Cordial Labeling Of One Point Union Of Double - Tail S_4 Garphs and invariance.

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1. Abstract: We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as double-tail graph. A double-tail graph is obtained by attaching a path P_m to a pair of adjacent vertices of given graph. It is denoted by double-tail(G,P_m) where G is given graph. We take G as S_4 and restrict our attention to m = 2 in P_m and consider upto four pendent edges each attached at a pair of adjacent vertices. Further we consider all possible structures of $G^{(k)}$ by changing the common point in one point union and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of $G^{(k)}$ under cordial labeling.For S_4 There are two types of double tail graphs. In type I we attach pendent edges to adjacent vertices of cycle C_4.In type II the pendent edges are fused at end of chord.In both cases we show invariance under cordial labeling.

Key words: cordial, one point union, double-tail graph, tail graph, cycle, labeling,

Subject Classification: 05C78

2. Introduction

3. The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [5] Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[9].I.Cahit introduced the concept of cordial labeling[5]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8].

Our focus of attention is on one point unions on shel related graphs obtained by fusing edges with it. For a given graph there are different one point unions (upto isomorphism) structures possible. It depends on which point on G is used to fuse to obtain one point union. We have shown that for G = bull on C_3,bull on C_4,C_3^+,C_4^+-e the different path union $P_m(G)$ follows invariance under cordial labeling [3]. We use the convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are. The graph whose cordial labeling is available is called as cordial graph. In this paper we define double-tail graph and obtain one point union graphs on it. Let G be a (p,q) graph. To one of it's pair of adjacent vertices we fuse t number of paths P_m. We denote this by double –tail(G,tPm).We choose m = 2 and t = 1,2.and discuss their one point union graph at different vertices of G and it's invariance under cordial labeling.

3. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path P_m is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, P_m). In this paper we fix G as C_3 and take P_m for m =2, 3, 4, 5.

3.2 Double tail graph : To any graph G we attach paths of equal length to adjacent pair of vertices .When these paths are gust an edge each then it is referred as bull graph.This graph is denoted by double-tail(G,Pm) when both tails are identical and equal to p_m .if tails are p_m and p_n then the graph is denoted by double-tail(G,p_n,p_m).It has p+m+n-2 vertices and q+m+n-1 edges where G is (p,q) graph. 3.3 Fusion of vertices. Let u ≠v be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.[6] 3.4 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$ 3.5 Ashel graph is obtained from a cycle by taking n-3 cords froma single vertex on C_4.It has n vertices (n> 3) and 2n-3 edges.

4. Results Proved: (For Type I)

Theorem 4.1 All non- isomorphic one point union on k-copies of graph obtained on G = double-tail(S_4,P_2) given by $G^{(k)}$ are cordial graphs for all k. Proof: From figure 4.1 it follows that there are six different structures on one point union of k copies of G possible depending on if we use vertex a, b, c, d ,e, f as common point.

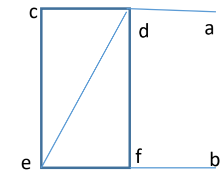

Fig 4.1 One point union to take at a, b, c, d

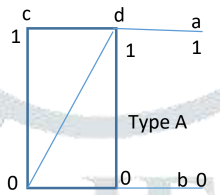

Fig 4.2 $v_f(0,1) = (3,3)$ $e_f(0,1)= (4,3)$

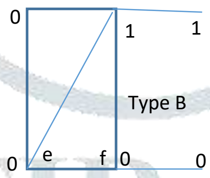

Fig 4.3 $v_f(0,1) = (4,2)$ $e_f(0,1)= (4,3)$

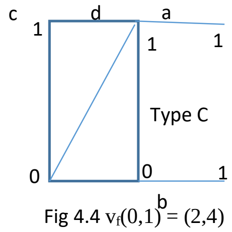

Fig 4.4 $v_f(0,1) = (2,4)$ $e_f(0,1)= (3,4)$

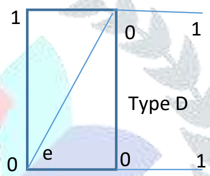

Fig 4.5 $v_f(0,1) = (3,3)$ $e_f(0,1)= (3,4)$

To take one point union at vertex a or d or c the type A or Type C label is used alternately. In $G^{(k)}$ where k = 2x the type A label and type C label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type C label is used for x times. The vertex of label common to all copies has label '1'. The label number distribution is $v_f(0,1) = (3+5x,3+5x)$ $e_f(0,1)= (4+7x,3+7x)$ for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (5+5(x-1),6+5(x-1))$ $e_f(0,1)= (7x,7x)$.

