# Prime Odd Mean Labeling of Some Special Graphs

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***Abstract:*** Let G be a Graph with p vertices and q edges an injective function $f : V(G) \rightarrow \{1,3,5,\ldots 2q+1\}$ such that gcd ( f(u),f(v) ) =1 and the induced edge labelings $f^*: E(G) \rightarrow \{2,4,6,\ldots 2p-2\}$ are defined

$$f(uv) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is called a prime mean labeling. A graph which admits an odd mean labelings are a prime odd mean labeling graph.

***Keywords:*** Prime Odd Mean Labeling, Bull graph, Caterpillar graph, Cycle, Kite graph, Ladder graph, Path, Peterson graph, Spider graph, Star graph, Tree.

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## 1 Introduction

In this paper we consider a finite, connected, undirected and simple graph. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively.
For various graph, Theoritic notations and terminology we follow D.B.West [7].
Mean labeling was introduced by Somasundaram and Ponraj [4].The notation of a Prime Labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6].
Many researchers have studied prime graph for example in Fu.H. (1994 P 181-186) [3] have proved that path $P_n$ on n vertices is a prime graph.
In Deretsky.T (1991 p359-369) [2] have proved that the $C_n$ on n vertices is a prime graph.
In this paper we introduced a new concept of prime Odd Mean Labeling. We have derived different graph families are satisfying the conditions of prime odd mean labeling.

**Definition 1.1**

A Graph G with p vertices and q edges are called a mean graph if there is an injective function f: $V(G) \rightarrow \{0,1,2,3\ldots q\}$ such that each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u)+f(v) is even and $\frac{f(u)+f(v)+1}{2}$ is odd, then the resulting edge labels are distinct. The graph which admits a *Mean labeling* is called a Mean graph.

**Definition 1.2**

A Prime labeling of a graph G is an injective function f: $V(G) \rightarrow \{1,2,3\ldots |V(G)|\}$ such that for every pair of adjacent vertices u and v, gcd (f(u),f(v)) =1. The graph which admits a *Prime labeling* is called a Prime graph.

**Definition 1.3**

The *Bull graph* is a planar undirected graph with 5 vertices and 5 edges in the form of a triangle with two disjoint pendant edges.

**Definition 1.4**

*Caterpillar graph* is a tree in which all the vertices are within distance 1 of a central path.

**Definition 1.5**

*Cycle* is a closed walk in which all the vertices and edges are distinct except u = v. A non-trivial closed path is a cycle and a cycle is also called a circuit.

**Definition 1.6**

The *Kite graph* is obtained by attaching a path of length 'm' with a cycle of length 'u' and it is denoted as $KI_{m,n}$. Kite graph is also known as the Dragon graph (or) Canoe paddle graph (or) Tadpole graphs.

**Definition 1.7**

The *Ladder graph* $L_n$ is a planar undirected graph with 2n vertices and n+2(n-1) edges (or) add the edges between corresponding vertices of two path of equal length.

**Definition 1.8**

A walk is a *path* if all it vertices and also edges are distinct. In addition if all its vertices are distinct then the trail is a path.

**Definition 1.9**

The *Peterson graph* is an undirected graph with 10 vertices and 15 edges. The Peterson graph is named after Julius Peterson.

**Definition 1.10**

A *spider* is a tree with one vertex of degree atleast 3 and all other with degree atmost 2.

**Definition 1.11**

A *Star* graph with n vertices is a tree with one vertex having degree n-1 and other n-1 vertices having degree 1. A star graph with n+1 vertices is $K_{1,n}$.**Definition 1.12**

A *tree* is an undirected graph in which any two vertices are connected by exactly one path.

## 2. Main Results

**Definition 2.1**

Let G be a graph with $|V(G)|$ = p vertices and $|E(G)|$ =q edges. A graph is called a prime odd mean labeling if the vertex labelings $f : V(G) \rightarrow \{1,3,5,\ldots..2q+1\}$ are defined by gcd (f(u),f(v))=1 and the induced edge labelings $f^*: E(G) \rightarrow \{2,4,6\ldots..2p-2\}$ are defined by

$$f(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

**Theorem 2.2:**
A Bull graph admits a Prime Odd Mean Labeling.

**Proof:**
Let G be a Bull graph. We define a vertex labeling $f : V(G) \rightarrow \{1,3,5,\ldots..2q+1\}$ as follows
$f(u_i) = 2i-1$, $i = 1,2,3,\ldots..,n$
such that gcd (f(u),f(v)) = 1 and the induced edge labelings are satisfied the condition of

$$\frac{f(u)+f(v)}{2} \quad \text{if } f(u)+f(v) \text{ is even}$$

$$\frac{f(u)+f(v)+1}{2} \quad \text{if } f(u)+f(v) \text{ is odd}$$

then all the edge labelings are distinct.
In view of the above labeled pattern of a Bull graph admits a Prime Odd Mean Labeling.

