# TWO STAGE SPECIALLY STRUCTURED FLOW SHOP SCHEDULING INCLUDING TRANSPORTATION TIME OF JOBS AND PROBABILITIES ASSOCIATED WITH PROCESSING TIMES TO MINIMIZE TOTAL WAITING TIME OF JOBS

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Abstract: In this paper, we study specially structured nx2 Flow Shop Scheduling model where the probabilities are considered with processing times of the machines. The time consumed in transportation of a job from machine-1 to machine-2 is also taken into account. The intention of the findings is to get optimal sequence of jobs in order to optimize the total waiting time of the jobs through iterative algorithm. The algorithm is also applied to a numerical example.

Key words: Waiting time of jobs, Transportation time, Flow Shop Scheduling, Processing time.

#### INTRODUCTION 1.

While following the chronological constraints and residing in the limitation of resources, the problem of deciding at what time to execute given jobs with the objective of optimizing some function is known as scheduling. Flow shop scheduling comes up with the idea that all the jobs follow the alike pre-described order of all the machines. In the present paper we talk about the n-job, 2machine specially structured Flow shop scheduling problem with the objective of minimization of total waiting time of jobs. When the jobs come for the processing, the waiting time for their turn on the first machine is considered to be zero. But in order to process a job on second machine they may have to wait for their turn for many reasons such as the previous job can take some time for the operation on second machine, machine take set up time, machine break down etc. This time which is consumed for their turn is known as the waiting time of the job.

# LITERATURE REVIEW

The Johnson's algorithm [1] for Flow Shop Scheduling problem for n-job, 2 and 3 machine to minimize the total elapsed time is popular among the analytical approaches that are used for solving 2-machine, n-job scheduling problem. Ignall E.et.al. [2] applied branch and bound technique for the permutation flow shop scheduling problem with the objective of minimization of makespan. Maggu P.L. et. al. [3] made an attempt to extend the study by introducing the concept of equivalent job for job block. Further studies are developed by Singh T. P. [4], by taking into account the transportation time, break down interval of machines. Rajendran C. et.al.[5] gave three heuristic algorithms to give consistently near optimal schedules to minimize the total flow time. Gupta D. [6] consider the optimality as to minimize the cost of rent of the machines, however it may increase the total elapsed time. Further Gupta D.et.al. [8], [9] extended the study by considering specially structured Flow shop scheduling models in two stage in which processing time structural relationship is well thought of with the purpose to minimize the cost which is consumed on rent of machines. Gupta D.et.al.[10],[11],[12] studied specially structured two stage Flow Shop Scheduling models with the objective to optimize the total waiting time of jobs.

The paper is an extension to the study done by Gupta D.et.al. [13] in the sense that we have taken into consideration the probabilities with the processing time.

#### **3.** PRACTICAL SITUATION

Flow shop scheduling occurs in various offices, service stations, banks, airports etc. Routine working in industries and factories have diverse jobs which are to be processed on various machines. The idea of minimizing the waiting time may be a reasonable aspect from managers of Factory /Industry perspective when he has minimum time bond with a profit-making party to complete the jobs.

#### 4. **NOTATIONS**

: Job sequence achieved by applying the algorithm proposed.  $A_k$ 

: Time taken by machine A to process k<sup>th</sup> job.

: Time taken by machine B to process kth job.  $b_k$ 

: Expected time of fictitious machine X to process k<sup>th</sup> job.  $X_{\mathbf{k}}$  $Y_k'$ : Expected time of fictitious machine Y to process k<sup>th</sup> job.

: Probability of k<sup>th</sup> job for processing on machine A.  $p_k$ 

: Probability of k<sup>th</sup> job for processing on machine B.  $q_{\mathbf{k}}$ 

: Transportation time taken to export k<sup>th</sup> job from machine A to machine B. t<sub>k.</sub>

 $T_{aX}$ : The completion time of job a on machine X.  $T_{aY}$ : The completion time of job a on machine Y.

 $W_{II}$ : Waiting time of job  $\mu$ .

W : Total waiting time of all the jobs.

