Cycle C_3 Related one point union Product cordial graphs

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Abstract: In this paper we discuss one point union graphs obtained from cycle related graphs. We show that $G^{(k)}$ is product cordial where G = FL(C_3), , bull of C_3, crown of C_3, double crown of C_3, C_3^{++}, tail(C_3,2P_2), C3 attached with 2 pendent edges attached at adjacent vertices and show them to be Product cordial under certain conditions.

Key words: labeling, cordial, product, wheel, crown. tail graph.

Subject Classification: 05C78

Introduction: The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [8], A dynamic survey of graph labeling by J.Gallian [7] and Douglas West.[8]. I.Cahit introduced the concept of cordial labeling [6].There are variety of cordial labeling available in labeling of graphs. Sundaram, Ponraj, and Somasundaram [9] introduced the notion of product cordial labeling. A product cordial labeling of a graph G with vertex set V is a function f from V to {0, 1} such that if each edge uv is assigned the label f(u)f(v), the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a product cordial labeling is called a product cordial graph. We use $v_f(0,1)$ = (a, b) to denote the number of vertices with label 1 are a in number and the number of vertices with label 0 are b in number. Similar notion on edges follows for $e_f(0,1)$ = (x, y).

A lot of work is done in this type of labeling so far. One interested in survey may refer Dynamic survey in Graph labeling by J. Gallian.We mention a very short part of it. Sundaram, Ponraj, and Somasundaram have shown that trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; Pm∪Pn; Cm∪Pn; Pm∪K1,n; Wm∪Fn (Fn is the fan Pn+K1); K1,m∪K1,n; Wm∪ K1,n; Wm ∪Pn; Wm ∪Cn; the total graph of Pn (the total graph of Pn has vertex set V (Pn)∪E(Pn) with two vertices adjacent whenever they are neighbors in Pn); Cn if and only if n is odd; $C_n^{(t)}$, the one-point union of t copies of C_n, provided t is even or both t and n are even; K2+mK1 if and only if m is odd; $C_m \cup P_n$ if and only if m+n is odd; $K_{m,n}$ ∪Ps if s > mn; Cn+2∪K1,n; Kn∪Kn,(n−1)/2 when n is odd; Kn∪Kn−1,n/2 when n is even; and P2 n if and only if n is odd. They also prove that $K_{m,n}$ (m,n > 2), $P_m \times P_n$ (m,n > 2) and wheels are not product cordial and if a (p,q)-graph is product cordial graph, then q = 6 (p−1)(p + 1)/4 + 1. In this paper We show that one point union of G = FL(C_3), , bull of C_3, crown of C_3, double crown of C_3, C_3^{++}, tail(C_3,2P_2), C_3 attached with 2 pendent edges attached at adjacent vertices and show them to be Product cordial under certain conditions.

3. Preliminaries:

3.1 Fusion of vertex. Let G be a (p, q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges. If u∈G_1 and v∈G_2, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as “u is identified with”.The concept is well elaborated in D. West [10].

3.2 Crown graph. It is $C_n \odot K_2$.At each vertex of cycle a n edge was attached. We develop the concept further to obtain crown for any graph. Thus crown (G) is a graph G $\odot K_2$.It has a pendent edge attached to each of it’s vertex. If G is a (p,q) graph then crown(G) has q+p edges and 2p vertices.

3.3 Flag of a graph G denoted by FL(G) is obtained by taking a graph G=G(p,q).At suitable vertex of G attach a pendent edge.It has p+1 vertices and q+1 edges.

3.4 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If

G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$ 3.5 A bull graph bull(G) was initially defined for a C_3-bull.It has a copy of G with an pendent edge each fused with any two adjacent vertices of G. For G is a (p,q) graph, bull(G) has p+2 vertices and q+2 edges.

3.6 A tail graph (also called as antenna graph) is obtained by fusing a path p_k to some vertex of G. This is denoted by tail(G, P_k). If there are t number of tails of equal length say (k-1) then it is denoted by tail(G, tp_k). If G is a (p,q) graph and a tail P_k is attached to it then tail(G, P_k) has p+k-1 vertices and q+k-1 edges.

