An Approach to Determination of Mathematical Modelling of An Electric Power System Using Linear Graph

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Abstract

The transformation of a physical system to mathematical base is very important due to analysis of the systems behaviour. In this study an electric power system is considered, and this system is characterized with first-order differential equations therefore state equations which define behaviour of the system for any instant "t" are obtained.

Keywords: Physical System, electric Power system.

1.Introduction

Exact solution and simulation of various engineering problems, "especially control engineering problems, depend on convenient mathematical models for elements and subsystems of the system considered. Process of the transformation of the system's behaviour to mathematical basis is called "mathematical modelling" [1,2].

Electric power systems are an integral part of the way of life in modern society. The electricity supplied by these systems has proved to be a very convenient, clean and safe form of energy. It runs our factories, warms and lights our homes, cooks our food, and powers our computers. Electricity is carrier of energy. Energy is neither naturally available in the electrical form nor is it consumed directly in that form. The advantage of the electrical form of energy is that it can be transported and controlled with relative ease and with a high degree of efficiency and reliability. An electric power system generally refers to the collection of components interconnected to undertake the entire process of converting various primary sources of energy (hydro, fossil, nuclear, etc.) to electrical energy, transmitting it to points of consumption, and driving various power utilization devices. The electric power industry began in the 1880s and has evolved into one of the largest industries. Large interconnected power systems have been formed in many parts of the world, covering vast geographical areas. These systems provide power to millions of industrial, commercial, and residential users with very high quality and reliability and at great affordability. To achieve this, power systems are designed and operated with well established criteria and procedures based on a wide range of engineering analyses, which require mathematical models appropriate for meeting the objectives of specific studies. Electric power systems are predominantly three-phase AC (alternating current) systems. As opposed to DC (direct current) systems, AC systems are more convenient for generation, transmission and consumption. In an AC system, voltage levels can be easily transformed, thus providing the flexibility of using different voltages for transmission, generation, and utilization; from the viewpoints of efficiency and powertransfer capability, the transmission voltages have to be high, but it is not practically feasible to generator and consume power at these voltages. As well, AC machines (generators and motors) are simpler and cheaper than DC machines. In a power system, electrical power is generated and transmitted in a balanced three-phase system. Industrial loads are invariably three-phase; single-phase residential and commercial loads are distributed nearly equally among the phases so as to effectively form a balanced three-phase system. The following sections describe the physical features of power systems, and introduce the mathematical models and simulations commonly used in engineering analyses of the steady state and dynamic performance of power systems.

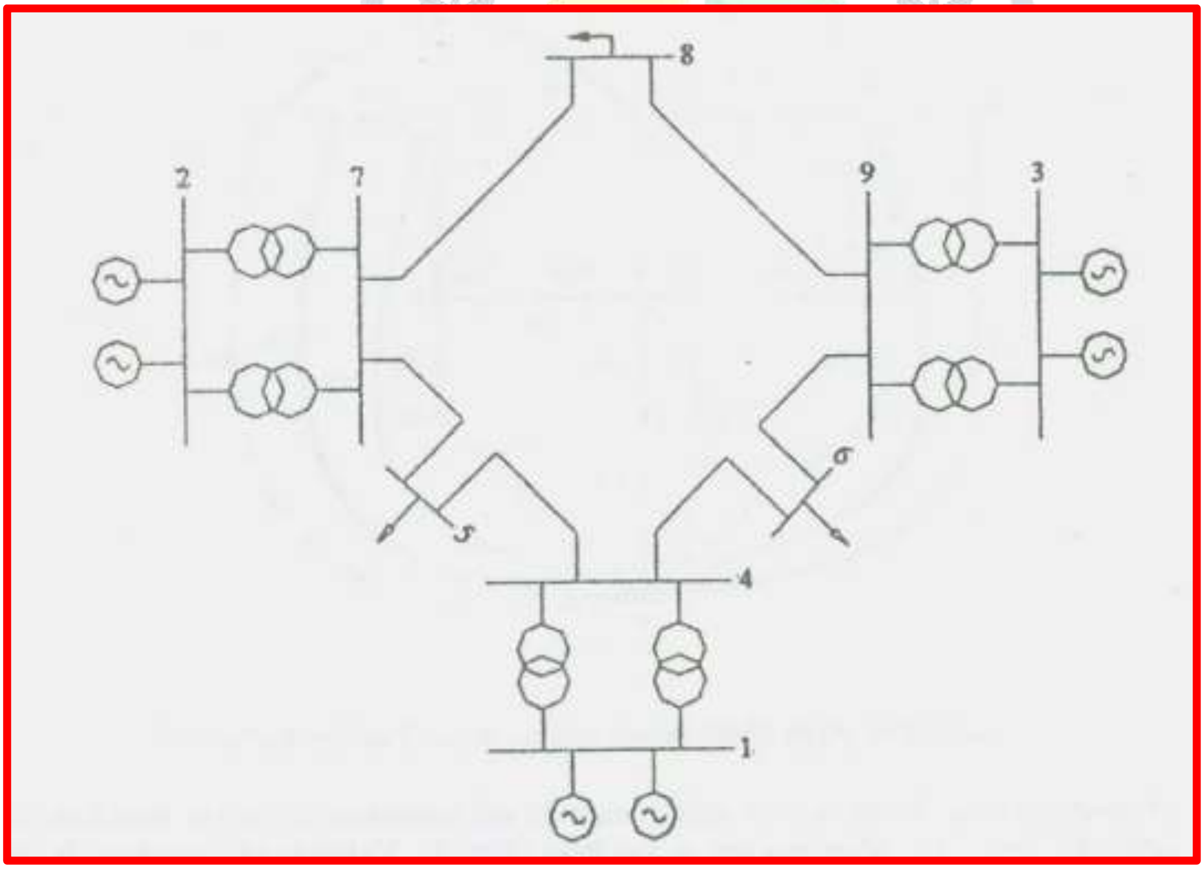
Basic Concepts Before the physical characteristics and modelling of power systems are discussed in detail, various electrical quantities associated with AC networks and their mathematical representation will be outlined in this section. In addition, the concepts of active power, reactive power and complex power are introduced. A clear conceptual understanding of these quantities is essential in the development and application of mathematical models of power systems.

