Fibonacci Antimagic Labeling of Some Special Graphs

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Abstract: In this paper we introduce Fibonacci Antimagic labeling . A bijective function
$f : V(G) \rightarrow \{F_0,F_1,....F_n\}$ where F_j is the j^{th} Fibonacci number (j = 0,1,...n) is said to the Fibonacci Antimagic labeling if the induced function
$f^* : E(G) \rightarrow \{1,2,...2P\}$ is defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labelings are distinct is called Fibonacci Antimagic labeling and a graph which admits Fibonacci Antimagic labeling is called a Fibonacci Antimagic graphs. Here we investigate Fibonacci Antimagic labeling of some special kind of graphs.

Keywords: Fibonacci Antimagic labeling, Tree, Path, Spider, Caterpillar, Unicyclic graph.

AMS Subject Classification (2010): 05C78

1. Introduction

In this paper we consider finite, connected, undirected and simple graphs only. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. For various graph theoretic notation and terminology by D.B. west [7]. Fibonacci graceful labeling was introduced by Kathiresan and Amutha [5] and different kind of Fibonacci labeling were studied by [1], [2], [3] and [6]. Antimagic labelings was introduce by N.Hartsfield and G.Ringel [4]. In this paper we introduce a new concept called Fibonacci Antimagic labelings.

Definition 1.1
The Fibonacci sequence is a set of numbers that starts with zero followed by a one and proceed based on the rule that each number is equal to the sum of the proceeding two numbers is called a *Fibonacci labelings*.
Definition 1.2 A graph G is called *antimagic* if the n edges of G can be distinctly labeled 1 through n in such a way that when taking the sum of the edge labels incident to each vertex, the sums will all be different.

Definition 1.3
A *Tree* is an undirected graph in which any two vertices are connected by exactly one path.

Definition 1.4
A Walk is a path if all its vertices and also edges are distinct. In addition, if all the vertices are distinct then the trail is a *path.*

Definition 1.5
Caterpillar is a tree with all vertices either on a single central path or distance 1 away from it. The central path may be considered to be the largest path in the caterpillar so that both end vertices have valency 1.

Definition 1.6
A *spider* is a tree with one vertex of degree atleast 3 and all others with degree atmost 2.
Definition 1.7
The graph $C_n \Theta mK_1$ is a *unicyclic graph* with p = q = n (m+1) obtained from the cycle C_n by attaching m - pendent edges at each vertex of the cycle C_n.

2. Main Results

Definition 2.1
Let G be a (p,q) graph. An bijective function $f : V(G) \rightarrow \{F_0,F_1,....F_n\}$ where F_j is the j^{th} Fibonacci number (j = 0,1,...n) is said to the Fibonacci Antimagic labeling if the induced function $f^* : E(G) \rightarrow \{1,2,...2P\}$ is defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labelings are distinct.

Theorem 2.2
Every path P_n , $n \geq 2$ admits a Fibonacci Antimagic labeling **.**
Proof

Let $G = P_n$ be a graph with n vertices and n - 1 edges.
Define a vertex labeling by $f : V(G) \to \{F_0, F_1, \ldots. F_n\}$ and the induced edge labeling
$f^*: E(G) \to \{1,2,\ldots 2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct. Therefore the resulting graph every path P_n, $n \geq 2$ admits a Fibonacci Antimagic labeling.

Example 2.3

The following is an example of the Fibonacci Antimagic labeling of the Path P_4 and P_5

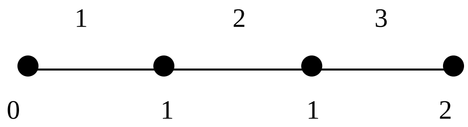

Fig (1.1) - P_4

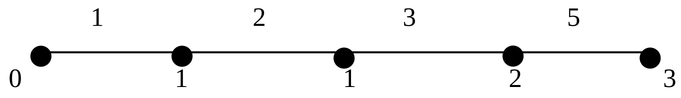

Fig (1.2) - P_5

Theorem 2.4

All trees are Fibonacci Antimagic labelings.

Proof

A tree has n vertices and n-1 edges.
Define a vertex labeling by $f : V(G) \to \{F_0, F_1, \ldots. F_n\}$ and the induced edge labeling
$f^* : E(G) \to \{1,2,\ldots 2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct.

Therefore the resulting all trees are Fibonacci Antimagic labeling.

Example 2.5

The following is an example of the Fibonacci Antimagic labeling of the tree.

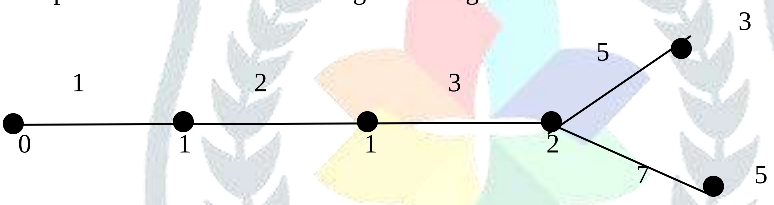

Fig (1.3)

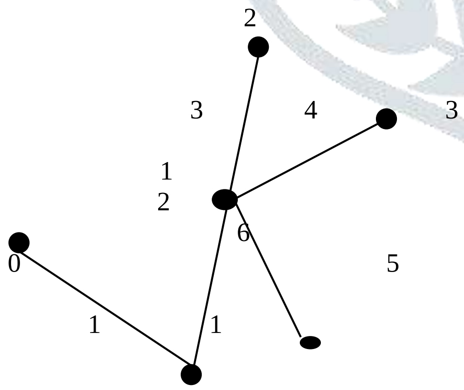

Fig (1.4)

Theorem2.6

The caterpillar graph admits a Fibonacci Antimagic labeling.

