Mathematical Modeling of Distribution Generator Penetration in Power System Network for Optimal Performance

Pallavi Bondriya[1] Yogendra Kumar[2] Arunkumar Wadhwani[3]
[1]Ph.D. Research Scholar, Rajeev Gandhi Technical University Bhopal, India
[2]Professor in EE Dept. MANIT, Bhopal, India, [3]Professor in EE Dept. MITS, Gwalior, India

Abstract**—***Distribution Generator (DG) penetration in distribution network near consumer end, offers number of advantages in comparison of conventional system. These potential advantages had attracted number of research scholars and research organization to work in the direction of placement optimization. An effort had been carried out in the concerning topic in this research article. DG placement is scholastic in nature, so any conventional or analytical technique will not result in needful solution. The system demands, advanced optimization technique for optimal placement of DG which includes its capacity and location. The optimization technique requires simplified mathematical model of application for optimizing the result. In this research article, we have developed mathematical model for DG integration in power system network. Constraints and condition had been included in model. Limitation and boundary conditions are specified and included. A multi objective function had been formulated. Both real and reactive power flow and power loss sensitivity factors are formulated, while the multi-objective optimization is formulated taking into consideration the three key factors, that is, (1) real power loss reduction index, (2) reactive power loss reduction index and (3) voltage profile improvement index. Both equality and inequality constraints are defined.*

Index Terms— Distribution Generation, Mathematical Model, Loss Sensitivity Factors, Equality & Inequality Constraints

I. INTRODUCTION

As there are many potential benefits by integrating the distributed generation (DG) units in a distribution network over conventional system. DG plays a vital role in a power system network. Renewable energy based DG units are located close to the consumers or load centers in order to improve voltage profile, reduce the network power losses and improves substation capacity release. Thus, while allocating DG units, care has to be taken in order to maximize the benefits. In this article, by installing DG, an optimal way of managing real and reactive power and improving the nodal voltages in primary distribution network is explained and a mathematical model is fabricated. The optimal rating of DG is computed by Optimization technique [1] to ensure reduction in power losses and to attain better voltage regulation. IEEE-33 & 69 bus radial distribution system is considered for DG penetration. Technical aspect of distribution generator [2] in distributed power system network had also been addressed in this article. Negative aspect, positive aspect, scope and limitations of distribution generation penetration are also summarized in brief. Problem formulation, formulations of the system sensitivity factors [3] and the multi objective optimization [4] is covered. The constraint to which the multi objective function is subjected to, are also defined here.

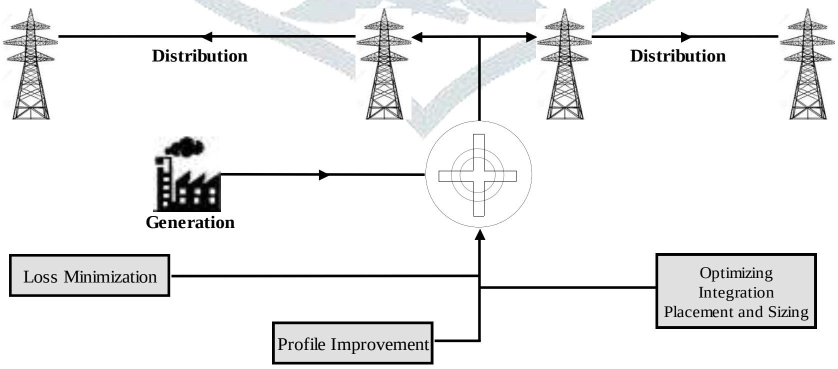

Figure-1. Advantages of Distribution Generator Penetration

Figure-1 shows a conceptual representation of DG Penetration. The installation of a DG, introduces another source in power system which influence the characteristic of distribution network. When the DG power is more than the downstream load, it sends power upstream reversing the direction of power flow, and at some point between the DG and substation, the real power flow is zero due to back flow of power from DG. The impact of DG on Distribution network includes; impact on voltage regulation, power losses, power security, harmonics, power quality, and environment [5]. Explanation of all the significant impacts of DG penetration is also given in the paper.

Distribution systems are employed with radial structure in order to obtain operational simplicity. By means of an interconnected transmission network, primary distribution substation receives power from generating stations. Radial Distribution

System (RDS) network is passive in nature [6] and transfers power to consumers from the substation. Thus, in RDS, the power flow is unidirectional. In case of distribution lines, due to high R /X ratio, high voltage drops, large power loss will occur. Everyday distribution networks are experiencing many changes in the load. At most of the nodes, RDS experience a sudden collapse in the voltage during critical load conditions because of low voltage stability index.

II. LITERATURE SURVEY

Although DG may have some benefits for the system such as improvements in power quality and system efficiency but there are many technical aspects and challenges that are still to be properly understood and addressed. For example, there is lack of suitable control strategies for networks with penetration of DG, while considering the interactions between the transmission and distributions systems. Since most of these studies have to be carried out based on simulations, adequate static and dynamic models for DG units and related interfaces and controls. These models should meet certain requirements to allow investigating relevant system stability and control issues, from both local and global system perspectives.

DG has been studied from the local/micro-grid point of view or the overall/global system point of view, as the level of penetration increases, the impact of DG is no longer restricted to the local load or distribution network where these units are connected, but may also have an impact on the transmission system as concluded by Leon, F.De and Ooi, B.T. [6]. DG penetration has been studied with respect to system control and stability by Reza, M [7], Azmy, A.M. and Erlich, I. [8] and J. G. Slootweg and W. L. Kling [9]; however, these studies do not fully consider the various kinds of DGs, and mainly concentrate on studying stability and control issues from the transmission system point of view. M. Reza [7] and Slootweg, J.G. and Kling, W.L. [9] have indicated the impacts of DG penetration level on power system transient stability which have been studied for different scenarios, and in [8], the impact of selected DG units, i.e. fuel cells (FCs) and micro turbines (MTs), on power system stability for various penetration levels are investigated. On the other hand, some studies have concentrated on the effect of DG units on the distribution network like Venkataramanan, G. and Indala, M.Ill. [10] and Xyngi, I. et. al [11]. Its analyzed that in [11] the stability analysis of a distribution network with selected DG units, i.e. wind generators, and MTs is presented. Kumar, A. et.al [12], [13] discusses about determination of Available transfer capability (ATC) in a competitive electricity market using AC distribution factors. Rao, R.S.et. al [14] discussed about reduction of power loss in distribution system when DG is penetrated in power system by network reconfiguration.

Luo, J. [15] employed an optimal power flow (OPF) technique to maximize DG capacity with respect to voltage and thermal constraints but short circuit levels, short circuit ratio, equipment ratings and losses are not considered in the paper. The effect of network sterilization is clearly demonstrated by comparison between allocating generation to buses individually rather than as a group. A technique is presented that is known as ‘reverse load-ability’in which the approach models fixed-power factor distributed generation as negative load and uses the optimal power flow to perform negative load shedding that effectively maximizes capacity. The technique is applied to an extensive distribution and sub-transmission network.

