c e^{-j\omega} + \ldots + a_{pc}^c e^{-j\omega p_c}\right|^2} \tag{4.6}$$

Where $P_r$, $P_c$ are the orders for the real and complex spectra. $P_{pr}, P_{pc}$ are prediction error power for the real and complex spectra.

The parameter set are determined from the Burg Estimation method for the reflection coefficient in conjunction with Levinson recursion. The parameter sets $\{a_1^r \ldots\ldots\ldots . a_{pr}^r, P_{pr}\}$ and $\{a_1^c \ldots\ldots\ldots\ldots\ldots, a_{pc}^c, P_{pc}\}$ are determined from the Burg estimation method for the reflection Coefficients in conjunction with Levinson recursion. It should be noted that only *pc* = 9 *pr* need be used since the complex conjugate poles of the real analysis are not modeled in the complex approach.

After the determination of the reflection coefficients, the dependence of the frequency estimate of a sinusoid in white noise on the phase of the sinusoid can be significantly reduced by using and analytic signal approach.

The analytic counterpart corresponding to the real valued signal is obtained using Hilbert Transform in section 4.2. Due to the infinite length there is reduction of estimation precision. This is the conventional methods to obtain the analytic signal.

Kay [1] used the frequency estimation using the Burg's maximum entropy method [2] and we used a new method of spectral estimation using wavelet method. In section 4.3 we have used wavelet method to obtain the analytic signal. Then the maximum entropy method is applied to the obtained analytic signal using wavelet method, for the frequency estimation. Wavelet method gives the more accurate result than Hilbert transform.

## V. SIMULATION AND RESULTS

Numerical simulation shows advantages of presented method in both in precision and anti-noise performance due to the localization property of Wavelet transform in time and space. Implementation is based on the generation of the analytic signal using Wavelet transform method. The analytic counterpart of real valued signal has been obtained has been found by the Hilbert transform and the wavelet transform. The dependence of [1] frequency of the sinusoid in white noise is the phase of sinusoid which can be reduced by using the wavelet method. The implementation of the wavelet transform to obtain the analytic signal is done. Frequency estimation using real data analytic signal by Hilbert transform and by wavelet transform has been given.

Frequency estimation has been done using $f = 1Hz$ and the size of data is 40 and sampled at the rate of 20times/sec.A41 point real data set is used with predictor order of 9 and SNR of 13dB. The same noise realization is used for each sinusoidal record varying phase.

Fig.() shows the frequency estimation using real data analytic signal using Hilbert transform and analytic signal using Wavelet transform.

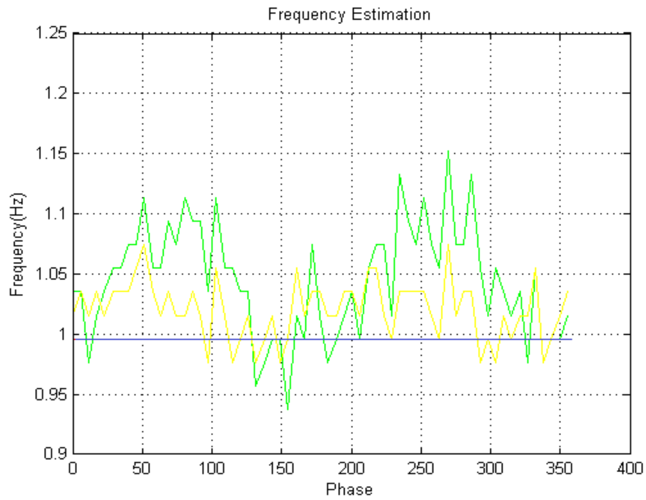


Fig.(2) Frequency Estimation
'Green' shown for real data
'Yellow' shown for analytic signal by Hilbert Transform
'Blue' shown for analytic signal by Wavelet Transform

The experiment is executed in MATLAB 2010 on an I3 Processor Laptop. Simulation result shows that percentage deviation of frequency estimation using real data can be as much as 9% and the percentage deviation of frequency estimation using analytic signal by Hilbert transform is 6%.and by the use of wavelet method of generation of analytic signal in only 0.3% which shows the superiority of the proposed method (i.e. Spectral estimation using analytic Signal) by wavelet method. A comparison is done to compare the effect of phase on the frequency estimate for the real and analytic signal.

## VI. CONCLUSION

There are many methods for the spectral estimation using analytic signal. Estimation of the frequency using analytic signal is done. We have seen that by varying the value of phase the frequency variation of the analytical signal is very less as compared to the real signal. Then the analytic signal is generated using Hilbert transform. The use of Haar wavelet for the generation of the analytic signal is reducing the percentage change in frequency deviation. The analytic signal is generated using the Haar wavelet.

By using the wavelet method of the generation of the analytical signal the frequency variation will be very less as compared to the real data and the signal will be less noisy. This means that the analytic signal is less noisy in comparison with the Kay's [1] method. Then the wavelet method is used to generate the analytic signal. The dependence of the frequency estimate of a sinusoid in white noise on the phase of the sinusoid can be significantly reduced by employing an analytic signal approach.

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