When one point union is taken at vertex e or fwe use type D and type B label alternately. In $G^{(k)}$ where k = 2x the type D label and type B label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type D label is used for x+1 times and type B label is used for x times. The vertex of label common to all copies has label '0'. The label number distribution is $v_f(0,1) = (3+5x,3+5x)$ $e_f(0,1)= (3+7x,4+7x)$ for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (6+5(x-1),5+5(x-1))$ $e_f(0,1)= (7x,7x)$.

When one point union is taken at vertex b we use type D and type C label alternately. In $G^{(k)}$ where k = 2x the type D label and type C label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type D label is used for x+1 times and type C label is used for x times. The vertex of label common to all copies has label '1'. The label number distribution is $v_f(0,1) = (3+5x,3+5x)$ $e_f(0,1)= (4+7x,3+7x)$ for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (5+5(x-1),6+5(x-1))$ $e_f(0,1)= (7x,7x)$.

Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality. # Theorem 4.1 All non- isomorphic one point union on k-copies of graph obtained on G = double-tail(S_4,2P_2) given by $G^{(k)}$ are cordial graphs for all k.

Proof: From figure 4.6 it follows that there are six different structures on one point union of k copies of G possible depending on if we use vertex a, b, c, d , e or f as common point. For the one point union at any of a, b, c, d vertices we fuse the type A and Type B label at respective vertex. In $G^{(k)}$ where k = 2x the type A label and type B label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The vertex of common label is 0.

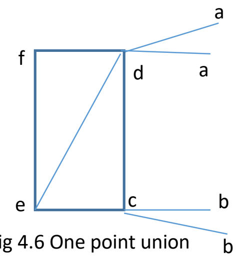

Fig 4.6 One point union to take at a, b, c, d,e,f

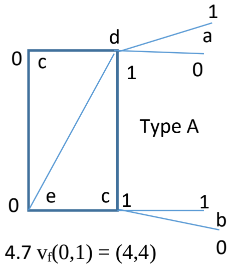

Fig 4.7 $v_f(0,1) = (4,4)$
$e_f(0,1)= (4,5)$

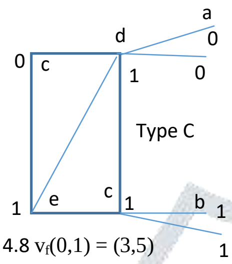

Fig 4.8 $v_f(0,1) = (3,5)$
$e_f(0,1)= (4,5)$

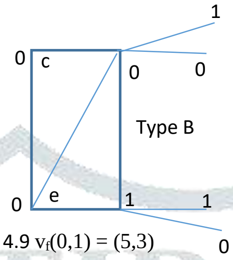

Fig 4.9 $v_f(0,1) = (5,3)$
$e_f(0,1)= (5,4)$

The label number distribution is $v_f(0,1) = (4+7x,4+7x)$ $e_f(0,1)= (4+9x,5+9x)$ for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (8+7(x-1),7+7(x-1))$ $e_f(0,1)= (9x,9x)$. When one point union is taken at vertex d or c we use type A label and type C label alternately. In $G^{(k)}$ where k = 2x the type A label and type C label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The vertex of common label is 1. The label number distribution is $v_f(0,1) = (4+7x,4+7x)$ $e_f(0,1)= (4+9x,5+9x)$ for k =2x+1 , x=0, 1, 2, … If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (7+7(x-1),8+7(x-1))$ $e_f(0,1)= (9x,9x)$. Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality. #

Theorem 4.3 All non- isomorphic one point union on k-copies of graph obtained on G = double- tail(S_4,3P_2) given by $G^{(k)}$ are cordial graphs. Proof: From Fig 4.10 it follows that we can take one point union at any of six vertices a, b, c, d, e, f. For the one point union at vertex a, b or c we use Type A label and type B label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type b label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The label number distribution is given by $v_f(0,1) =(5+9x,5+9x)$, $e_f(0,1)=(6+11x,5+11x)$ where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1) = (10+9(x-1),9+9(x-1))$, $e_f(0,1) = (9x,9x)$.