Forbes, M. [16] in his research article proposed a new graph-theory and improved genetic algorithm based on practical method to solve the optimal sectionalize switch placement problem. The proposed method determines the best locations of sectionalize switching devices in distribution networks considering the effects of presence of distributed generation (DG) in fitness functions and other optimization constraints, providing the maximum number of customers to be supplied by distributed generation sources in islanded distribution systems after possible faults. The proposed method is simulated and tested on several distribution test systems in both cases of with DG and without DG. The results of the simulation validates the proposed method for switch placement of the distribution network in the presence of distributed generation. In his article the stability and profile issue were not addressed as expected.

Kazem, H. and Shayanfar, A. [17] in their research article developed a new method of generalized pattern search and genetic algorithm for optimal placement and sizing of Distributed Generation (DG) and capacitor for the loss reduction in distribution networks. Kazeem, B. et.al [18] showed that deployment of distributed generation (DG) is gaining more acceptances in improving power distribution networks. Sub-optimal DG placement in distribution systems increases power losses, degrade voltage profile and cause stability problems. Due to the nonlinear and complex dynamics of the power systems, this paper proposes a neuro-genetic technique for the optimal placement of DGs.

Mohamed Tolba, A. [19] explained about integration of renewable Distributed Generation (DG) such as Photovoltaic (PV) system and wind turbine (WT) in distribution networks. His article introduces a new robust and effective hybrid PSOGSA optimization algorithm that proposed to detect the optimal location with convenient size of DG units for minimizing system power losses and operating cost besides improving voltage stability. In this technique optimization is performed in two stages. In first stage, Loss Sensitivity Factors (LSFs) are employed to select the most candidate buses for DG placement and in the second stage, the PSOGSA is implemented to deduce the optimal sitting and sizing of DG from the elected buses. The proposed technique has been applied on 33-bus and 69-bus IEEE standard radial distribution systems. The numerical results have confirmed the superiority with high performance of the proposed technique to find the optimal solutions of DG units allocation. Various optimization techniques have been developed for optimal placement of Dg in distribution system. Still a research gap exists in placement of DG which is indicated in Table 1.

Table-1. Research Gap in Distribution Generation Placement Technology

Title and Authors	Finding	Research Gap
"An improved PSO algorithm based on statistics for distribution network reconfiguration to increase the penetration of distributed generations" By Zhu, J. et al. [20]	This paper explores a way to promote the maximum capacity of distributed generations (DGs) that a distributed network could consume through network reconfiguration. The research implements particle swarm optimization (PSO) algorithm based on statistics. The algorithm is improved by introducing a scenarios library.	Optimal location and size of the Distributed generator is evaluated. Article does not consider about the stability of the power system after the DG penetration in network and the quality of power is not discussed.
"Optimal coordination of directional over current relays in distribution systems with distributed generation based on a hybrid PSO-LP algorithm" by Papaspiliotopoulos,V.A. et al.[21]	Research presents method for solving problem of directional over current relays coordination in distribution grids with high penetration of distributed generation, using hybrid particle swarm optimization- linear programming algorithm. The purpose of the proposed technique is to calculate the values of pickup currents (PU) and time dial (TD) settings of each DOCR to minimize relays operating times.	The location and the size of the distribution generator is not discussed. Only focus about the over current protection in the case of contingencies. Performance, efficiency and losses is totally omitted in the research.
"Sitting and sizing of Distributed Generation in distribution network to improve of several parameters by PSO algorithm" Sedighi, M. et al., [22]	Research presents a method for optimal sitting and sizing of DG in distribution systems. In this paper. The objective was to reach optimal distributed generation allocation and sizing for several parameters, include: voltage profile improvement, loss reduction, and THD (Total Harmonic Distortion) reduction in distribution networks. Particle Swarm Optimization (PSO) is used as the solving tool.	The Research did not discuss about the performance of the system after penetration of DG in the application. Protection in over current is not discussed. Effect on the quality of power deliver at load end is not discussed.

III. IMPACT OF DG PENETRATION

All the significant impacts of DG penetration are summarized below:

1. Voltage Regulation
2. System Losses
3. Power Security
4. Power Quality
5. System Harmonics
6. Short Circuit Levels of System

III.I Impact of DG on Voltage Regulation

In a distribution system voltage regulation takes place at substation with the aid of **(1)** load tap changing transformers, **(2)** regulators on distribution feeders and **(3)** shunt capacitor on feeders or along the line in a radial distribution. The magnitude of real and reactive power flows as well as the voltage profile along a feeder changes when distributed generation is integrated into a distribution system. The change in power flow makes the distribution network active and non-unidirectional. Depending on system characteristic, location and size of DG, its impact on voltage regulation can either be negative or positive. In Figure 2, the integration of DG shows an improved voltage profile, but for the situation without DG, possible solutions include but are not limited to, moving the DG unit to the optimal location side of the network, while the second solution is adding regulator controls to compensate for the DG output.

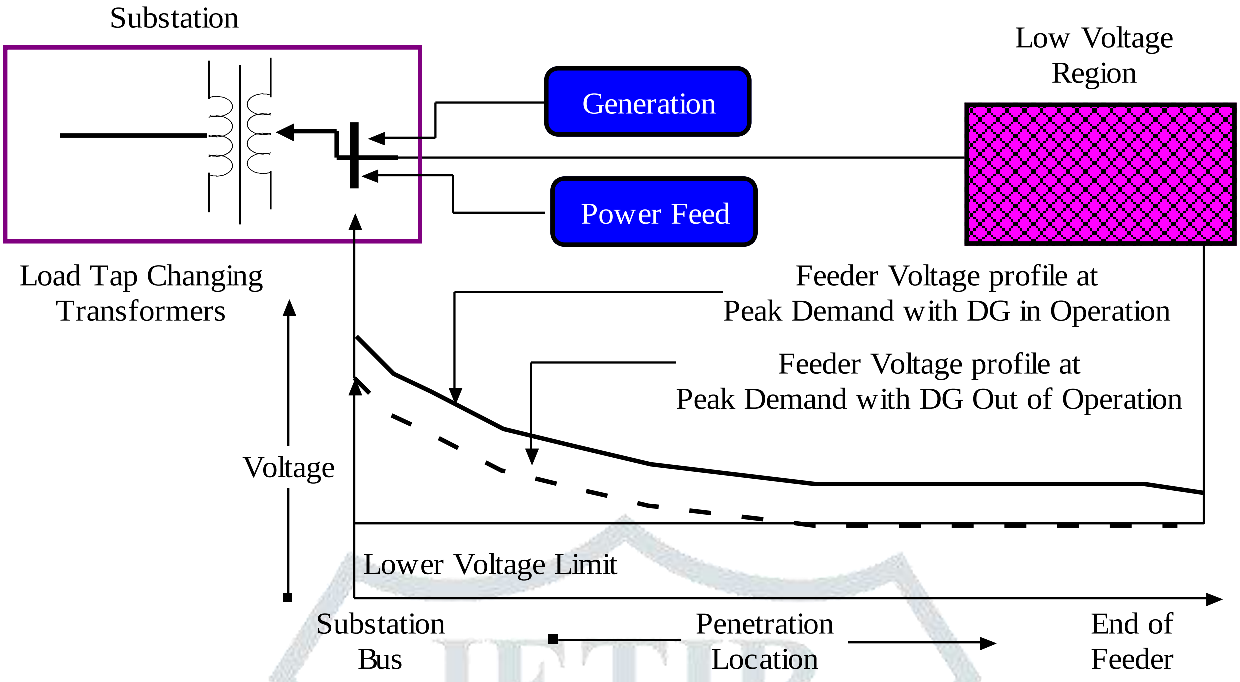

Figure-2 Voltage profiles with and without DG

If the aggregate capacity increases to critical thresholds, then voltage regulation analysis is required to make sure that the feeder voltage will be fixed within suitable limits. Figure-2 depicts the voltage profiles with and without DG.

