A note on Cordial Labeling Of One Point Union Of Graphs Related To **triple -Antena** Of C_8 and invariance.

Mukund V.Bapat[1]

1. ***Abstract:*** We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as triple-antena graph. A triple-antena graph also called as triple tail graph is obtained by attaching a path P_m to any three vertices which forms a path p_3 in given graph C_8. It is denoted by triple-tail(G,P_m) where G is given graph and all the three tails may or may not be identical to p_m. We take G as C_8 and restrict our attention to m = 2, and two edges attached at each vertex of P_3 on C_8.We have taken care that the sum of pendent edges on all the three vertices of path is same , in this case upto 2. Further we consider all possible structures of $G^{(k)}$ by changing the common point and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of $G^{(k)}$ under cordial labeling.

Key words: cordial, one point union, triple-tail graph, cycle, labeling, vertex.
Subject Classification: 05C78

2. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [4] , Graph Theory by Harary [5], A dynamic survey of graph labeling by J.Gallian [7] and Douglas West.[8].I.Cahit introduced the concept of cordial labeling [3]. $f:V(G)\rightarrow\{0,1\}$ be a function. From this label of any edge (uv) is given by $|f(u)-f(v)|$. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3{}^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [7].

Our focus of attention is on one point unions on C_8 graphs. For a given graph there are different one point unions (upto isomorphism) structures possible in $G^{(k)}$. It depends on which point on G is used to fuse to obtain one point union. It is called as invariance under cordial labeling. We use the convention that $v_f(0,1) = (a,b)$ to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1) = (x,y)$ we mean the number of edges labeled with o are x and number of edges labeled with 1 arey. The graph whose cordial labeling is available is called as cordial graph. In this paper we define triple -tail graph and obtain one point union graphs on it. For this we consider $C_5$and t-pendent edges attached to each of any three vertices forming a path on C_7.($t\leq 3$).In this paper we discuss the graphs obtained from C_8 by fusing an edge each or fusing two edges at at each consecutive three vertices.

3. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path P_m is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, P_m).

3.2 double-tail graph of G is denoted by double-tail(G,Pm).It is obtained by attaching (fusing) path

P_m to a pair of adjacent vertices of G .It has q+2m-2 edges and p + 2m-2 vertices.(m≥ 2)

3.3 Fusion of vertices. Let u ≠v be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.[4]

3.4 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$

3.4 triple-tail graph of G is denoted by triple-tail(G,P_m).It is obtained by attaching (fusing) path Pm to each of three vertices of G that forms a path P_3 .It has q+3m-3 edges and p + 3m-3 vertices.(m≥ 2)

Results Proved:

Theorem 4.1 All non- isomorphic one point union on k-copies of graph obtained on G = triple – tail(C_8,p_2) given by $G^{(k)}$ are cordial graphs.

Proof: From fig.4.1 it follows that there are six non-isomorphic structures of one point union possible at vertices a, b, c ,d, e, f

Define f:V(G)→{0,1} that gives us labeled copies of G as given below..We extend the same to f : $V(G^{(k)})$:→{0,1} to obtain cordial labeling of $G^{(k)}$. When the one point union is taken at a, d , b or e then type A and type B label are fused alternately at vertex desired vertex of these vertices.. The first copy being type A .

