Stability of the Equilibrium Position of the Centre of Mass of an extensible Cable Connected Satellites System under the influence of air resistance in circular orbit.

Dr. Dinesh Tiwari
Ex-Research Scholar,
Department of Mathematics,
J.P. University, Chapra, India.

Abstract : In a linear motion of a system of two cable connected satellites, one stable equilibrium point exists when perturbative forces like solar radiation pressure, shadow of the earth, oblateness of the earth, air resistence and earth's magnetic force act simultaneously. Many research workers obtained the stable point in case of any one of above mentioned perturbative forces. We have obtained one stable point of equilibrium in case of perturbative force air resistance acting on the motion of two cable-connected satellites. Liapunav's theorem has been exploited to test the stability of the equilibrium point.

I. Introduction

The present paper is concerned with the stability of the equilibrium point of the centre of mass of the system of two satellites connected by a light, flexible and extensible cable under air resistance in case of circular orbit, Beletsky, V.V. is the pioneer worker in this field.

II. Equations of motion

The equations of motion of one of the two satellites moving along keplerian elliptic orbit in Nechvill's coordinates may be obtained by using Lagramje's equations of motion of first kind in the form:

$$\left.\begin{aligned} x''-2y'-\rho x+f\rho\rho' &= -\overline{\lambda_\alpha}\left[1-\frac{\ell_0}{r}\right]x \\ y''-2x'-f\rho^2 &= -\overline{\lambda_\alpha}\left[1-\frac{\ell_0}{r}\right]y \end{aligned}\right\} \qquad \text{-------- (1)}$$

Where

$$\rho=\frac{1}{1+e\cos v};r=\sqrt{x^2+y^2}$$

V= True anomaly of the centre of mass of the system.

$$f=\frac{a_1p^3}{\sqrt{\mu p}}=\text{Air resistance parameter}$$

$\overline{\lambda_\alpha}=\frac{p^3}{\mu}\lambda_\alpha{}'1_0$ is the natural length of the cable connecting two satellites.

Here dashes denotes differentiations with respect to true anomaly v

The condition of constraint is given by

$$x^2+y^2\le\frac{\ell_0^2}{\rho^2} \qquad \text{-------- (2)}$$

For circular orbit of the centre of mass of the system, we must have e=0 and so

$$\rho=\frac{1}{1+e\cos\upsilon}=1 and \rho'=0$$

Putting ρ=1 and ρ'=0 in (1), we get the equations of motion in the form.

$$\left.\begin{aligned} x''-2y'-3x &= -\overline{\lambda}_\alpha\left[1-\frac{\ell_0}{r}\right]x \\ and \quad y''+2x' &= -\overline{\lambda}_\alpha\left[1-\frac{\ell_0}{r}\right]y \end{aligned}\right\} \quad \text{---------- (3)}$$

Where $r=\sqrt{x^2+y^2}$

The condition of constant given by (2) takes the form:

$$x^2+y^2 \le \ell_0^2 \quad \text{----------- (4)}$$

We also assume that in case of circular orbit, the true anomaly v for the elliptic orbit will be replaced by τ whose value is as follows:

$$\tau = \omega_0 t \quad \text{---------- (5)}$$

Where ω_0 is the angular velocity of the centre of mass of the system in case of circular orbit and t is the time.

Hence (3) can be written as

$$\left.\begin{aligned} x''-2y'-3x &= -\overline{\lambda}_\alpha\left[1-\frac{\ell_0}{r}\right]x \\ y''+2x'+f &= -\overline{\lambda}_\alpha\left[1-\frac{\ell_0}{r}\right]y \end{aligned}\right\} \quad \text{--------- (6)}$$

When in (4), the inequality sign holds, then the free motion of the system will take place otherwise the motion will be constrained.

Therefore, we have three types of motion

(i) Free motion (in case of loose string)

(ii) Constrained motion (in case of tight string)

(iii)Evolutional motion (combination of free and constrained motion)

We find that the equations of motion given by (6) do not contain time t explicitly. Hence, there must exist Jacobi's integral for the problem.

Multiplying the first equation of (6) by 2x' and the second equation of (6) by 2y' and adding them together and then integrating, we get Jacobi's integral in the form.

$$x'^2+y'^2-3x^2+2fy+\overline{\lambda}_\alpha\left[\left(x^2+y^2\right)-2\ell_0\sqrt{x^2}+y\right]=h \quad \text{---------- (7)}$$

Where *h* is the constant of integration

Equation given by (7) can be written as

$$x'^2+y'^2=3x^2-2fy-\overline{\lambda}_\alpha\left[\left(x^2+y^2\right)-2\ell_0\sqrt{x^2+y^2}\right]+h \quad \text{------------ (8)}$$

The curve of zero velocity can be obtained in the form.

$$3x^2-2fy-\overline{\lambda}_\alpha\left[\left(x^2+y^2\right)-2\ell_0\sqrt{x^2+y^2}\right]+h=0 \quad \text{---------- (9)}$$

Hence we conclude that the satellite of mass m_1 will move inside the boundaries of different curves of zero velocity represented by (9) of (8) for different values of Jacobian constant *h*.

III. Equilibrium point of the problem

We have obtained the system of equations given by (6) for the motion of the system in rotating frame of reference. It has been assumed that the system is moving with effective constraints and the connecting cable of the two satellites of masses m_1 and m_2 respectively will always remain tight.