#### 5.1 PROBLEM FORMULATION

The machines A and B are dealing out n jobs in the sort AB. ak and bk are the relevant processing times together with probabilities  $p_k$  and  $q_k$  of the  $k^{th}$  job correspondingly,  $t_k$  is the transportation time of  $k^{th}$  job from machine A to machine B. Our goal is to come across a best possible sequence {A<sub>i</sub>} of jobs with minimum total waiting time of jobs. Expected processing time of  $k^{th}$  job on machine A and B are defined as  $a_k' = a_k \times p_k$ ,  $b_k' = b_k \times q_k$ . Define the two fictitious machines X and Y with processing times  $X'_k$  and  $Y'_k$  defined as  $X'_k = a'_k + t_k$  and  $Y'_k = b'_k + t_k$  satisfying processing times structural relationship  $\max X'_k \leq \min Y'_k$ 

Table 5 1: Mathematical model of the problem

Job	Machine A		Transportation time		Machine B	
J	a <sub>k</sub>	$p_k$	$t_k$	10 1	$b_k$	$q_k$
1.	$a_1$	$p_1$	$t_1$	45	$b_1$	$q_1$
2.	$a_2$	p <sub>2</sub>	t <sub>2</sub>	46	$b_2$	$q_2$
3.	a <sub>3</sub>	p <sub>3</sub>	t <sub>3</sub>		b <sub>3</sub>	$q_3$
1.	5	الحب	1		V.	1 : Y
n.	a <sub>n</sub>	p <sub>n</sub>	t <sub>n</sub>	1,279	b <sub>n</sub>	$q_n$

#### 5.2 **ASSUMPTIONS**

In the given flow shop scheduling the following assumptions are made

- There are n number of jobs (J) and two machines (A & B). 1)
- 2)  $\max X'_k \leq \min Y'_k$
- 3)  $\sum_{k=1}^{n} p_k = \sum_{k=1}^{n} q_k = 1$
- Each of the jobs firstly processed on machine A than on machine B. 4)
- 5) Each job is not dependent on any other job.
- 6) Machines break down interval, set up times are not considered for calculating waiting time.
- Pre- emption is not allowed i.e. the process can't be interrupted until a job which is started on a 7) machine can't be fully completed.
- **Lemma:** Two machines X, Y are handing out n jobs in sort XY among no passing permissible. X'<sub>k</sub> and Y'<sub>k</sub> are the dealing out times of job k (k = 1,2,3,...,n) on both machines correspondingly satisfying processing times structural relationship Max  $X_k' \leq \text{Min } Y_k'$  then for the n job sequence  $C: \mu_1, \mu_2, \mu_3, \dots \dots \mu_n$ ,  $T_{\mu_n Y} = X_{\mu_1}' + Y_{\mu_1}' + Y_{\mu_2}' \dots + Y_{\mu_n}'$

**Proof.** Using mathematical Induction hypothesis on n:

Consider 
$$S(n)$$
:  $T_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} ... + Y'_{\mu_n}$ 

$$T_{\mu_1 X} = X'_{\mu_1}, T_{\mu_1 Y} = X'_{\mu_1} + Y'_{\mu_1}$$
 S(1) is true.

Assume the result holds for less than n jobs,  $T_{\mu_n Y} = Max(T_{\mu_n X}, T_{\mu_{n-1} Y}) + Y'_{\mu_n}$ 

As Max  $X'_k \leq Min Y'_k$ 

Consequently,  $T_{\mu_n Y} \stackrel{\cdot \cdot \cdot}{=} X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} \ldots + Y'_{\mu_n}$ 

S(n) is true for all  $n \in N$ 

**Lemma.** Following the similar notations as used in 5.3 Lemma, for n job sequence  $C: \mu_1, \mu_2, \mu_3, \dots \dots \mu_n$ 5.4

$$W_{\mu_1}=0\,$$

$$W_{\mu_n} = X'_{\mu_1} + \sum_{r=1}^{n-1} x_{\mu_r} - X'_{\mu_n}$$

 $x_{\mu_r}$  is defined as  $x_{\mu_r}=Y'_{\mu_r}-X'_{\mu_r},~\mu_r\,\varepsilon\,(1,2,3,\dots,n)$  Proof.  $W_{\mu_1}=0$ 