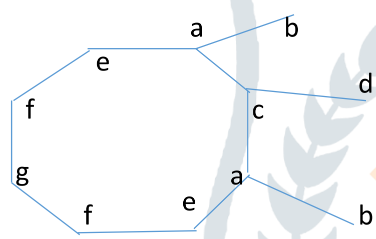

Fig 4.1 one point union may be taken at a, b, c, d, e, f, g

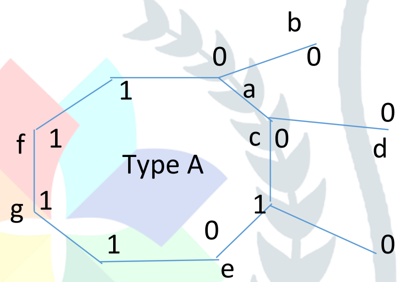

Fig 4.2 $v_f(0,1) = (6,5)$ $e_f(0,1) = (65)$

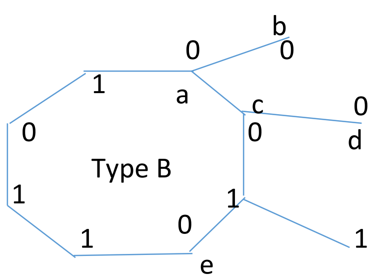

Fig 4.3 $v_f(0,1) = (6,5)$ $e_f(0,1) = (5,6)$

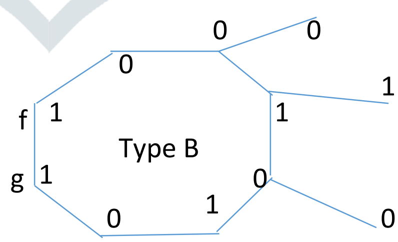

Fig 4.4 $v_f(0,1) = (6,5)$ $e_f(0,1) = (5,6)$

To obtain one point union of k copies of G at any of the vertices a, b,,c, d, e when k =1 we use type A label. For k>1 For one point union at the vertices a, b, c, d, e we fuse type A and type B label. When k = 2x there will be x copies of type A and type B each. When k = 2x+1 there will be x+1 copies of type A label and x copies of type B label. The label number distribution is $v_f(0,1)$ =(5k + 1,5k) for all k , $e_f(0,1)$= (6+11x, 5+11x).when k = 2x+1, x= 0,1, 2, .. The label number distribution is when k = 2x, x= 1, 2, ..we have labels on edge are $e_f(0,1)$= (11 + 11(x-1),11+11(x-1)).In this case the common vertex is with label 0

When one point union is taken at point c or g or f and k= 1 Type A label is used. For k>1 , for k = 2x there will be x copies of type A and type C each. When k = 2x+1 there will be x+1 copies of type A label and x copies of type C label are used. The label number distribution is $v_f(0,1)$ =(5k + 1, 5k) for all k , $e_f(0,1)$= (6+11x, 5+11x).when k = 2x+1, x= 0,1, 2, .. The label number distribution is when k = 2x, x= 1, 2, ..We have labels on edge are $e_f(0,1)$= (11 + 11(x-1),11+11(x-1)).In this case the common vertex is with label 1. Thus we observe that All non- isomorphic one point union on k-copies of graph obtained on G = triple –tail(C_8,p_2) given by $G^{(k)}$ are cordial graphs. #

Theorem 4.2 All non- isomorphic one point union on k-copies of graph obtained on G =triple-tail(C_8,$2P_2$) given by $G^{(k)}$ are cordial graphs.

Proof: From fig 4.5 it follows that there are 6 non-isomorphic structure at points a, b, c, d, e, f and g possible. At these points one can obtain one point union of k copies of graph.

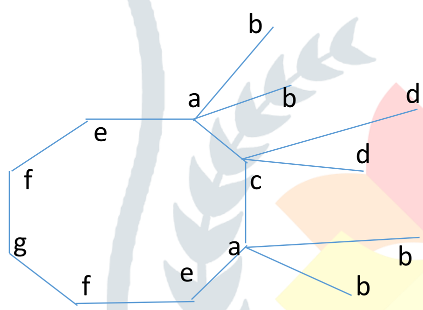

Fig 4.5 one point union may be taken at a, b, c, d, e, f, g

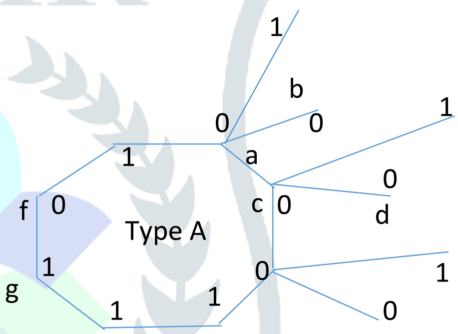

Fig 4.6 $v_f(0,1)$ = (7,7) $e_f(0,1)$ = (7,7)