For the one point union at vertex d,e we use Type A label and type C label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type C label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type C label is used for x times. The label number distribution is given by $v_f(0,1) =(5+9x,5+9x)$, $e_f(0,1)=(6+11x,5+11x)$ where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1) = (9+9(x-1),10+9(x-1))$, $e_f(0,1) = (9x,9x)$. For the one point union at vertex f we use Type D label and type B label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type D label and type B label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type D label is used for x+1 times and type C label is used for x times. The label number distribution is given by $v_f(0,1) =(5+9x,5+9x)$, $e_f(0,1)=(6+11x,5+11x)$ where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1) = (10+9(x-1),9+9(x-1))$, $e_f(0,1) = (9x,9x)$.

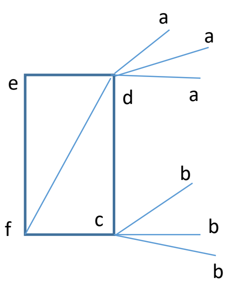

Fig 4.10 One point union to take at a, b, c, d,e,f

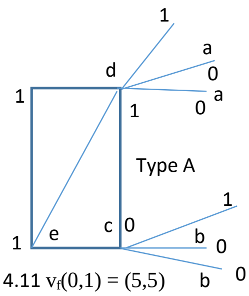

Fig 4.11 $v_f(0,1) = (5,5)$
$e_f(0,1)= (6,5)$

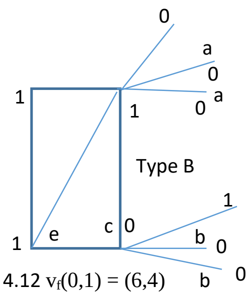

Fig 4.12 $v_f(0,1) = (6,4)$
$e_f(0,1)= (5,6)$

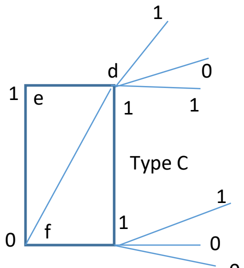

Fig 4.13 $v_f(0,1) = (4,6)$
$e_f(0,1)= (5,6)$

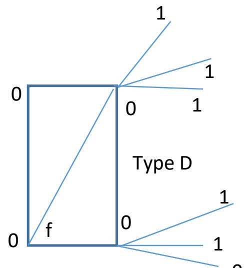

Fig 4.14 $v_f(0,1) = (5,5)$
$e_f(0,1)= (6,5)$

Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality. #

Theorem 4.4 All non- isomorphic one point union on k-copies of graph obtained on G = double- tail(S_4,$4P_2$) given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.15 it follows that we can take one point union at any of six vertices a, b, c, d, e, f. For the one point union at vertex a, b or c we use Type A label and type B label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type B label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The label number distribution is given by $v_f(0,1)$ =(6+11x,6+11x) , $e_f(0,1)$=(6+13x,7+13x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (12+11(x-1),11+11(x-1)), $e_f(0,1)$ = (13x,13x).The label of common vertex being 0.

For the one point union at vertex f or d we use Type C label and type E label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type C label and type E label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type C label is used for x+1 times and type E label is used for x times. The label number distribution is given by $v_f(0,1)$ =(6+11x,6+11x) , $e_f(0,1)$=(6+13x,7+13x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (12+11(x-1),11+11(x-1)), $e_f(0,1)$ = (13x,13x).The label of common vertex being 1.

For the one point union at vertex e we use Type A label and type E label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type E label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type E label is used for x times. The label number distribution is given by $v_f(0,1)$ =(6+11x,6+11x) , $e_f(0,1)$=(6+13x,7+13x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (12+11(x-1),11+11(x-1)), $e_f(0,1)$ = (13x,13x).The label of common vertex being 1.