$$W_{\mu_n} = Max(T_{\mu_n X}, T_{\mu_{n-1} Y}) - T_{\mu_n X}$$

$$\begin{split} &=X'_{\mu_1}+Y'_{\mu_1}+Y'_{\mu_2}...+Y'_{\mu_{n-1}}-X'_{\mu_1}-X'_{\mu_2}...-X'_{\mu_n}\\ &=X'_{\mu_1}+\sum_{\substack{r=1\\n-1\\n-1}}(Y'_{\mu_r}-X'_{\mu_r})-X'_{\mu_n}\\ &=X'_{\mu_1}+\sum_{r=1}(x_{\mu_r})-X'_{\mu_n} \end{split}$$

Theorem.

Following the similar notations as used in 5.3 Lemma, for the n job sequence  $C: \mu_1, \mu_2, \mu_3, \dots \mu_n$ , the total waiting time (W) is given by the following formula  $W = nX'_{\mu_1} + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{k=1}^{n} X'_k$  where  $y_{\mu_r} = (n-r)x_{\mu_r}$ ;  $\mu_r \varepsilon (1, 2, 3, ..., n)$ 

**Proof**. From Lemma 5.4 we have

$$W_{\mu_1} = 0$$

For n = 2,

$$W_{\mu_2} = X'_{\mu_1} + \sum_{r=1}^{1} x_{\mu_r} - X'_{\mu_2}$$

For n = 3,

$$W_{\mu_3} = X'_{\mu_1} + \sum_{r=1}^2 x_{\mu_r} - X'_{\mu_3}$$

Continuing in this way

$$W_{\mu_n} = X'_{\mu_1} + \sum_{r=1}^{n-1} x_{\mu_r} - X'_{\mu_n}$$

$$W = \sum_{i=1}^n W_{\mu_i}$$

$$W = n X_{\mu_1}' + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{k=1}^{n} X_k'$$

**Theorem:** For a natural number 'n' and real numbers  $x_1, x_2, x_3, \dots x_n$  if

 $x_{\mu_1} \leq x_{\mu_2} \leq x_{\mu_3 \leq \dots \dots} \leq x_{\mu_n} \Rightarrow nx_{\mu_1} + (n-1)x_{\mu_2} + (n-2)x_{\mu_3} + \dots \dots + x_{\mu_n} \text{ is minimum for } \mu \in S_n \text{, permutation group of n-supplies}$ 

**Proof:** Using mathematical Induction hypothesis on n:

For n=1, the result holds trivially

Assume the result holds for less than n real numbers

For 
$$x_{\mu_1} \le x_{\mu_2} \le x_{\mu_3} \le ... ... \le x_{\mu_n}$$

$$nx_{\mu_1} + (n-1)x_{\mu_2} + (n-2)x_{\mu_3} + \dots + x_{\mu_n}$$

$$= (n-1)x_{\mu_1} + (n-2)x_{\mu_2} + (n-3)x_{\mu_3} + \dots + x_{\mu_{n-1}} + \sum_{k=1}^{n} x_{\mu_k}$$

As  $\sum_{k=1}^{n} x_{\mu_k}$  is constant, by induction hypothesis  $nx_{\mu_1} + (n-1)x_{\mu_2} + (n-2)x_{\mu_3} + \cdots + x_{\mu_n}$  is minimum for  $\mu \in S_n$ .

#### 6.1 **ALGORITHM**

**Step 1:** Calculate expected processing times,  $a'_k$  and  $b'_k$  on machines A and B defined as follows:

$$a'_k = a_k \times p_k, b'_k = b_k \times q_k.$$

**Step 2:** Define the fictitious machines X and Y with processing times  $X'_k$  and  $Y'_k$  as in the study made by Aggarwal S.et.al.[7], Gupta D. et.al.[9], [13], as follows:

 $X_k' = a_k' + t_k$  and  $Y_k' = b_k' + t_k$  and verify the structural relationship, Max  $X_k' \le Min \ Y_k'$ 

**Step 3:** Find  $x_k = Y'_k - X'_k$ , k=1,2,3,....,n

**Step 4:** Assemble the jobs in increasing order of  $x_k$ . Assuming the schedule found be  $(\mu_1, \mu_2, \mu_3, ... ... \mu_n)$ 

**Step 5:** Locate  $a = \min\{X'_k\}$ 

If  $X'_{\mu_1}=a$ , then schedule noticed previously in step 4 is the requisite favorable schedule.