Main Results:

4.1 Theorem 1. Let G' = FL(C_3). Then G = G'(k) is product cordial iff k is congruent to (2 mod 4). Proof: There are different structures possible on G depending on the vertex of G' (vertex common to all copies) used to obtain G. From fig 4.1 it follows that there are three structures possible on G by taking one point union on vertex a, b or at c. The point of fusion is 'x'.

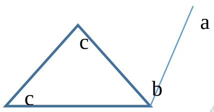

Fig 4.1 non isomorphic structures are possible at point a, b or c.

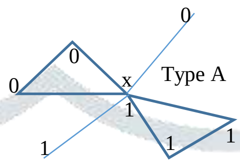

Fig 4.2 vertex labels are shown. $v_f(0,1) = (3, 4)$; $e_f(0,1)= (4,4)$

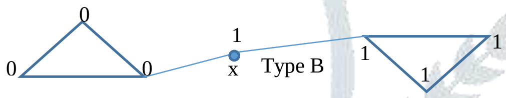

Fig 4.3 vertex labels are shown. $v_f(0,1) = (3, 4)$; $e_f(0,1)= (4,4)$

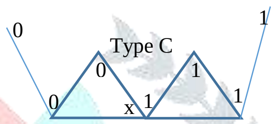

Fig 4.4 vertex labels are shown. $v_f(0,1) = (3, 4)$; $e_f(0,1)= (4,4)$

In structure 1 type A is used and the vertex common to all graphs is x whose label is '1'. In structure 2 type B is used and the vertex common to all graphs is x whose label is '1'. In structure 3 type B is used and the vertex common to all graphs is x whose label is '1'. In all structures the label number distribution is $v_f(0,1) = (3x, 3x+1)$; $e_f(0,1)= (4x,4x)$ for G= $(G')^{(2x)}$.#

Theorem 2. Let G' = bull(C_3). Then G = $G'^{(k)}$ is product cordial iff k is congruent to (2 mod 4). Proof: One can see that there are three possible non –isomorphic structures on the graph G.

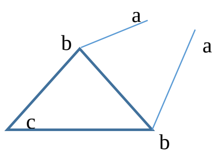

Fig 4.5 points for pairwise non isomorphic $G^{(k)}$

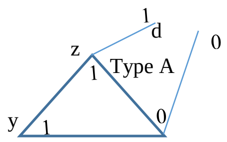

Fig 4.6 : $v_f(0,1) = (3,2)$; $e_f(0,1) = (5,5)$

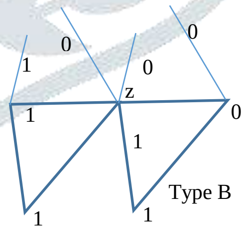

Fig 4.7: labeled copy of $(Bull(C_3)^{(2x)})$: $v_f(0,1) = (4, 5)$; $e_f(0,1)= (5,5)$

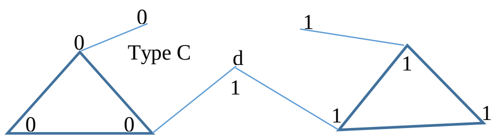

Fig 4.8 : labeled copy of $(Bull(C_3)^{(2x)})$: $v_f(0,1) = (4, 5)$; $e_f(0,1)= (5,5)$

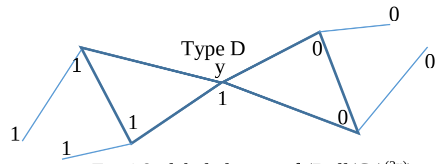

Fig 4.9 : labeled copy of $(Bull(C_3)^{(2x)})$: $v_f(0,1) = (4, 5)$; $e_f(0,1)= (5,5)$