III.II Impact of Distributed Generation on Losses

Power losses on feeders are another impact of distributed generation on distribution network. Optimal allocation of the DG units is an important criterion that must be analyzed to achieve a better reliability of distribution system. Capacitor allocation on distribution network is similar to DG placement and sizing for losses reduction. The main difference between both situations is that DG may contribute both active power and reactive power (P and Q). On the other hand, capacitor banks only contribute with reactive power flow (Q). Generators in system operate with a power factor range between 0.85 lagging and unity, but the presence of inverters and synchronous generators provides a contribution to reactive power compensation.

If feeders have high losses, adding a number of small capacity DGs will show an important positive effect on the losses and have a great benefit to the system. But if higher units are added, they must be connected considering the feeder capacity limits. Feeder capacity may be restricted as overhead lines and cables have thermal characteristic which should not exceed.

III.III Impact of Distributed Generation on Power Quality

Distributed Generation can also result in excess voltage. The voltage of substation distribution lines is controlled by a programmed timer or line drop compensator. Single distribution transformer has several feeder lines, and the voltage in these lines is adjusted in a block. Additionally, a static voltage regulator compensates the voltage midway along the line in heavy power-flow or long transmission lines. The load of each feeder should be balanced proportionally to utilize these voltage control systems. If there are many DG connections concentrated on a specific line, the gap in the power flow between feeder lines increases because of the back-flow from DG. This is of particular concern; when generating systems that depend on natural conditions, such as wind power or solar photovoltaic generators, are interconnected to the local system.

III.IV Impact of Distributed Generation on Harmonics

A wave that does not follow a “pure” sinusoidal wave is considered as harmonically distorted wave. They can be caused by non-linearity in transformer exciting impedance or loads such as fluorescent lights, AC to DC conversion equipments, variable-speed drives, switch mode power equipment, arc furnaces, and other equipments. DG can be a source of harmonics to the network. Harmonics produced can be from either the generation unit itself or from the power electronics equipment such as inverters. In the case of inverters, their contribution to the harmonic currents is in part due to the SCR power inverters that produce high levels of harmonic currents. Nowadays, inverters are designed with IGBT technology that use pulse width modulation to generate the injected “pure” sinusoidal wave. This new technology produces a cleaner output with fewer harmonics that should satisfy the IEEE 1547-2003 standards. This problem of harmonics is usually caused by resonance with capacitor banks, or problems with equipment that are sensitive to harmonics. In the worst case, the equipment at the DG may need to be disconnected as a consequence of the extra heating caused by the harmonics.

III.V Impact of DG on Short Circuit Levels of the Network

The presence of DG in a system affects the short circuit levels of the network. It creates an increase in the fault currents when compared to normal conditions at which no DG is installed in the network. The influence of DG on faults depends on some factors such as the generating size of the DG, the distance of the DG from the fault location and the type of DG. Many small units or a few large units are installed in the system; they can alter the short circuit levels sufficiently. This could affect the reliability and safety of the distribution system. In this case if the fault current is large enough, the fuse may no longer coordinate with the feeder circuit breaker during a fault. If the DG is located between the utility substation and the fault, a decrease in fault current from the utility substation may be observed. This decrease needs to be investigated for minimum tripping or coordination problems.

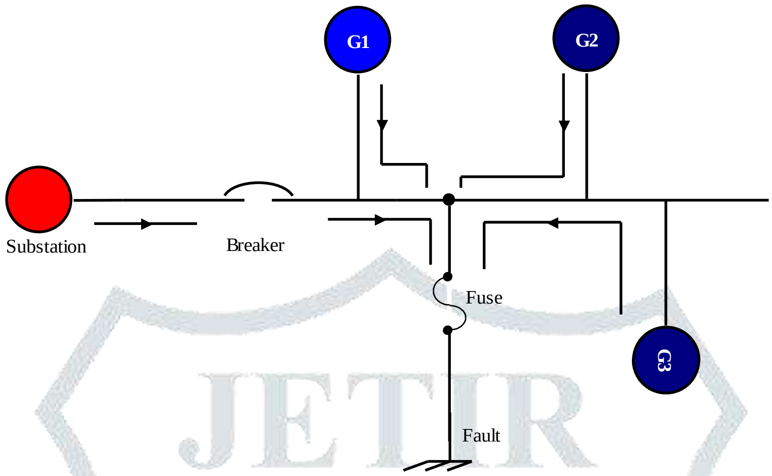

Figure-3. Fault contribution due to embedded DG in the system

On the other hand, if the DG source is strong compared to the utility substation source, it may have a significant impact on the fault current coming from the utility substation. This may cause failure to trip. The type of the DG also affects the short circuit levels. During the first few cycles the contribution is equal from the induction generator and self excited synchronous generator, while after the first few cycles the synchronous generator is the most fault current contributing DG type. Figure-3 shows a fault contribution due to embedded DG in the system.

III.V Impact of Distributed Generation on Reliability

DG units can have a positive impact on distribution system reliability if they are properly coordinated with the rest of the network. A common example of DG use is as generation backup, in which the unit operates in the case of main supply interruption. A utility can install DG to provide additional capacity to feeders or substations. DG can also be used to improve restoration capability of the distribution networks, that is, DG can eliminate network constraints during the restoration process. Impact on reliability is of great importance to power system optimization.

III.VI Impact of Distributed Generation on the Environment

Renewable DG such as wind, solar PV and other low carbon like Micro CHP system have a positive impact on the environment by effective reduction of emission and warming. Aside from the market attractiveness, environmental friendliness is one of the major criteria that support DG operation. Towards eco-friendly operation improvement, there is need to program the central controller (CC) to make operational decisions based on the net lowest emission production, in view of both local emission and displaced emission from micro sources. The structure of emission tariffs is a combined function of season, time and location so the tariffs would be most attractive at worst pollution time and location. This would send signal to the central controller to operate the micro source optimally for minimizing emission.

IV. INTERFACING DISTRIBUTED GENERATION WITH POWER GRID

Interface technology may comprise of a transformer and converters, which is the technology used at the point of common connection commonly refers as PCC. Various types of interfacing technologies are used; the use of power electronics converters and generators is common. Interfacing technology main objective is to meet the energy requirements of the grid. DG interfacing technologies can be classified into four categories:

1. DG direct machine coupling
2. DG full power electronics coupling
3. DG partial power electronics coupling
4. Modular or distributed generation power electronics coupling

IV.I DG direct machine coupling

Mechanical power can be converted into electrical more efficiently using direct machines coupling to the grid without any transitional stage. The nature of mechanical power supply dictates the type of machines coupling to be used in any case. Synchronous machine is the right candidate for constant mechanical power that rotates at fixed speed while induction machines is suitable for strongly variable power that rotates at variable speed. Wind energy source is good example of these machines, connected as shown in Figure -4.