If  $X'_{u_1} \neq a$ , then move on to step 6

Step 6: Consider the different sequence of jobs  $A_1, A_2, A_3, \dots, A_n$ . Where  $A_1$  the schedule described in step 3, schedule  $A_i$  ( $i = 1, 2, 3, 3, \dots, A_n$ ).  $2,3,\ldots,n$ ) can be achieved by placing i<sup>th</sup> job in the schedule  $A_1$  to the initial position and rest of the schedule remaining same.

Step 7: Calculate the total waiting time W for each and every of the schedules  $A_1, A_2, A_3, \dots, A_n$  using the formula derived in 5.5 Theorem as follows:

```
W = nX'_{\mu_1} + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{k=1}^{n} X'_k
                                                                                                                         (6.1)
   where y_{\mu_r} = (n - r)x_{\mu_r}, \mu_r \in (1, 2, 3, ..., n)
   As \sum_{k=1}^{n} X_k' is constant for the problem, we can now say that X_{\mu_1}' is \min\{X_k'\} and x_{\mu_1} \leq x_{\mu_2} \leq x_{\mu_3} \leq \cdots \ldots \leq x_{\mu_n}
   Using the 5.6 Theorem, the schedule with minimum total waiting time is the required optimal schedule.
6.2 C++ PROGRAM FOR THE GIVEN ALGORITHM
   #include<iostream.h>
   #include<conio.h>
   #include<iomanip.h>
   void main()
            clrscr();
             float p[30], q[30], a[30], b[30], mul_a[30], mul_b[30], sum_p, sum_q, t[30], x_k[30], y_k[30], xk[30], tempx, tempy,
   tempsort, kcopy[30],temp,job1[30],job2[30],job3[30],job4[30],minx,tem,temx;
             int i,j,num,job[30],jobx,minjobx;
             cout<<"Input the number of jobs\n";
             cin>>num;
            cout<<"Enter the processing times of Machine A\n";
             for(i=0;i< num;i++)
                      cin>>a[i];
             }
             abc:
             sum p=0.0;
            cout<<"\nEnter the probabilities for machine A\n":
             for(i=0;i< num;i++)
                      cin >> p[i];
                      sum_p=sum_p+p[i];
                      if(sum_p>1.0001)
                               cout<<"sum of probabilities is greater than 1\n enter the values again";
                               goto abc;
            if(sum_p<.9999)
                      cout<<"sum of probabilities is less than 1\n enter the values again";
                      goto abc;
             cout<<"The total sum of probabilities\t"<<sum_p<<"\n";
             cout<<"After multiplication ak and pk we will get\n";
             cout<<"Job\tak\t \tpk\t \ta'k\n";
             for(i=0;i< num;i++)
                      mul_a[i]=a[i]*p[i];
                      cout < i+1 < "\t" << a[i] << "\t" << p[i] << "\t" << setprecision(2) << mul_a[i] << "\n";
            cout<<"Enter the processing times of Machine B\n";
             for(i=0;i< num;i++)
             {
                      cin >> b[i];
             }
            abcd:
            sum_q=0.0;
             cout<<"\nEnter the probabilities for Machine B\n";
             for(i=0;i< num;i++)
```

```
{
         cin>>q[i];
         sum_q=sum_q+q[i];
         if(sum_q>1.0001)
                  cout<<"sum of probabilities is greater than 1\n enter the values again";
                  goto abcd;
}
if(sum_q<0.9999)
         cout<<"sum of probabilties is less than 1\n enter the values again";
         goto abcd;
cout << "The total sum of probabilities \t" << sum_q << "\n";
cout<<"After multiplication bk and qk we will get\n";
cout << "Job \tbk \t \tgk \t \tb'k \";
for(i=0;i< num;i++)
         mul_b[i]=b[i]*q[i];
         cout < i+1 < "\t" < b[i] < "\t" < q[i] < "\t" < setprecision(3) < mul_b[i] < "\n";
cout<<"\nEnter the transportation times\n";
for(i=0;i< num;i++)
{
         cin>>t[i];
}
cout<<"After entering transportation times Fictitious Machine X\n";