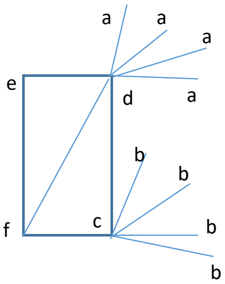

Fig 4.15 One point union to take at a, b, c, d, ,e, f

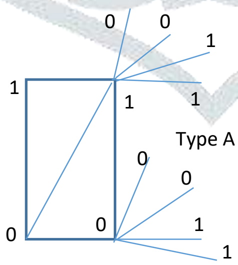

Fig 4.16 $v_f(0,1) = (6,6)$
$e_f(0,1)= (6,7)$

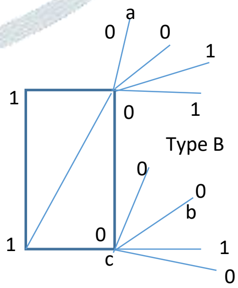

Fig 4.17 $v_f(0,1) = (7,5)$
$e_f(0,1)= (7,6)$

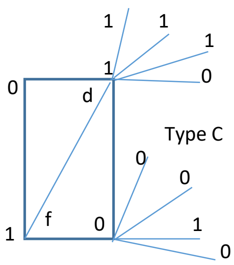

Fig 4.18 $v_f(0,1) = (6,6)$
$e_f(0,1)= (6,7)$

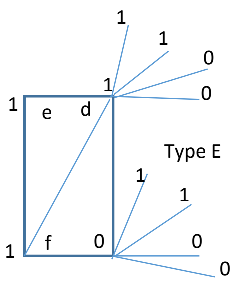

Fig 4.19 $v_f(0,1) = (5,7)$
$e_f(0,1)= (7,6)$

Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality of $G^{(k)}$ #

Main Results (For Type II)

Theorem 4.5 All non-isomorphic one point union on k-copies of graph obtained on G = double-tail(S_4,P_2) given by $G^{(k)}$ are cordial graphs for all k.

Proof: From figure 4.20 it follows that there are three different structures on one point union of k copies of G possible depending on if we use vertex a, b, c as common point.

c

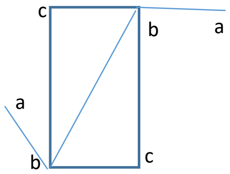

Fig 4.20 One point union to take at any of three points a, b, c

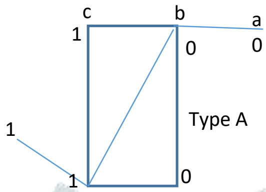

Fig 4.21 $v_f(0,1)$ = (3,3) $e_f(0,1)$= (4,3)

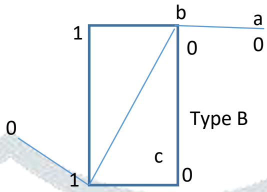

Fig 4.22 $v_f(0,1)$ = (4,2) $e_f(0,1)$= (3,4)

To take one point union at vertex a or b or c the type A or Type B label is used alternately. In $G^{(k)}$ where k = 2x the type A label and type B label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The vertex of label common to all copies has label '0'. The label number distribution is $v_f(0,1)$ = (3+5x,3+5x) $e_f(0,1)$= (4+7x,3+7x) for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1)$ = (6+5(x-1),5+5(x-1)) $e_f(0,1)$= (7x,7x). Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality. #

Theorem 4.6 All non- isomorphic one point union on k-copies of graph obtained on G = double-tail(S_4,$2P_2$) given by $G^{(k)}$ are cordial graphs for all k.

Proof: From figure 4.23 it follows that there are three different structures on one point union of k copies of G possible depending on if we use vertex a, b, c as common point. For the one point union at vertex b we fuse the type A and Type B label at vertex b. In $G^{(k)}$ where k = 2x the type A label and type B label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The vertex of common label is 0.