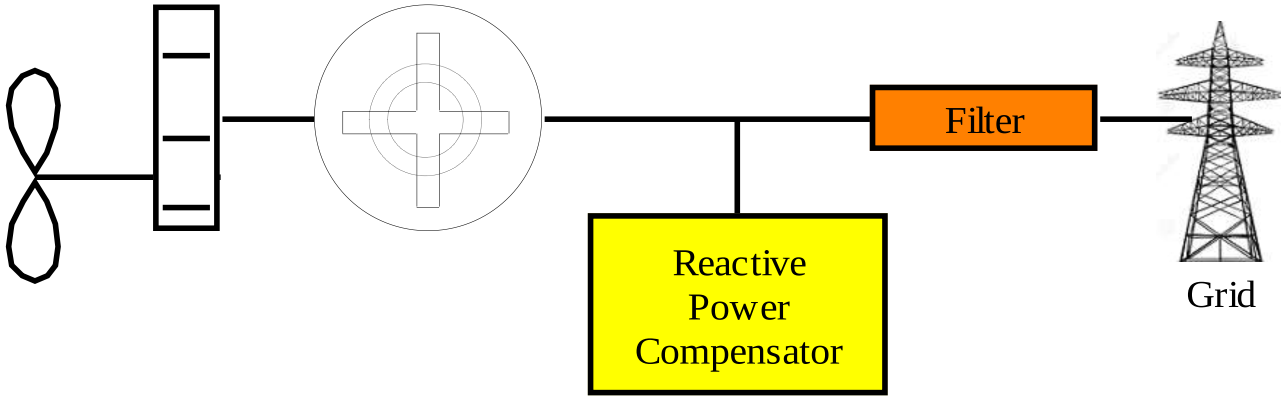

Figure-4. Direct induction generator coupling for a wind turbine

IV.II Full Power Electronics Coupling

The energy supplied by the distributed generator is condition by the power electronics interface to accommodate the grid requirements and to improve the performance of the energy source. However, the ability of power electronics equipment to transform power from one form to another with use of electronics controlled switches is called power electronics converter. In the coupling process, direct current power converters to alternating current power that matches the grid requirements using the power electronic converters. Such converters can be called an inverter or DC/AC converters. With a source that produce DC energy, the power electronics may consist of one DC/AC converter or an intermediate DC/DC conversion stage can be added to achieve a specific goal, for instance, in an attempt to stabilize the output voltage so that the maximum available power is extracted as in the case of PV systems. Figure-5 shows full power electronics interfaced wind turbine with induction generator.

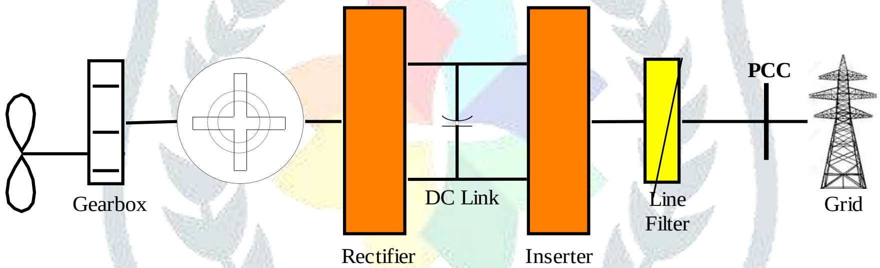

Figure-5. Full power electronics interfaced wind turbine with induction generator

IV.III Partial Power Electronics Coupling

Coupling arrangement for grid integration is a partial power electronics interface, in which the rating of the converter is proportional to a certain percentage of the apparent power of the DG.

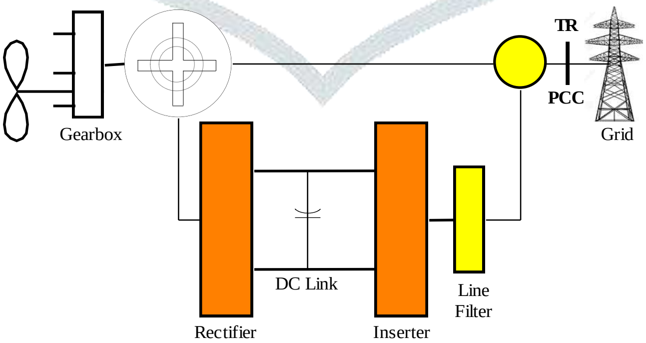

Figure-6. Double-fed induction generator connection of a wind turbine

A partially rated power electronics converter at the connection point of a wind farm is usually needed to mainly provide a voltage dip ride-through capability, which is a required feature regarding different grid codes, and possible reactive power support. Figure-6 shows double-fed induction generator connection of a wind turbine.

IV.IV Power Electronics Interface

Distributed power electronics interfaces refer to a number of distributed generations that are connected to the same local grid through power electronics converters. If such units belong to the same owner, their operation can be coordinated in a way to achieve certain benefits such as regulating the local voltage. The module-integrated photovoltaic system, shown in Figure 7 is also a type of distributed active interface structure that has been developed in order to increase the efficiency and reliability of the solar power cells. This is made possible when different solar cells, in an array or a cluster are exposed to different irradiation.

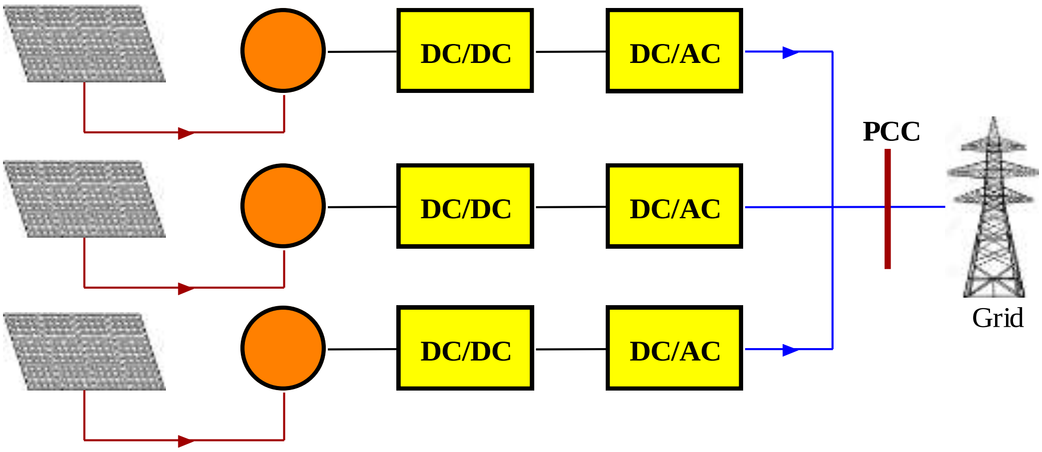

Figure-7. Photovoltaic array interface through modular conversion

Another example is a wind farm with full power electronics-interfaced wind turbines. Implementing a proper control architecture that makes use of the distributed controllability of the converters makes such setup reliable, reconfigurable, and self-healing. This can be concluded that DG impacts Distribution Network differently, both negatively and positively depending on fact to be considered, which can result in economical or technical benefit. Positive impact can be achieved by proper integration of the Distributed Generation via optimal allocation. Also, various Distributed generation integration technologies, through which better power loss reduction and improved voltage can be achieved has been reviewed.