cout<<"Job\ta'k\t\tTrans\t\tX'k\n";
float sumx_k=0;
for(i=0;i< num;i++)
{
         jobx=i+1;
         x_k[i]=mul_a[i]+t[i];
         sumx_k=sumx_k+x_k[i];
         cout<<jobx<<"\t"<<mul_a[i]<<"\t+\t"<<t[i]<<"\t=\t"<<x_k[i]<<"\n";
cout<<"After entering transportation times Fictitious Machine Y\n";
cout<<"Job\tb'k\t\tTrans\t\tY'k\n";
for(i=0;i<\!num;i++)
{
         y_k[i]=mul_b[i]+t[i];
         cout << i+1 << "\t" << mul_b[i] << "\t" +\t" << t[i] << "\t" +\t" << y_k[i] << "\n";
}
tempx=x_k[0];
for(i = 0; i < num; ++i)
{
         if(tempx < x_k[i])
         tempx=x_k[i];
cout << "Maximum X'k = "<< tempx << "\n";
tempy=y_k[0];
for(i = 0; i < num; ++i)
{
         if(tempy> y_k[i])
         tempy=y_k[i];
```

```
}
cout << "Minimum Y'k=" << tempy << "\n";
if(tempx<=tempy)
        cout<<"Processing time structural relationship is satisfied\n";
        cout << "Job \txk= Y'k-X'k \n";
         for(i=0;i< num;i++)
                 xk[i]=y_k[i]-x_k[i];
                 cout << i+1 << "\t" << xk[i] << "\n";
                 kcopy[i]=xk[i];
                 job1[i]=i+1;
         for(i=0;i< num;i++)
                 for(j=i+1;j< num;j++)
                          if(kcopy[i]>kcopy[j] || job1[i]>job1[j] )
                                   tempsort=kcopy[i];
                                   kcopy[i]=kcopy[j];
                                   kcopy[j]=tempsort;
                                   temp=job1[i];
                                   job1[i]=job1[j];
                                   job1[j]=temp;
         float sum=0.0;
        int index;
        int j1=0;
         int j=num-1;
        index=job1[0];
        sum=sum+(x_k[index-1]*num);
        cout << "Schedule A1 is:";
         for(i=0;i<num;i++,j--)
                 cout << job1[i] << "\t";
                 j1=job1[i];
                 sum=sum +(j*xk[j1-1]);
                 job2[i]=job1[i];
         float sumfinal[30];
        sumfinal[0]=sum-sumx_k;
        cout<<"\nWaiting Time W="<<sumfinal[0];
         minx=x_k[0];
        minjobx=1;
         for(i=0;i< num;++i)
                 if(minx>x_k[i])
                          minx=x_k[i];
                          minjobx=i+1;
```

```
cout <<"\nMinimum X'k="<<minx;</pre>
        if(minjobx==job1[0])
                 cout<<"\nSchedule A1 is the required Schedule";
        else
                 cout<<"\nOther Possible schedules are:\n";</pre>
                 int aa=num,pp=1,m=0,j;
                 while(aa>1)
                          sum=0;
                          int templl=job2[0];
                          job2[0] = job2[pp];
                          job2[pp]=templl;
                          index=job2[0];
                          cout << "A" << pp+1;
                          sum=sum+(x_k[index-1]*num);
                          j=num-1;
                          for(i=0;i<num;i++,j--)
                                  cout << "\t" << job2[i];
                                  j1=job2[i];
                                  sum=sum +(j*xk[j1-1]);
                          sumfinal[m+1]=sum-sumx_k;
                          cout<<"\nWaiting Time W = "<<setprecision(2)<<sumfinal[m+1]<<"\n";
                          aa--;
                          m++;
                 float minsum;
                 minsum=sumfinal[0];
                 for(i=1;i < num;++i)
                          if(minsum>sumfinal[i])
                                  minsum=sumfinal[i];
                 cout <<"\nMinimum W = "<<minsum;</pre>
                 cout<<"\nSchedule With Minimum Waiting Time W is the Required Schedule";
         }
        cout<<"Processing time structural relationship is not satisfied";
getch();
```