The equilibrium positions of the system are given by the constant values of the coordinates in the rotating frame of reference.

Now, let x = x_0 and y=y_0 give the equilibrium position where x_0 and y_0 are constants

Hence, x'=0=x" and y'=0=y"

Thus, equations given by (6) take the form:

$$\left.\begin{aligned} -3x_0 &= -\overline{\lambda_\alpha}\left[1-\frac{\ell_0}{r_0}\right]x_0 \\ f &= -\overline{\lambda_\alpha}\left[1-\frac{\ell_0}{r_0}\right]y_0 \\ where \quad r_0 &= \sqrt{x_0^2+y_0^2} \end{aligned}\right\} \quad \text{----------- (10)}$$

From 10, we get the equilibrium point as

$$\left[0, \frac{\overline{\lambda_\alpha}\ell_0 - f}{\overline{\lambda_\alpha}}\right] \quad \text{------------- (11)}$$

It can be easily seen that the equilibrium position given by (11) gives a meaningful value of Hook's modulus of elasticity.

IV. Stability of the system

We shall study the stability of equilibrium point given by (11) of the system in the sense of Liapunov. For this,

Let a = x = 0 and $b = y = \frac{\overline{\lambda_\alpha}\ell_0 - f}{\overline{\lambda_\alpha}}$

Let δ_1 and δ_2 be the small variations in x_0 and y_0 respectively. For the given position of equilibrium (o, b) given by (11). Hence, we get

$$\left.\begin{aligned} &x = 0+\delta_1 and \quad y = b+\delta_2 \\ &\therefore x' = \delta_1' and \quad y' = \delta_2' \\ &x'' = \delta_1'' and \quad y'' = \delta_2'' \end{aligned}\right\} \quad \text{---------- (12)}$$

$$\left.\begin{aligned} &\delta_1'' - 2\delta_2' - 3\delta_1 = -\overline{\lambda_\alpha}\left[1-\frac{\ell_0}{r}\right]\delta_1 \\ &and\ \delta_2'' + 2\delta_1' + f = -\overline{\lambda_\alpha}\left[1-\frac{\ell_0}{r}\right](b+\delta_2) \end{aligned}\right\} \quad \text{------------ (13)}$$

Where $r = \sqrt{\delta_1^2 + (b+\delta_2)^2}$

Multiplying the first equation and the 2nd equation of (13) by $2\delta_1$' and $2(b+\delta_2)$' respectively and adding these together and then integrating, we get Jacobi's integral in the form.

$$\delta_1^{12} + \delta_2^{12} - 3\delta_1^2 + 2f(b+\delta_2) + \overline{\lambda_\alpha}\left[\delta_1^2 + (b+\delta_2)^2\right]$$
$$-2\overline{\lambda_\alpha}\ell_0\left[\delta_1^2 + (b+\delta_2)^2\right]^{1/2} = h \quad \text{---------- (14)}$$

Where h is the constant of integration.

To test the stability in the sense of Liapunov, we take Jacobi's integral given by (14) as Liapunov's function v (δ_1', δ_2', δ_1, δ_2) and is obtained expanding the terms of (14) as

$$
\begin{aligned}
v\left(\delta_1', \delta_2', \delta_1, \delta_2\right) = \delta_1'^2 + \delta_2'^2 & \left[\overline{\lambda_0} - 3 - \frac{\overline{\lambda}_\alpha \ell_0}{b} \right] \\
& + \delta_2^2 \left[\overline{\lambda_0} \right] \\
& + \delta_2 \left[2f + 2b\overline{\lambda_\alpha} - 2\overline{\lambda}_\alpha \ell_0 \right] \\
& + 2fb - 2b\overline{\lambda_\alpha} \\
& + O(3) \\
& = h
\end{aligned}
\quad \text{--------- (15)}
$$

Where O(3) stands for third and higher order terms in δ_1 and δ_2 By Liapunov's theorem on stability, it follows that the only criterion for given equilibrium position (0, b) to be stable is the V defined by (15) must be positive definite and for this the following three conditions must be satisfied:

$$
\left.
\begin{aligned}
&(i) && 2f + 2b\overline{\lambda} - 2\overline{\lambda_\alpha}\ell_0 = 0 \\
&(ii) && \overline{\lambda_\alpha} - 3 - \frac{\overline{\lambda_\alpha}\ell_0}{b} > 0 \\
&(iii) && \overline{\lambda_\alpha} > 0
\end{aligned}
\right\} \quad \text{------------ (16)}
$$

(i) Can be seen to be satisfied if we put $b = \frac{\overline{\lambda_\alpha}\ell_0 - f}{\overline{\lambda_\alpha}}$

and (iii) can be seen as

(ii)

$$
b = \frac{\overline{\lambda_\alpha}\ell_0 - f}{\overline{\lambda_\alpha}} > 0 and \overline{\lambda_\alpha} > 0
$$

Thus, all the above three conditions given in (18) are satisfied. Hence, we conclude that the equilibrium point (0, b) given by (13) of the system is stable in the sense of Liapunov.

References

[1] Beletsky; V.V. : Kosmicheskie Isslodovna, Russia p.p. 827-840; 1969

[2] Sharma; B. : The motion of a system of two cables connected satellites in the atmosphere Ph.D. thesis, submitted to B.U.; Muz; 1974

[3] Thakur; H.K.: The motion of a system of two satellites connected by extensible cable, Ph.D. thesis, submitted to B.U. : Muz: 1975.