We can take one point union on any of the three vertices as shown in figure 4.5. Define f: V(G)→{0,1} as follows. Using f we get labeled copies as shown in Type A , Type B , type C, type D. For k =1 and k = 2 figures above explains the matter. When k = 2x we have to repeatedly fuse respective type of labeling for x times at vertex z, vertex y or at vertex d to obtain $G'^{(2x)}$.The label number distribution is $v_f(0,1) = (4x, 4x+1)$; $e_f(0,1)= (5x,5x)$.To obtain the labeled copy of $G'^{(2x+1)}$ first obtain labeled copy of $G'^{(2x)}$ .With this fuse type A label at vertex d on it with vertex d of $G'^{(2x)}$,(obtained from type C label) fuse type A label at vertex z on it with vertex z of $G'^{(2x)}$, (obtained from type B label) fuse type A label at vertex y on it with vertex y of $G'^{(2x)}$. (obtained from type D label).Thus We get a labeled copy of $G'^{(2x+1)}$. The label number distribution is $v_f(0,1) = (4x+3, 4x+2)$; $e_f(0,1)= (5x+5,5x=5)$.Thus the graph is having product cordial label. Theorem 4.3 One point union of k copies of C_3^+ , G = $(C_3^+)^{(k)}$ is product cordial graph for all k provided the point common to all copies is the pendent vertex and common point is degree three then k is equal to 1 or even number only.

Proof: From figure it follows that on $C_3^{+(k)}$ there are only two structures possible up to isomorphism

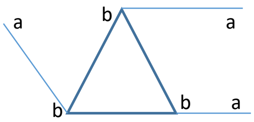

Fig 4.10 Only two non-isomorphic structures are possible at 'a' and 'b'

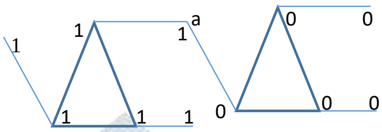

Fig 4.11 Type A label.: $v_f(0,1) = (5,6)$; $e_f(0,1) =(6,6)$

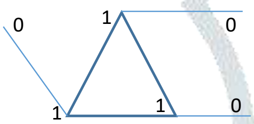

Fig 4.12 : Type C label. Labeled copy.$v_f(0,1) = (3,3)$; $e_f(0,1) =(3,3)$

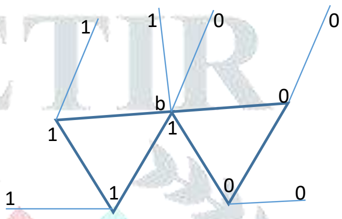

Fig 4.13 Type B label.: $v_f(0,1) = (5,6)$; $e_f(0,1) =(6,6)$

Define f: V(G)→{0,1} as follows. Using f we get labeled copies of $G'^{(2)}$as shown in fig 4.11 and 4.12, fig 4.13.When k = 1 Use type C label. S structure 1 . The common point is vertex 'b'. First obtain labeled copy of G for k = 2x. This is done by fusing type B label at point b for x-1 times. The resultant graph is $C_3^{+(2x)}$ and label number distribution is $v_f(0,1) = (5x,5x+1)$; $e_f(0,1) =(6x,6x)$. To obtain a labeled copy for k = 2x+1 first obtain labeled copy for k= 2x. Append it with type C label at vertex b. The resultant graph is $C_3^{+(2x+1)}$ and label number distribution is $v_f(0,1) = (5x+3,5x+3)$; $e_f(0,1) =(6x+3,6x+3)$.

For k is odd number greater than 1, if we have to take union point on G as pendent vertex then if it's label is '0' then it produces $e_f(0)$greater than 2 by $e_f(1)$. If we label the common vertex as '1' then also condition on edges is not satisfied. When k = 2x fuse the type A labeling with type A label at point 'a' for (x-1) times to get G . The resultant graph is $C_3^{+(2x)}$ and label number distribution is $v_f(0,1) = (5x,5x+1)$; $e_f(0,1) =(6x,6x)$.That completes the proof. # Theorem 4.4 Let G' be Tail(C_3,$2P_2$) obtained from attaching two pendent edges at a vertex of C_3,then G= $(G')^{(K)}$ is product cordial for all k and all pairwise non-isomorphic structures obtained by taking different vertices on G' as common point.