Proof

Let G be a caterpillar graph.
Define a vertex labeling by $f : V(G) \to \{F_0, F_1, \ldots. F_n\}$ and the induced edge labeling
$f^* : E(G) \to \{1,2,\ldots 2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct.

Therefore the resulting caterpillar graph admits a Fibonacci Antimagic labeling.

Example 2.7
The following is an example of the Fibonacci Antimagic labeling of the caterpillar graph.

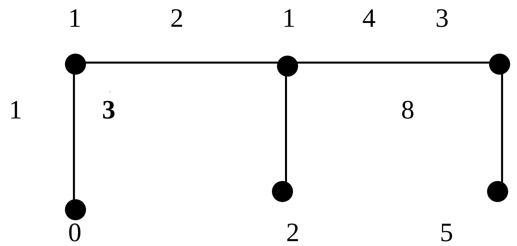

Fig (1.5) - T(X_1, X_2, X_3)

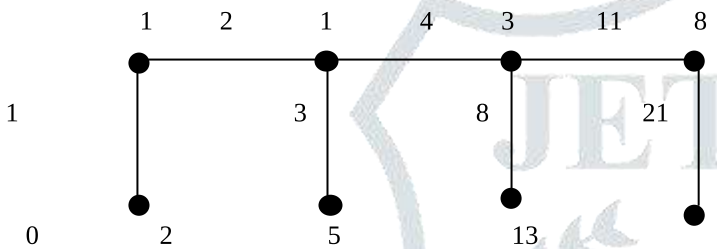

Fig (1.6) - T(X_1,X_2,X_3,X_4)

Theorem 2.8
The spider graph admits a Fibonacci Antimagic labeling.

Proof
Let $G=S_{n,m}$ be a spider graph with n spokes in which each spoke is a path on length m.
Define a vertex labeling by $f : V(G) \rightarrow \{F_0,F_1,....F_n\}$ and the induced edge labeling
$f^* : E(G) \rightarrow \{1,2,...2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct.Therefore the resulting spider graph admits a Fibonacci Antimagic labeling.

Example 2.9
The following is an example of the Fibonacci Antimagic labeling of the spider graph.

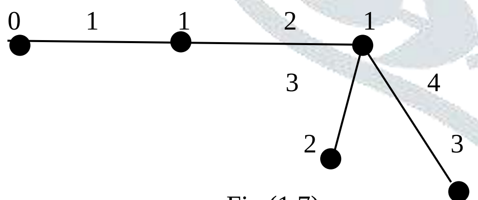

Fig (1.7)

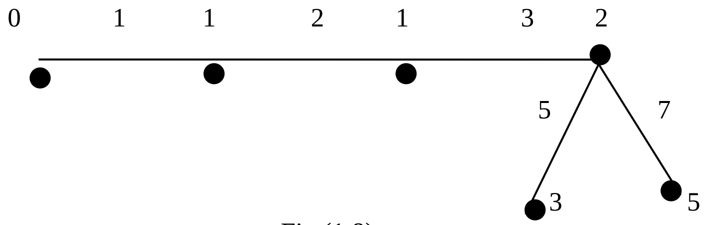

Fig (1.8)

Theorem 2.10
Every unicyclic graph admits a Fibonacci Antimagic labeling.

Proof
Let G be a unicyclic graph with p vertices and q edges.
Define a vertex labeling by $f : V(G) \rightarrow \{F_0,F_1,....F_n\}$ and the induced edge labeling
$f^* : E(G) \rightarrow \{1,2,...2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct.

Therefore the resulting every unicyclic graph admits a Fibonacci Antimagic labeling.

Example 2.11
The following is an example of the Fibonacci Antimagic labeling of the unicyclic graph.

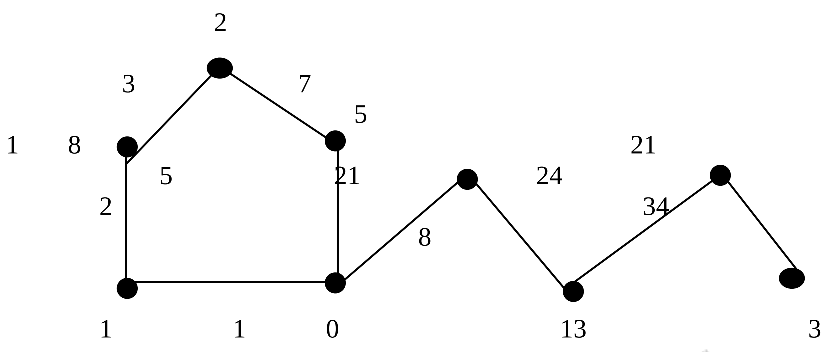

Fig 1.9 (C_5 at P_4)

3. Conclusion
In this paper we have shown that path, tree, spider, caterpillar and unicyclic graphs are fibonacci Antimagic labeling. In future, the same process will be analysed for other graphs.

References

[1]. David W. Bange and Anthony E. Barkauskas, (1980). Fibonacci graceful graphs.
[2]. Gross, J. And J.yellan, (1999). Graph theory and its application, CRC press.
[3]. J.A. Gallian , A dynamic survey of graph labelling , The Electronics Journal of Combinatorics, 16(2013)DS 61-308.
[4]. N.Hartsfield and G.Ringel, Labeling graphs, pearls in graph theory , academic press (1990),103-121.
[5]. M.Kathiresan and S.Amutha, Fibonacci graceful graphs, Ars combinatorial, 97(2010),41-50.
[6]. Rokad,A.H. and G.V. Ghodasara (2016). Fibonacci cordial labelling of some special graphs.133-144.
[7]. Dougles B.West, Introduction to graph theory, Second Edition, University of Illions, Urbana, 2003.