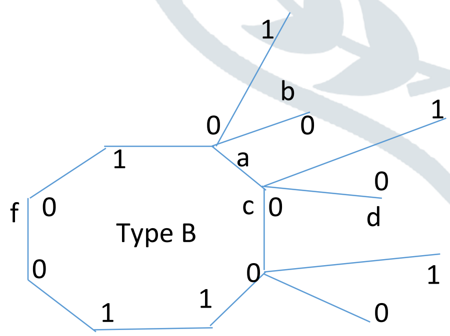

Fig 4.7 $v_f(0,1)$ = (8,6) $e_f(0,1)$ = (7,7)

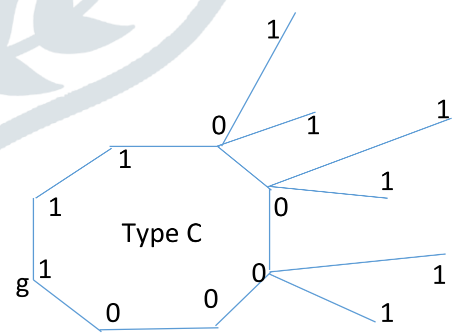

Fig 48 $v_f(0,1)$ = (6,8) $e_f(0,1)$ = (7,7)

Define f:V(G)→{0,1} that gives us labeled copies of G as above. We extend the same f : V($G^{(k)}$):→{0,1} to obtain cordial labeling of $G^{(k)}$. To obtain one point union at points a or b or c or d or f we fuse type A label with type B label at one of these required points. When k= 1 we use type A label .When k = 2x type

A and type B are used x times each. When k = 2x+1 then type A label is used x+1 times and type B label for x times to obtain $G^{(K)}$.

The label distribution is $v_f(0,1) =(7+13x,7+13x)$ for all k and $e_f(0,1)= (7k,7k)$ .when k = 2x+1, x= 0,1, 2, .. The label number distribution is $v_f(0,1)= (14+13(x-1),13+13(x-1))$.when k = 2x, x= 1, 2, … . The common vertex label is 0.

The one point union at point g is taken then type A and Thpe C label is used.When k = 1 only type A label is used. When k = 2x type A and type C are used x times each. When k = 2x+1 then type A label is used x+1 times and type C label for x times to obtain $G^{(K)}$.The label distribution is $v_f(0,1) =(7+13x, 7+13x)$ for all k and $e_f(0,1)= (7k, 7k)$ .when k = 2x+1, x= 0,1, 2, .. The label number distribution .when k = 2x, x= 1, 2, … $v_f(0,1)= (13+13(x-1),14+13(x-1))$. The common vertex label is 1. Thus even if we change point common to all copies in $G^{(k)}$ the cordiality is preserved.

Conclusions: In this paper we define some new families obtained from C_8.We take a copy of C_8 and to any three of it's adjacent vertices fuse t pendent edges each. We call this as triple-tail (G,tP_m) graph.. We show that

1) All non- isomorphic one point union on k-copies of graph obtained on G = triple-tail(C_8,P_2) given by $G^{(k)}$ are cordial graphs.

2) All non- isomorphic one point union on k-copies of graph obtained on G = triple-tail$(C_8,2P_2)$ given by $G^{(k)}$ are cordial graphs.

It is necessary to investigate the cordiality and invariance for one point union graph for the general case when t pendent edges are attached at each three vertices of C_8.

References:

[1] Bapat Mukund, Ph.D. thesis submitted to university of Mumbai. India 2004.
[2] Bapat Mukund V. Some Path Unions Invariance Under Cordial labeling, IJSAM feb.2018 issue.
[3] I.Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin., 23 (1987) 201-207.
[4] J. Clark and D. A. Holton, A first look at graph theory; world scientific.
[5] Harary, Graph Theory, Narosa publishing ,New Delhi
[6] Yilmaz, Cahit, E-cordial graphs, Ars combina, 46,251-256.
[7] J.Gallian, Dynamic survey of graph labeling, E.J.C 2017
[8] D. WEST, Introduction to Graph Theory, Pearson Education Asia.

[1]Mukund V. Bapat, Hindale, Tal: Devgad, Sindhudurg
Maharashtra, India 416630
mukundbapat@yahoo.com