V. MATHEMATICAL MODELLING

In this section of address problem formulation, reactive power flow sensitivity factors formulation and real power loss sensitivity factors formulation. Multi objective optimization function formulation is also done taking important index in to considerations and these are (1) Real power loss reduction index (2) Reactive power loss reduction index (3) Voltage profile improvement index. Equality constraints and Inequality constraints are also defined for optimization. The mathematical modeling had been carried out for following segment of system under consideration:

1. Load Modeling
2. Renewable DG Modeling
3. Total System Power Loss
4. System Power Flow Sensitivity Factors
5. System Power Loss Sensitivity Factors
6. Objective Function
7. Formulation of Multi-objective Function

V.I . Load Modeling

The distribution system under study is assumed to follow normalized 24 hour load profile. Constant P and Q load in load characteristic incorporate through load factor LF.

$$\text{Load } \textbf{Factor} = LF = \sum_{time(t)=1hr}^{time(t)=24hr} \frac{p.u.demand(t)}{24} \cdots\cdots\cdots\cdots\cdots\cdots Equation - 1$$

V.II. Renewable DG Modeling

Most common DGs in industrial practice are wind power and solar PV cell. The outputs of wind DG and PV-based DG are assumed to follow the normalized average output curve. The capacity factor that is CF of each type of DG is estimated as the ratio of the area under the output curve in p.u. to the total duration of operation that is 24 hour.

$$\text{Capacity Factor} = CF = \sum_{time(t)=1hr}^{time(t)=24hr} \frac{p.u.DG\_Output(t)}{24} \cdots\cdots\cdots\cdots\cdots\cdots Equation - 2$$

Wind energy is extracted through wind turbine blades and then transferred through a gearbox and the rotor hub to the mechanical energy in the shaft. The shaft drives the generator to convert the mechanical energy to electrical.

$$Machenical\ Power = P_m = C_p(\lambda\beta)\frac{(\rho A v^3)}{2} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots Equation - 3$$

In the above expression that is equation 3 the notation used are as listed below:
ρ = Air density
A = Area of rotor
P_m = Maximum power
C_p = Performance Coefficient
λ = Speed Ratio

β = Blade Pitch Angle
w_r = Angular Frequency of Rotational Turbine

The output power from PV module at (s) can be expressed as follows:

$$P_{PVO}(s) = N \times FE \times V_y \times I_y \cdots\cdots\cdots\cdots\cdots Equation - 4$$

$$FF = \frac{V_{MPP} \times I_{MPP}}{V_{oc} \times I_{sc}};\ V_y = V_{oc} - K_v \times T_{cy} \cdots\cdots\cdots\cdots Equation - 5$$

$$I_y = s[I_{sc} + K_i \times (T_{cy} - 25)];\ T_{cy} = T_A + s\left(\frac{N_{OT} - 20}{0.8}\right) \cdots\cdots\cdots Equation - 6$$

The notations used in equation 4,5 and 6 are explained below.
N = Number of Modules
T_{cy} = Cell Temperature
T_A = Ambient Temperature
K_i = Current Temperature Coefficients
K_v = voltage Temperature Coefficients
N_{OT} = Nominal Operating Temperature of cell
FF = Fill Factor
V_{oc} = Open Circuit Voltage
I_{sc} = Short Circuit Current
V_{MPP} = Voltage at Maximum Power Point
I_{MPP} = Current at Maximum Power Point

V.III. Total System Power Loss

Total power loss of the system is summation of reactive power loss and active power loss.

$$Total\ System\ Power = \sqrt{P_{loss}^2 + Q_{loss}^2} \cdots\cdots\cdots\cdots Equation - 7$$

Total active power loss in a Radial distribution system is given by P_{loss}

$$P_{loss} = \left\{\sum_{i=1}^{n} (I_i)^2 * R_i\right\} \cdots\cdots\cdots\cdots\cdots Equation - 8$$

Total reactive power loss in a distribution system is also given by Q_{loss}

$$Q_{loss} = \left\{\sum_{i=1}^{n} (I_i)^2 * X_i\right\} \cdots\cdots\cdots\cdots\cdots Equation - 9$$

The following notations are used in equation 8 and 9 are:
$\boldsymbol{n}$ = Number of lines at bus i
$\boldsymbol{I_i}$ = current at bus i in amps
$\boldsymbol{R_i}$ = line resistance in ohms,
$\boldsymbol{X_i}$ = line reactance at bus i in
The total system power loss is also formulated as exact loss formula which includes real and reactive power losses respectively. Exact reactive and active power losses at any time in the system at ith location and in the nth bus is a complex function of numbers of variables.

$$P_{loss} = \sum_{i=1}^{n}\sum_{j=1}^{n}[a_{ij}(P_iP_j + Q_iQ_j) + b_{ij}(Q_iP_j + P_iQ_j)] \cdots\cdots\cdots Equation - 10$$

$$Q_{loss} = \sum_{i=1}^{n}\sum_{j=1}^{n}[c_{ij}(P_iP_j + Q_iQ_j) + d_{ij}(Q_iP_j + P_iQ_j)] \cdots\cdots\cdots Equation - 11$$

In equations 10 and 11, the notations used are as listed below:

$$a_{ij} = \frac{R_{ij}}{V_iV_j}\cos(\delta_i - \delta_j)$$

$$b_{ij} = \frac{R_{ij}}{V_iV_j}\sin(\delta_i - \delta_j)$$

$$c_{ij} = \frac{X_{ij}}{V_iV_j} * \cos(\delta_i - \delta_j)$$

$$d_{ij} = \frac{X_{ij}}{V_iV_j} * \sin(\delta_i - \delta_j)$$

$\boldsymbol{n}$ = Bus Number
$\boldsymbol{a_{ij}}$, $\boldsymbol{b_{ij}}$, $\boldsymbol{c_{ij}}$ and $\boldsymbol{d_{ij}}$ = Function of loss coefficient between bus $\boldsymbol{i}$ *and* $\boldsymbol{j}$
$\boldsymbol{P_i}$ = Real Power Flow at bus in kW
$\boldsymbol{Q_i}$ = Reactive Power Flow at bus i in kVAR
$\boldsymbol{P_j}$ = Real Power Flow at bus j in kW
$\boldsymbol{Q_j}$ = Reactive Power Flow at bus j in kVAR
$\boldsymbol{R_{ij}}$ = Resistance of the line connecting bus $\boldsymbol{i}$ and $\boldsymbol{j}$ in Ohms
$\boldsymbol{X_{ij}}$ = Reactance of the line connecting bus $\boldsymbol{i}$ and $\boldsymbol{j}$ in Ohms
$\boldsymbol{V_i}$ *and* $\boldsymbol{V_j}$ = Bus Voltage magnitude at bus $\boldsymbol{i}$ and $\boldsymbol{j}$ in PU
$\boldsymbol{\delta_i}$ *and* $\boldsymbol{\delta_j}$ = Bus Voltage Angle at bus $\boldsymbol{i}$ and $\boldsymbol{j}$
The total impedance between the bus i and bus j is given by

$$Z_{ij} = R_{ij} + jX_{ij} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 12$$