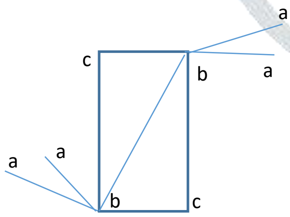

Fig 4.23 One point union to take at any of vertices a, b, c

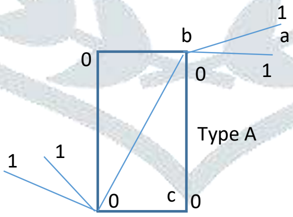

Fig 4.24 $v_f(0,1)$ = (4,4) $e_f(0,1)$ = (5,4)

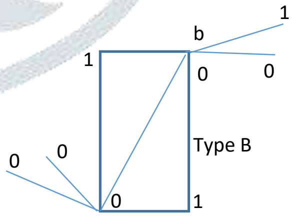

Fig 4.25 $v_f(0,1)$ = (5,3) $e_f(0,1)$ = (4,5)

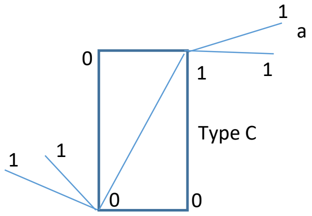

Fig 4.26 $v_f(0,1) = (3,5)$ $e_f(0,1) = (4,5)$

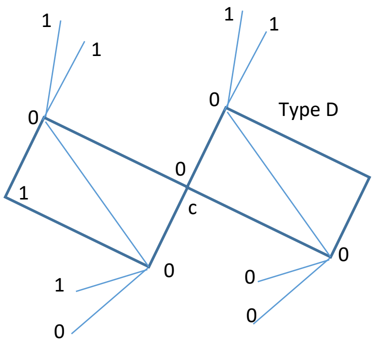

Fig 4.27 $v_f(0,1) = (8,7)$ $e_f(0,1) = (9,9)$

The label number distribution is $v_f(0,1) = (4+7x,4+7x)$ $e_f(0,1)= (5+9x,4+9x)$ for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (8+7(x-1),7+7(x-1))$ $e_f(0,1)= (9x,9x)$. When one point union is taken at vertex a we use type A label and type C label alternately. In $G^{(k)}$ where k = 2x the type A label and type C label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The vertex of common label is 1. The label number distribution is $v_f(0,1) = (4+7x,4+7x)$ $e_f(0,1)= (5+9x,4+9x)$ for k =2x+1 , x=0, 1, 2, … If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (7+7(x-1),8+7(x-1))$ $e_f(0,1)= (9x,9x)$.

When one point union $G^{(k)}$ is taken at vertex c we use type A label. For k= 2x we fuse type D label for x times at point c. At this stage we have $v_f(0,1) = (1+7x,7x)$ $e_f(0,1)= (9x,9x)$ .For k = 2x+1 we first obtain labeling for k = 2x and fuse it with type A label at vertex c. The label number distribution is $v_f(0,1) = (4+7x,4+7x)$ $e_f(0,1)= (9x+5,9x+4)$ Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality. #

Theorem 4.7 All non- isomorphic one point union on k-copies of graph obtained on G = double- tail(S_4,3P_2) given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.28 it follows that we can take one point union at any of six vertices a, b, c, For the one point union at any of these vertex a, b or c we use Type A label and type B label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type b label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times.

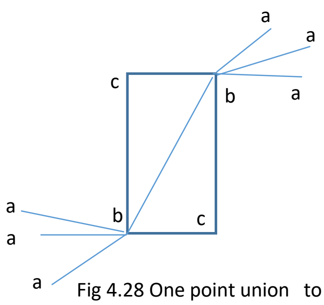

Fig 4.28 One point union to take at a, b, c

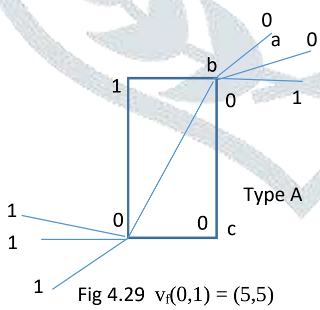

Fig 4.29 $v_f(0,1) = (5,5)$ $e_f(0,1) = (5,6)$

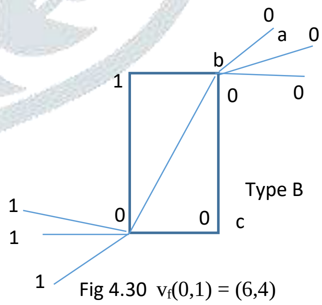

Fig 4.30 $v_f(0,1) = (6,4)$ $e_f(0,1) = (6,5)$

The label number distribution is given by $v_f(0,1) =(5+9x,5+9x)$, $e_f(0,1)=(5+11x,6+11x)$ where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1) = (10+9(x-1),9+9(x-1))$, $e_f(0,1) = (11x,11x)$. Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality. #