Proof: From fig 4.13 it is clear that we can take one point union at five points but only at three points vertex a, vertex b, vertex c results in pairwise non isomorphic structures.

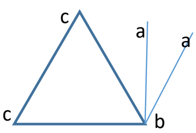

Fig 4.14 Three structures at three points a, b, c.

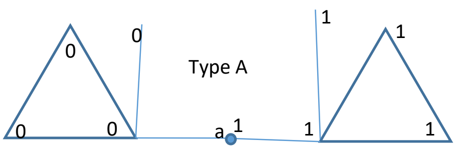

Fig 4.5 $(G')^{(2)}$ $v_f(0,1) = (4,5)$; $e_f(0,1) =(5,5)$

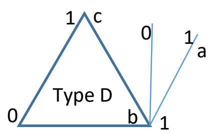

Fig 4.16: $(G)^{(1)}$ $v_f(0,1) = (2,3)$; $e_f(0,1) =(3,2)$

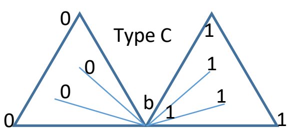

Fig 4.17: $(G)^{(2)}$ $v_f(0,1) =(4,5)$; $e_f(0,1) =(5,5)$

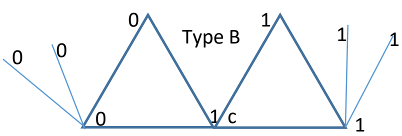

Fig 4.18: $(G')^{(2)}$ $v_f(0,1) = (4,5)$; $e_f(0,1) = (5,5)$

Define f: V(G)→{0,1} as follows. Using f we get labeled copies of $G'^{(2)}$as Type A , Type B , Type C and when k = 1 we have Type D label. To obtain a labeled copy of $G'^{(k)}$ we first obtain a labeled copy of $G'^{(2x)}$.In structure 1 'a' is the common point to all copies of G'. We fuse Type A label repeatedly for (x-1) times to obtain a labeled copy of $G'^{(2x)}$. In structure 2 'b' is the common point to all copies of G' we fuse Type C label repeatedly for (x-1) times to obtain a labeled copy of $G'^{(2x)}$. In structure 3 'c' is the common point to all copies of G' we fuse Type B label repeatedly for (x-1) times to obtain a labeled copy of $G'^{(2x)}$. To obtain labeled copy of $G'^{(2x+1)}$ fuse a copy of type D label at respective point on it (vertex a for structure 1 or vertex b for structure 2 or vertex c for structure 3) with $G'^{(2x)}$ .In all cases we get label number distribution as follows: When k = 2x we have, $v_f(0,1) = (4x,4x+1)$; $e_f(0,1) =(5x,5x)$ and when k = 2x+1 we have $v_f(0,1) = (4x+2,4x+3)$; $e_f(0,1) =(5x+3,5x+2)$. Thus the family of graphs is product cordial. **Theorem 4.5** Let G' be a graph obtained from cycle C_3 by attaching two pendent vertices each at two vertices of C_3.Let G be the one point union of G' is given by G = $G'^{(k)}$.Then G is product cordial. Proof: There are three possible pair wise non- isomorphic structures on G'.Structure 1 is obtained if vertex a is the common point on G. Structure 2 is obtained if vertex b is the common point on G. Structure 3 is obtained if vertex a is the common point on G. This is shown in figure 4.18. Define f: V(G)→{0,1} as follows. Using f we get labeled copies of G' as Type A and Type B. Both are product cordial. For all three structures we use Type A label on G when k is of type 2x+1.for x= 0,1,2.. We use type B label when k is of type 2x. x= 1, 2, ..In all the three structures the label number distribution is $v_f(0,1) = (3+6x,4+6x)$; $e_f(0,1) =(4+7x, 3+7x)$ for k = 2x.And when k = 2x+1 we have $v_f(0,1) = (6x,6x+1)$; $e_f(0,1) =(7x,7x)$.