}

} else

# **6.3 NUMERICAL ILLUSTRATION**

Assume 5 jobs 1, 2, 3, 4, 5 has to be processed on two machines A and B with processing times ak, bk and pk, qk are their respective probabilities and t<sub>k</sub> is the transportation time of k<sup>th</sup> job from machine A to machine B

Table 6.1: Processing time matrix

Job	Machine A		Transportation time	Machine B	
J	$a_k$	$p_k$	t <sub>k</sub>	$\mathbf{b_k}$	$\mathbf{q_k}$
1.	6	0.2	4	12	0.2
2.	7	0.2	3	21	0.2
3.	12	0.2	2	34	0.2
4.	11	0.3	3	22	0.2
5.	13	0.1	2	24	0.2

Intention is to achieve a most favorable schedule, minimizing the total waiting time for the jobs.

## **Solution**

As per step 1- Expected processing time  $a_k \& b_k$  on machine A & B are calculated in the following table Table 6.2: Expected processing time matrix

Job	Machine A	Transportation time	Machine B
J	$\mathbf{a}_{\mathbf{k}}^{\prime}$	t <sub>k</sub>	$\mathbf{b_k'}$
1.	1.2	4	2.4
2.	1.4	3	4.2
3.	2.4	2	6.8
4.	3.3	3	4.4
5.	1.3	2	4.8

As per step 2: Defining the fictitious machines X and Y with processing times  $X'_k = a'_k + t_k$  and  $Y'_k = b'_k + t_k$  respectively.

Table 6.3					
Job	Machine X	Machine Y			
J	X' <sub>k</sub>	Y' <sub>k</sub>			
1.	5.2	6.4			
2.	4.4	7.2			
3.	4.4	8.8			
4.	6.3	7.4			
5.	3.3	6.8			

Processing time structural relationship Max  $\overline{X'_k}$ 

As per step 3- Find  $x_k = Y'_k - X'_k$ 

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Job	Machine X	Machine Y	All Marie
J	$X'_{\mathbf{k}}$	$Y'_{\mathbf{k}}$	$\mathbf{x_k}$
1.	5.2	6.4	1.2
2.	4.4	7.2	2.8
3.	4.4	8.8	4.4
4.	6.3	7.4	1.1
5.	3.3	6.8	3.5

As per step 4: The schedule thus found be 4, 1, 2, 5, 3.

**As per step 5-**  $a=3.3 \neq X'_{4}$ 

As per step 6-Considering the different sequence of jobs:

 $A_1: 4, 1, 2, 5, 3; A_2: 1, 4, 2, 5, 3; A_3: 2, 4, 1, 5, 3; A_4: 5, 4, 1, 2, 3; A_5: 3, 4, 1, 2, 5$ 

As per step 7- Calculating the total waiting time (W) for the schedules obtained in step 6 using the formula in Equation (6.1)

Here,  $\sum_{k=1}^{5} X'_{k} = 23.6$ 

For the schedule  $A_1$ : 4, 1, 2, 5, 3,

W = 25

For the schedule  $A_2$ : 1, 4, 2, 5, 3,

W = 19.6

For the schedule  $A_3$ : 2, 4, 1, 5, 3

W = 18.8

For the schedule  $A_4$ : 5, 4, 1, 2, 3

W = 15.4

For the schedule  $A_5$ : 3, 4, 1, 2, 5

W = 24.5

Hence schedule A<sub>4</sub>: 5, 4, 1, 2, 3 is the required schedule with minimum total waiting time.

### CONCLUSION

The present study deals with the flow shop scheduling model with the main idea to optimize the total waiting time of jobs. However it may increase the other costs like machine idle cost or penalty cost of the jobs, yet the idea of minimizing the waiting time is a matter that cannot be avoided in the cases when there is a minimum time contract with the customers. The study can be extended by introducing various parameters like weightage of jobs, set up time of machines, break down interval of machines etc.

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