V.IV. System Power Flow Sensitivity Factors

System power flow sensitivity can be explained as the change in power flow in a transmission or distribution line connected between two buses say bus i and bus j due to a unit change in the power injected at any bus in the system. The complex power injected by a source into a bus, say i^{th} bus of a power system is given by

$$S_j = P_i + jQ_i = V_iJ_i; \quad where\ i = 1, 2, 3, 4 \cdots\cdots\cdots \boldsymbol{upto\ n}$$

where **:**
$\boldsymbol{V_i}$ is the voltage at the $\boldsymbol{i^{th}}$ bus with respect to ground
$\boldsymbol{J_i}$ is the source current injected into the bus
The complex conjugate of the above equation is considered as below for analysis

$$S_i^* = P_i + jQ_i = V_i^*J_i; \quad where\ i = 1, 2, 3, 4 \cdots\cdots\cdots \boldsymbol{upto\ n}$$

The source current is given by

$$J_i = \sum_{j=1}^{n} Y_{ij}V_j; \qquad where\ i = 1, 2, 3, 4 \cdots\cdots\cdots upto\ n$$

Thus by substituting this equation into the complex conjugate equation of power injection we have:

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij}V_j\,; \qquad where\ i = 1, 2, 3, 4 \cdots\cdots\cdots upto\ n$$

From the representations given above the real and reactive powers can be expressed in general as shown:

$$P_i = |V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)\,; \;\; where\ i = 1, 2, 3, 4 \cdots\cdots\cdots upto\ n$$

$$Q_i = -|V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j); \;\; where\ i = 1, 2, 3, 4 \cdots\cdots\cdots upto\ n$$

V.IV.I. Change in Real Power Flow Analysis

The real power flow in a line k connecting two buses, bus i and bus j can be expressed as below:

$$P_i = V_iV_jV_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) + V_i^2Y_{ii} \cos\theta_{ii} \cdots\cdots\cdots\cdots\cdots \; Equation - 13$$

Vj = Voltage Magnitudes at buses i and j respectively
δj = Voltage angles at buses i and j respectively
Yii = Magnitude of ith element of the $YBus$ matrix
Yij = Magnitude of the $ijth$ element of the $YBus$ matrix
θii = Angle of the ith element of the $YBus$ matrix
θij = Angle of the $ijth$ element of the $YBus$ matrix
Mathematically, the real power flow sensitivity can be written as

$$Real\ Power\ Flow\ Sensitivity = \begin{bmatrix} \frac{\Delta P_{ji}}{\Delta P_n} \\ \frac{\Delta Q_{ji}}{\Delta Q_n} \end{bmatrix} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 14$$

Using Taylor series approximation while ignoring the second and higher order terms the change in real line flow can be expressed as:

$$\Delta P_{ji} = \frac{\partial P_{ij}}{\partial \delta_i} * \Delta\delta_i + \frac{\partial P_{ij}}{\partial \delta_i} * \Delta\delta_j + \frac{\partial P_{ij}}{\partial \delta_i} * \Delta V_i + \frac{\partial P_{ij}}{\partial \delta_i} * \Delta V_j \cdots\cdots\cdots \; Equation - 15$$

The coefficients appearing in the above equation can be obtained using the partial derivatives of real power flow with respect to variables δ and V as shown below:

$$\frac{\partial P_{ij}}{\partial \delta_i} = -V_iV_jV_{ij} \sin(\theta_{ij} + \delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 16$$

$$\frac{\partial P_{ij}}{\partial \delta_j} = V_i V_j V_{ij} \sin(\theta_{ij} + \delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 17$$

$$\frac{\partial P_{ij}}{\partial V_j} = V_j Y_{ij} \cos(\theta_{ij} + \delta_{ij}) - 2V_j Y_{ij} \cos(\theta_{ij}) \cdots\cdots \; Equation - 18$$

$$\frac{\partial P_{ij}}{\partial V_j} = -V_j Y_{ij} \cos(\theta_{ij} + \delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 19$$

V.IV.II. Change in Reactive Power Flow Analysis

The reactive power flow in a line k connecting two buses, bus i and bus j can be expressed as below:

$$Q_i = -V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) - V_i^2 \, Y_{ij} \sin \theta_{ii} \cdots\cdots \; Equation - 20$$

Vj = Voltage magnitudes at buses i and j respectively

δj = Voltage angles at buses i and j respectively

Yii = Magnitude of i^{th} element of the Y_{Bus} matrix

Yij = Magnitude of the ij^{th} element of the Y_{Bus} matrix

θii = Angle of the i^{th} element of the Y_{Bus} matrix

θij = Angle of the ij^{th} element of the Y_{Bus} matrix

Mathematically, the reactive power flow sensitivity can be written as:

$$Reactive\ Power\ Flow\ Sensitivity = \begin{bmatrix} \frac{\Delta Q_{ji}}{\Delta P_n} \\ \frac{\Delta Q_{ji}}{\Delta Q_n} \end{bmatrix} \cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 21$$

Using Taylor series approximation while ignoring second and higher order terms the change in reactive line flow can be expressed as:

$$\Delta Q_{ji} = \frac{\partial Q_{ij}}{\partial \delta_i} * \Delta\delta_i + \frac{\partial Q_{ij}}{\partial \delta_i} * \Delta\delta_j + \frac{\partial Q_{ij}}{\partial \delta_i} * \Delta V_i + \frac{\partial Q_{ij}}{\partial \delta_i} * \Delta V_j \cdots\cdots \; Equation - 22$$

The coefficients appearing in the above equation can be obtained using the partial derivatives of reactive power flow with respect to variables and V as shown;

$$\frac{\partial Q_{ij}}{\partial \delta_i} = -V_i V_j V_{ij} \sin(\theta_{ij} + \delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 23$$

$$\frac{\partial Q_{ij}}{\partial \delta_j} = V_i V_j V_{ij} \sin(\theta_{ij} + \delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 24$$

$$\frac{\partial Q_{ij}}{\partial V_j} = V_j Y_{ij} \cos(\theta_{ij} + \delta_{ij}) - 2V_j Y_{ij} \cos(\theta_{ij}) \cdots\cdots\cdots \; Equation - 25$$

$$\frac{\partial Q_{ij}}{\partial V_j} = -V_j Y_{ij} \cos(\theta_{ij} + \delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 26$$

V.IV.III. Formulating the Power Flow Sensitivity Factors

The power flow sensitivity factors represent the change in the real reactive power flow over a distribution line that connects bus-i and bus-j due to the change in active and reactive power injected at any other bus-n. The equations for the changes in the line flows can be arranged in matrix form and expressed as:

$$\begin{bmatrix} \Delta P_{ij} \\ \Delta V_j \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{ij}}{\partial \delta} & \frac{\partial P_{ij}}{\partial V} \\ \frac{\partial Q_{ij}}{\partial \delta} & \frac{\partial Q_{ij}}{\partial \delta} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \cdots\cdots\cdots\cdots\cdots\cdots \; Equation - 27$$

V.V. System Power Loss Sensitivity Factors

Both real & reactive power loss sensitivity factors can be calculated from Figure -8 given below:

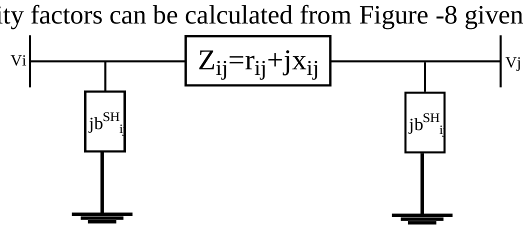

Figure-8. Circuit diagram of a line lumped model

V.V.I. Change in Real Power Loss Analysis

The active power loss of the line lumped model circuit shown is figure above can be expressed as below:

$$P_{l(ij)} = g_{ij}(V_i^2 + V_j^2 - 2V_i V_j \cos \delta_{ij})$$

Thus the total active power loss in the circuit can be expressed as;

$$P_{L(total)} = \sum_{l=1}^{nl}\left[g_{ij}\left(V_i^2 + V_j^2 - 2V_iV_j\cos\delta_{ij}\right)\right]$$

nl is the number of lines of the network;
g_{ij} = Conductance of the line i-j
V_i = Node voltage of bus i
V_j = Node voltage of bus j
δ_{ij} = Phase angle difference between the buses i and j
Mathematically, the reactive power loss sensitivity can be written as

$$Real\ Power\ Loss\ Sensitivity = \begin{bmatrix} \frac{\Delta P_{L(ji)}}{\Delta P_n} \\ \frac{\Delta P_{L(ji)}}{\Delta Q_n} \end{bmatrix} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ Equation - 28$$

Using Taylor series approximation while ignoring second and higher order terms the change in real power loss can be expressed as:

$$\Delta P_{L(ji)} = \frac{\partial P_{L(ij)}}{\partial \delta_i} * \Delta\delta_i + \frac{\partial P_{L(ij)}}{\partial \delta_i} * \Delta\delta_j + \frac{\partial P_{L(ij)}}{\partial \delta_i} * \Delta V_i + \frac{\partial P_{L(ij)}}{\partial \delta_i} * \Delta V_j \cdots\ Equation - 29$$

$$\frac{\partial P_{L(ij)}}{\partial \delta_i} = -2g_{ij}V_iV_j\sin(\delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ Equation - 30$$

$$\frac{\partial P_{L(ij)}}{\partial \delta_j} = -2g_{ij}V_iV_j\sin(\delta_{ij}) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ Equation - 31$$

$$\frac{\partial P_{L(ij)}}{\partial V_j} = -2g_{ij}(V_i - V_j\cos\delta_{ij})$$

$$\frac{\partial P_{L(ij)}}{\partial V_j} = -2g_{ij}(V_j - V_i\cos\delta_{ij})$$

The sensitivity power factor of real power loss can be now expressed as below:

$$\alpha_i = \frac{\sigma P_L}{\sigma P_i} * 2\sum_{l=1}^{nl}\left(a_{ij}P_j - b_{ij}Q_j\right)$$

V.V.II. Change in Reactive Power Loss Analysis

The reactive power loss in a line l connecting two buses, bus i and bus j can be expressed as:

$$Q_{L(ij)} = b_{ij}\left(V_i^2 + V_j^2 - 2V_iV_j\cos\delta_{ij}\right)$$

$$Q_{L(total)} = \sum_{l=1}^{nl} -b_{ij}^{st}\left[\left(V_i^2 + V_j^2\right) - b_{ij}\left(V_i^2 - V_j^2 - 2V_iV_j\cos\delta_{ij}\right)\right]$$

n_l =Number of lines of the network
b_{ij}^{st} = Shunt Susceptance of the line i-j
V_i = Node voltage of bus i
V_j = Node voltage of bus j
δ_{ij} = Phase angle difference between the buses i and j
Mathematically, the reactive power loss sensitivity can be written as:

$$Reactive\ Power\ Loss\ Sensitivity = \begin{bmatrix} \frac{\Delta P_{L(ij)}}{\Delta P_n} \\ \frac{\Delta P_{L(ij)}}{\Delta Q_n} \end{bmatrix} \cdots\cdots\cdots\cdots\ Equation - 32$$

Using Taylor series approximation while ignoring second and higher order terms the change in real power loss can be expressed as:

$$\Delta Q_{L(ij)} = \frac{\partial Q_{L(ij)}}{\partial \delta_i}\Delta\delta_i + \frac{\partial Q_{L(ij)}}{\partial \delta_i}\Delta\delta_j + \frac{\partial Q_{L(ij)}}{\partial V_i}\Delta V_i + \frac{\partial Q_{L(ij)}}{\partial V_i}\Delta V_j \cdots\cdots\ Equation - 33$$

The coefficients used in above equation are as described below:

$$\frac{\partial Q_{ij}}{\partial \delta_i} = -2b_{ij}V_iV_j\sin(\delta_{ij})$$

$$\frac{\partial Q_{L(ij)}}{\partial \delta_i} = 2b_{ij}V_iV_j\sin(\delta_{ij})$$

$$\frac{\partial Q_{L(ij)}}{\partial V_i} = -2\left[b_{ij}^{sh}V_i + b_{ij}\left(V_i - V_j\cos\delta_{ij}\right)\right]$$

$$\frac{\partial Q_{L(ij)}}{\partial V_i} = -2\left[b_{ij}^{sh}V_i + b_{ij}\left(V_j - V_i\cos\delta_{ij}\right)\right]$$

V.V.III. Formulating Power Loss Sensitivity Factors

The real power loss sensitivity factors represent the change in the real power loss over a transmission or distribution line connected between bus-i and bus-j due to the change in active power or reactive power injected at any other bus-n while the reactive power loss sensitivity factors represent the change in the reactive power loss over a transmission or distribution line connected between bus-i and bus-j due to the change in reactive power or real power injected at any other bus-n. The equations for the changes in the line flows can be arranged in matrix form and expressed as:

$$Power\ Loss\ Sensitivity\ Factor = \begin{bmatrix} \Delta P_{L(ij)} \\ \Delta Q_{L(ij)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{ij}}{\partial \delta} & \frac{\partial P_{ij}}{\partial V} \\ \frac{\partial Q_{ij}}{\partial \delta} & \frac{\partial Q_{ij}}{\partial \delta} \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

By using the load flow Jacobian matrix; the above expression can be re-write as below:

$$Full\ Power\ Loss\ Sensitivity\ Factor = \begin{bmatrix} \Delta P_{L(ij)} \\ \Delta Q_{L(ij)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{L(ij)}}{\partial \delta} & \frac{\partial P_{L(ij)}}{\partial V} \\ \frac{\partial Q_{L(ij)}}{\partial \delta} & \frac{\partial Q_{L(ij)}}{\partial \delta} \end{bmatrix} [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Above equation gives change in both real and reactive power flow and thus using this equation the real and reactive power flow sensitivity factors can be determined.

V.VI. Objective Function

The multi-objective index for the performance calculation of distribution systems for DG size and location planning with load models considers the below mentioned indices by giving a weight to each index.
The generalized objective function can be written as below:

$$\boldsymbol{Minimize\ f(x)} = \{f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), \cdots\cdots\cdots\ upto\ f_k(x)\}$$

Where
$f_1(x), f_2(x), f_3(x), f_4(x), f_5(x)$are variables in the objective function Subject to following constrain:

$$\boldsymbol{Equality\ Constrain}: g(x) = g_i(x), g_2(x), g_3(x), g_4(x)\ \cdots\cdots\cdots\ upto\ g_k(x) = 0$$
$$\boldsymbol{Inequality\ Constrain}: h(x) = h_j(x), h_2(x), h_3(x), h_4(x)\ \cdots\cdots\cdots\ upto\ h_k(x) = 0$$

V.VI.I. Operational Constraints Formulation

The above formulated multi-objective function is minimized subject to various operational constraints so as satisfy the electrical requirements for the distribution network. The operational constraints are classified into two types:

[1] Equality Constraints $g_k(x) = 0$
[2] Inequality Constraints $h_k(x) = 0$

Where, k is the number of objective functions and x is the vector of control variables.