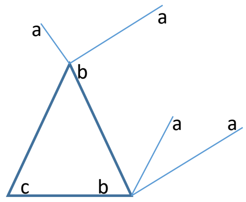

Fig 4.19 three structures at vertices 'a', 'b', 'c'

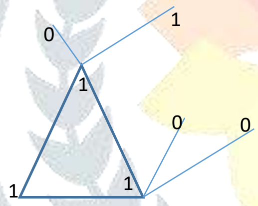

Fig 4.20: $v_f(0,1)$ = (3,4); $e_f(0,1)$ = (4,3):

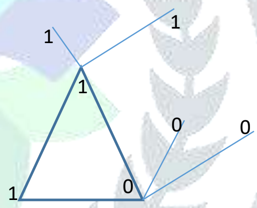

Fig 4.21: $v_f(0,1)$ = (4,3); $e_f(0,1)$ = (4,3):

This shows that the graph is product cordial. Also the function f defined is not vertex sensitive in the sense that the same function f works for all structures. Theorem 4.5 Let G' be a graph obtained from cycle C_3 by fusing two pendent vertices each at each vertex of C_3.G= $G'^{(k)}$ is product cordial on all structures . Proof: Define a function f: V(G)→{0,1}as follows. On using f we get two types of labeled copies of G' type A and Type B namely. figure 4.21 explains that there are two non-isomorphic structures on G'

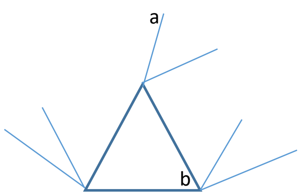

Fig 4.22 at 'a' an at 'b' two structures are possible,

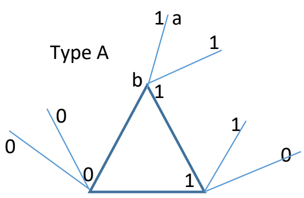

Fig 4.23: $v_f(0,1)$ = (4,5); $e_f(0,1)$ = (5,4):

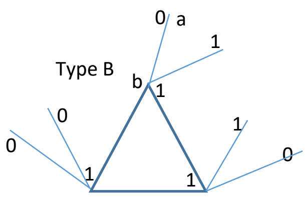

Fig 4.24: $v_f(0,1)$ = (4,5); $e_f(0,1)$ = (4,5):

When k is of type k = 2x+1; x= 0,1, 2, ..Use type A label. When k = 2x; x= 1, 2, .. use Type B label to obtain G.For all possible two structures the label number distribution is is $v_f(0,1) = (4+8x,5+8x)$; $e_f(0,1) =(5+9x, 4+9x)$ for k = 2x. And when k = 2x+1 we have $v_f(0,1) = (8x,8x+1)$; $e_f(0,1) =(9x,9x)$.This completes the proof.

Conclusions: In this paper one point union of C_3 related graphs are discussed and are shown to be product cordial. We show that

1) $G' = FL(C_3)$. Then $G = G'(k)$ is product cordial iff k is congruent to (2 mod 4)

2) $G' = bull(C_3)$. Then $G = G'^{(k)}$ is product cordial iff k is congruent to (2 mod 4).

3) One point union of k copies of C_3^+ , $G = (C_3^+)^{(k)}$ is product cordial graph for all k provided the point common to all copies is the pendent vertex and if common point is degree three then k is equal to 1 or even number only.

4) Let G' be Tail(C_3,$2P_2$) obtained from attaching two pendent edges at a vertex of C_3, then $G= (G')^{(K)}$ is product cordial for all k and all pairwise non-isomorphic structures obtained by taking different vertices on G' as common point.

5) G' be a graph obtained from cycle C_3 by fusing two pendent vertices each at each vertex of C_3. $G= G'^{(k)}$ is product cordial on all structures.

Thus it is interesting to study the cycles with pendent edges fused at some or all vertices of it for product cordiality.

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