Theorem 4.8 All non- isomorphic one point union on k-copies of graph obtained on G = double- tail(S_4,4P_2) given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.12 it follows that we can take one point union at any of threevertices a, b, c. For the one point union at vertex a or b we use Type A label and type B label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type B label each is used for x times. If k = 2x+1 (x =

0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The label number distribution is given by $v_f(0,1)$ =(6+11x,6+11x) , $e_f(0,1)$=(7+13x,6+13x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (12+11(x-1),11+11(x-1)), $e_f(0,1)$ = (13x,13x).The label of common vertex being 0.

For the one point union at vertex c we use Type A label and type C label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type C label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type C label is used for x times. The label number distribution is given by $v_f(0,1)$ =(6+11x,6+11x) , $e_f(0,1)$=(7+13x,6+13x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (11+11(x-1),12+11(x-1)), $e_f(0,1)$ = (13x,13x).The label of common vertex being 1.

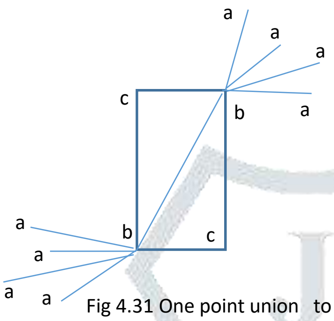

Fig 4.31 One point union to take at a, b, c

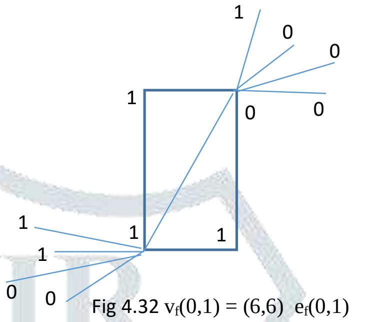

Fig 4.32 $v_f(0,1)$ = (6,6) $e_f(0,1)$ = (7,6)

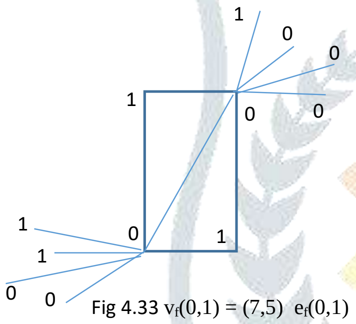

Fig 4.33 $v_f(0,1)$ = (7,5) $e_f(0,1)$ = (6,7)

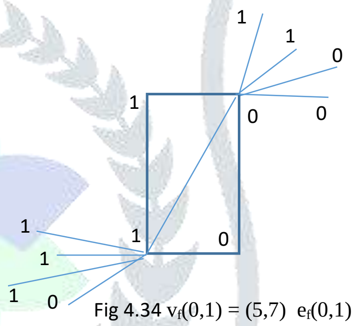

Fig 4.34 $v_f(0,1)$ = (5,7) $e_f(0,1)$ = (6,7)

Thus the graph is cordial and we can define $G^{(K)}$ at any of the point on G and still we get cordial labeling. That explains invariance under cordiality of $G^{(k)}$ #

Conclusions: In this paper we define some new families obtained from S_4 and fusing two adjacent vertices with pendent edges upto four. There are two types of adjacent edges in S4 . One is on cycle S_4 and the other is chord. In both these cases we define double path union (S_4,tP_2) for t upto 4 and discuss and obtain cordial labeling and show thir invariance. For both type of double path union (S_4,tP_2) we prove following results.

1) All non- isomorphic one point union on k-copies of graph obtained on G =double - tail(S_4,P_2) also called $G^{(k)}$ are cordial graphs.

2) All non- isomorphic one point union on k-copies of graph obtained on G =double- tail(S_4,$2P_2$) given by $G^{(k)}$ are cordial graphs.

3) All non- isomorphic one point union on k-copies of graph obtained on G =double- tail(S_4,$3P_2$) given by $G^{(k)}$ are cordial graphs.

4) All non- isomorphic one point union on k-copies of graph obtained on G =double- tail(S_4,$4P_2$) given by $G^{(k)}$ are cordial graphs.

For both type od designs it is necessary to investigate the graph for any number t >4.

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