**Theorem 2.3:**
The Caterpillar graph is a Prime Odd Mean Labeling.

**Proof:**
Let G be denote the Caterpillar graph. Let us define a vertex labeling $f : V(G) \rightarrow \{1,3,5,\ldots..2q+1\}$. Let $u_1,u_2,\ldots.u_n$ be the vertex of the central path and $a_1,a_2,\ldots\ldots,a_n$ be the vertices of pendent edge joining to the central path. We have

$f(u_i) = 2i-1,\ 1 \le i \le n$
$f(a_i) = 2p+,\ 1 \le i \le n$
such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then all the edge labelings are distinct.
Therefore the resulting of Caterpillar graph is a Prime Odd Mean Labeling.

**Theorem 2.4:**
The cycle $C_n$, $n \ge 3$ and n is odd then admits a prime odd mean labeling.
**Proof:**
Let G = $C_n$ be graph with $p_n$ vertices and $q_n$ edges. We define a vertex labeling
f: V(G) $\rightarrow$ {1,3,5,…..2q+1} as follows
$f(u_i) = 2i-1$ , i =1,2,3,……n
such that gcd(f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then all the edge labelings are distinct.
In view of the above labelled pattern of a Cycle $C_n$ admits a Prime Odd Mean Labeling.

**Theorem 2.5:**
A Kite Graph $KI_{n,m}$ admits a Prime Odd Mean Labeling.
**Proof:**
Let G=$KI_{n,m}$ be a Kite graph. We define a vertex labeling f: V(G) $\rightarrow$ {1,3,5,…..2q+1} as follows
$f(u_i) = 2i-1$ , i =1,2,3,……n
such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then all the edge labelings are distinct.
Hence proved a kite graphs $KI_{n,m}$ admits a prime odd mean labeling.

**Theorem 2.6:**
A ladder graph $L_n$ admits a prime odd mean labeling.
**Proof:**
Let G=$L_n$be a ladder graph. Let us define a vertex labeling f: V(G) $\rightarrow$ {1,3,5,…..2q+1}. Let $u_1,u_2,\ldots u_n$ be the vertex of the central path and $a_1,a_2,\ldots a_n$ be the vertices of pendent edge joining to the central path. We have
$f(u_i) = 2i-1$ , $1 \le i \le n$
$f(a_i) = 2p+i$, $1 \le i \le n$
such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

thenthe edge labelings are distinct.
Thus we proved a ladder graph $L_n$ admits a prime odd mean labeling.

**Theorem 2.7:**
Every path $P_n$ is a prime odd mean labeling.
**Proof:**
Let G=$P_n$ be a path. Let us define a vertex labeling f: V(G) $\rightarrow$ {1,3,5,…..2q+1} as follows
$f(u_i) = 2i-1$ , i =1,2,3,……n

such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then all the edge labelings are distinct.
Hence proved a path $P_n$ is a prime odd mean labeling.

**Theorem 2.8:**

The Petersen graph is a prime odd mean labeling.

**Proof:**

Let G be a Petersen graph with $u_1,u_2,u_3,u_4,u_5$ be the internal vertices and $u_6,u_7,u_8,u_9,u_{10}$ be the external vertices. Let us define a vertex labeling f: V(G) → {1,3,5,…..2q+1} as follows

$f(u_i) = 2i-1$ , i =1,2,3,……n

such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then the edge labelings are distinct.
Theorefore the resulting of Petersen graph is a prime odd mean labeling.

**Theorem 2.9:**

A spider graph admits a prime odd mean labeling.

**Proof:**

Let G be a spider graph. Let us define a vertex labeling f: V(G) → {1,3,5,…..2q+1} as follows

$f(u_i) = 2i-1$ , i =1,2,3,……n

such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then all the edge labelings are distinct.
In view of the above labelled pattern of spider graph admits a prime odd mean labeling.

**Theorem 2.10:**

A star graph $S_n$ admits a prime odd mean labeling.

**Proof:**

Let $G=S_n$be a star graph. Let us define a vertex labeling f: V(G) → {1,3,5,…..2q+1}. Let $u_1$ be the central of the graph.

$f(u_1)=1$

$f(u_i) = 2i-1$ , i =1,2,3,……n

such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then all the edge labelings are distinct.
Thus we proved a star graph $S_n$ admits a prime odd mean labeling.

**Theorem 2.11:**

A tree is a prime odd mean labeling.

**Proof:**

Let G be a tree. Let us define a vertex labeling f: V(G) → {1,3,5,…..2q+1}as follows

$f(u_i) = 2i-1$ , i =1,2,3,……n

such that gcd (f(u),f(v)) =1 and the induced edge labelings are satisfied the condition of

$\frac{f(u)+f(v)}{2}$ if f(u)+ f(v) is even

$\frac{f(u)+f(v)+1}{2}$ if f(u)+ f(v) is odd

then all the edge labelings are distinct.
Hence proved a tree is a prime odd mean labeling.

**3. Conclusion**

In this article we have investigated Prime Odd Mean labeling of Paths, Bull Graph, Caterpillar, Cycles, Kite Graphs, Ladder Graphs, Petersen Graphs, Spider Graphs, Star Graphs and Tree. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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