V.VI.II. Real Power Loss Reduction Index

A common strategy for sizing and placement of DG is to minimize system power loss of the power system. Real power loss reduction factor index per node is defined as the ratio of percentage reduction in real power loss from base case when a DG is installed at bus i. Real Power Loss Reduction Index (PLRI) is expressed as

$$\boldsymbol{Real\ Power\ Loss\ Reduction\ Index\ (PLRI)} = f_1 = \frac{P_{L(base)} - P_{L(dg)}}{P_{L(base)}}$$

Where;
$P_{L(base)}$ = Active Power Loss before DG Installation
$P_{L(dg)}$= Real Power Loss in the System after DG Installation

V.VI.III. Reactive Power Loss Reduction Index

In order to determine the effect of DG on reactive power losses, Reactive Power Loss Reduction Factor Index is incorporated in the objective function. This refers to the ratio of percentage reduction in reactive power loss from base case when a DG is installed at bus i. Reactive Power Loss Reduction Index (QLRI) is expressed as

$$\boldsymbol{Reactive\ Power\ Loss\ Reduction\ Index\ (RLRI)} = f_1 = \frac{Q_{L(base)} - Q_{L(dg)}}{Q_{L(base)}}$$

where**;**
$Q_{L(base)}$ = Reactive Power Loss before DG Installation
$Q_{L(dg)}$= Reactive Power Loss in the System after DG Installation

V.VI.IV. Voltage Profile Improvement Index

In a power system the voltage at each bus should be within the acceptable range and the line flow within the limits. These limits are important so that integration of DG into the system does not increase the cost for voltage control or replacement of existing lines. The Voltage Profile Improvement Index help identify the size-location pair which gives higher voltage deviations from the base voltage. The Voltage Profile Improvement Index (VPII) is defined as;

$$\textbf{Voltage Profile Improvement Index (VPII)} = f_3 = \frac{1}{\lambda + max_1^n(|1 - V_{(DG)}|)}$$

$V_{(DG)}$ = Voltage value after DG installation.
λ = Scalar Value.

V.VII. Formulation of Multi-objective Function

Multi-Objective Function (MOF) require to achieve the performance calculation of distributed systems for DG size and location is given by;

$$MOF = w_1 PLRI + w_2 QLRI + w_3 VPII$$

Where, w_1, w_2 and w_3 are the respective weights assigned to each factor.
The total sum of the weight assigned to all parameters of (MOF) should be equal to one;
That is:

$$|w_1| + |w_2| + |w_3| = 1$$

These weights are indicated to give the corresponding importance to each impact index for the penetration of DG depending on the required analysis. The weights vary according to the requirement concerns individual industry or corporate. The index whose impact outperforms the others in terms of importance and benefits is given a larger weight and vice versa.

V.VII.I. Equality Constraints

Real and Reactive power constraints are similar to the non-linear power flow equations. The constraints for power balance can be formulated as follows:

$$P_i = P_{DGi} + P_{Di}$$
$$Q_i = Q_{DGi} + Q_{Di}$$

where:
P_i = Real power flow at bus i in ***kW***
Q_i = Reactive power flow at bus i in ***kVAR***
PDG_i = Real power generation from DG placed at bus$i$in ***kW***
QDG_i = Reactive power generation from DG placed at bus i in ***kVAR***
PD_i = Real power demand at bus i in ***kW***
QD_i = Reactive power demand at bus i in ***kVAR***
P_i is also written as $P_i = -\frac{1}{[a_{ij}]}\sum_{j=1}^{n}\sum_{j\neq 1}\left[(a_{ij})P_j - (b_{ij})Q_j\right]$
The above equation is rewritten as equation for optimal sizing of DG

$$P_{DGi} = P_{Di} + \left[\frac{1}{[a_{ij}]}\sum_{j=1}^{n}\sum_{j\neq 1}\left[(a_{ij})P_j - (b_{ij})Q_j\right]\right]$$

Consequently for reactive power

$$Q_{DGi} = Q_{Di} + \left[\frac{1}{[c_{ij}]}\sum_{j=1}^{n}\sum_{j\neq 1}\left[(c_{ij})P_j - (b_{ij})Q_j\right]\right]$$

Where,
$\boldsymbol{Pi}$ = Real power flow at bus i in kW , $\boldsymbol{Qi}$ = Reactive power flow at bus i in kVAR.
$\boldsymbol{PDGi}$ = Real power generation from DG placed at bus$i$in kW
$\boldsymbol{QDGi}$ = Reactive power generation from DG placed at bus i in kVAR
PD_i = Real power demand at bus i in kW, $\boldsymbol{QDi}$ = Reactive power demand at bus i in kVAR

V.VII.II. Inequality Constraints

The inequality constraints are those associated with the bus voltages and DG to be installed. Inequality constraints lie between acceptable limits to satisfy the objective function.

V.VII.II.I. Bus Voltage limits

This includes the upper and lower voltage magnitude limit, V_{imin} and V_{imax} at bus i. Bus Voltage magnitude is to be kept within acceptable operating limits throughout the optimization process, then the bus voltage limit is given by

$$V_{imim} \leq V_i \leq V_{imix}$$

where;
$\boldsymbol{V_{imin}}$ and $\boldsymbol{V_{imax}}$ are lower and upper bound bus voltage limits in p. u.

V.VII.II.II DG Capacities

The DG capacity of each unit should be defined around its nominal value. So that, each DG unit is maintained within an acceptable limit. This includes the upper and lower real and reactive power generation limits of distributed generators (DGs) connected at bus **i**. The capacity of DG is given as follows:

$$P_{DGmin} \leq P_{DGi} \leq P_{DGmax}$$
$$Q_{DGmin} \leq Q_{DGi} \leq Q_{DGmax}$$

where;
P_{DGmin} = Minimum real power generation from DG capacity in **kW**
P_{DGmax} = Maximum real power generation from DG capacity in **kW**
Q_{DGmin} = Minimum reactive power generation from DG capacity in **kVAR**
Q_{DGmax} = Maximum reactive power generation from DG capacity in **kVA**R

VI. CONCLUSION

In this article the impact of DG penetration in the context of all integral parties had been discussed. Numbers of technique are suggested for interfacing between Power system and DG Installation. Facilitation and vulnerabilities of individual techniques is also focused. Mathematical modeling is presented for DG in instigation with Power System Network. Power flow sensitivity factors and system power loss sensitivity factors have also been formulated to be used in getting the combined sensitivity factors. The combined sensitivity factors will be used to get the candidate buses for DG allocation. The multi-objective function to be used in calculating fitness for solutions during optimization has also been formulated in this paper.

VII. CONCLUSION

DG penetration in power system application is one of the most attracting technology to achieve enhance performance of the power application. Penetration of DG is extremely sensitive and multidimensional process, and numbers of parameter affects the performance of DG in power application. We have considered few of the performance parameter in the presented research article. The work can be extended to incorporate other important parameter in performance analysis. Negative impacts of DG which can be eliminated with optimal allocation of DG, this aspect can also be taken into planning strategy. The Multi-objective function can be improved by taking into consideration other power system parameters like stability issues. More future work can be done to make AI optimization technique accommodate more variability of renewable energy source.

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