2. Determination of state Equations of a Physical System:

When state-equation of the physical systems are determined, the methods which are known from Circuit Analysis Theory in Electrical Engineering may be successively used. Where, in order to obtain state-equations, linear graph theory, very important technique is used [3-8]. During analysis, it is assumed that the solution of a physical system given is possible. As general definition, in a proper system, voltage and current sources may not mesh and cut, respectively. Therefore, in the system graph for a proper system, a tree that graph elements of voltage sources and current sources correspond to branch and link, respectively may be chosen [9-12]. If possible, all capacitance-type elements are put in branches of the tree and all inductance-type elements are out of the tree. This tree used for writing the equations is called "proper tree". State variables of the system are the voltages of capacitors in the branches and the currents of inductors out of the tree. Therefore, first-order differential equations which consists of unknown voltages and currents are written.

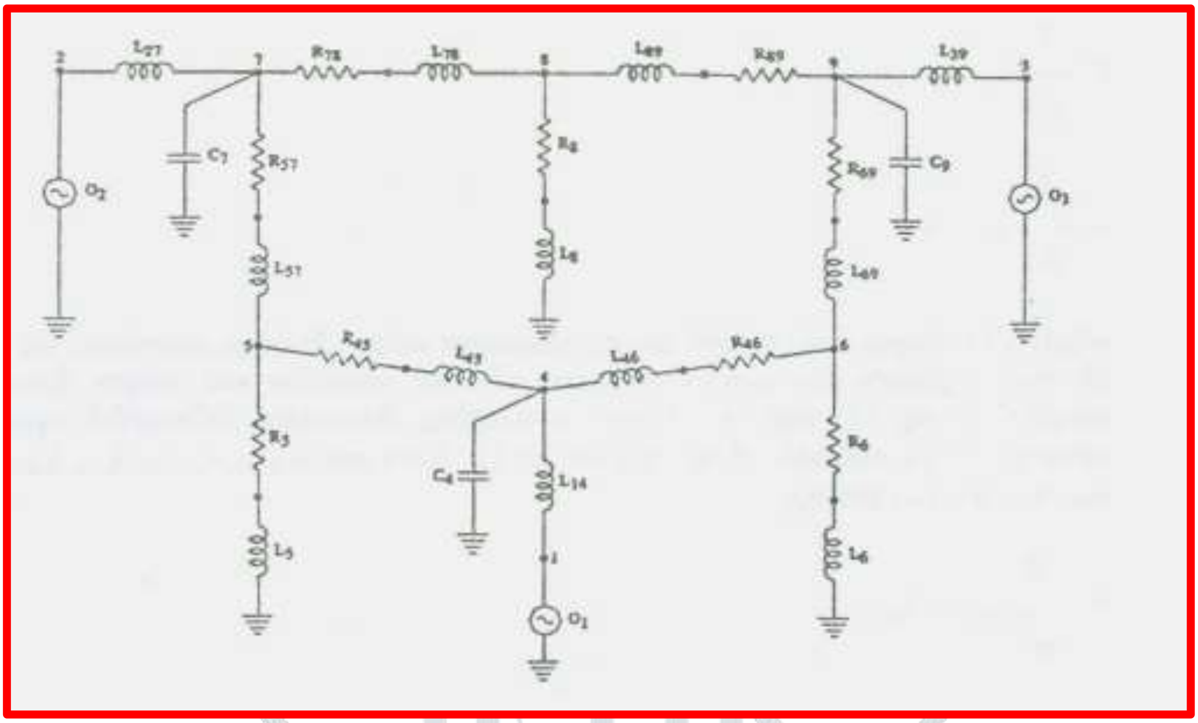
3. A Sample For Determination of Mathematical Model:

Here electric power system is shown in fig.(1).is considered as physical system.



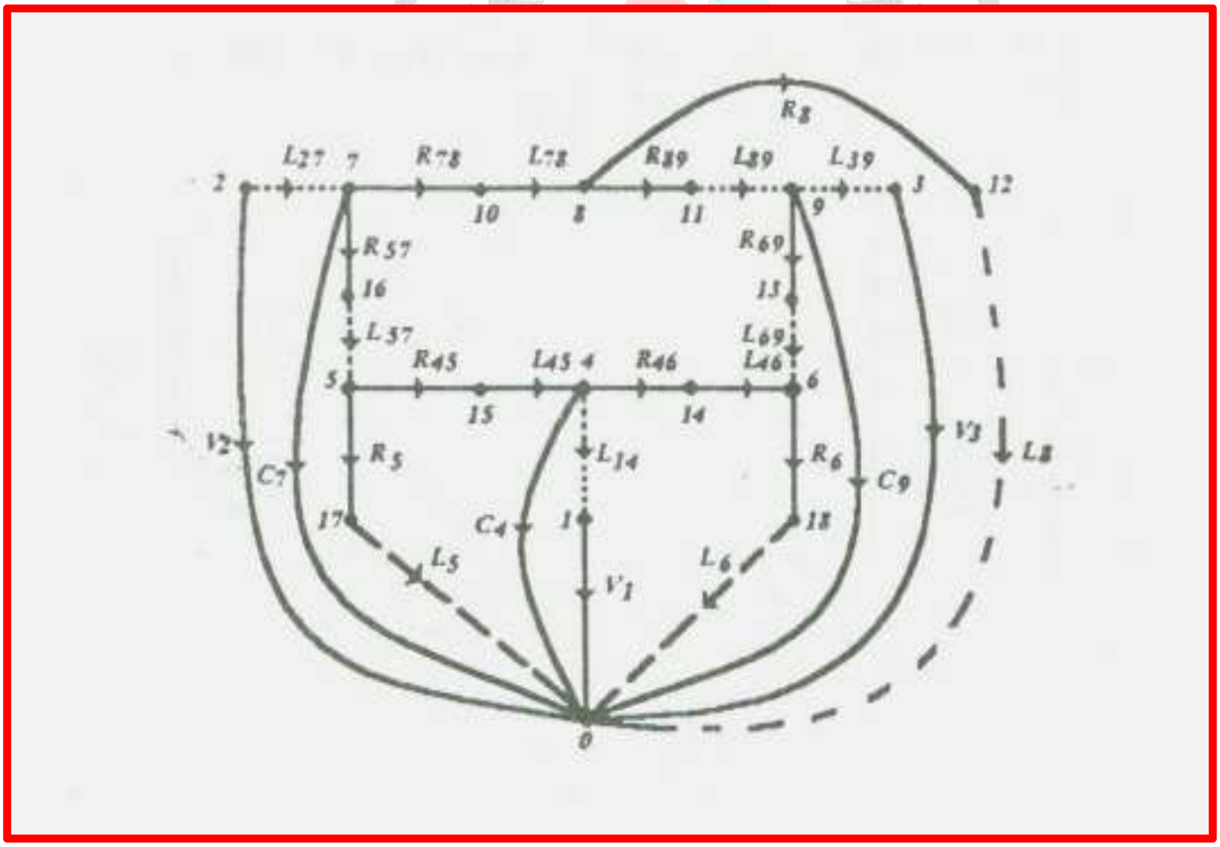
Fig(1) Illustration of Single Line Diagram of Power System Considered.

After the system's Parameters are defined as Circuit elements, the circuit in Fig.(2) is drawn.



Fig(2) Illustration of Power System As An Electrical Circuit

Later's circuit graph is drawn as shown in fig(3). As follows.



Fig(3) The Graph of Power System and chosen " Proper Tree"

A proper tree is chosen so that voltage sources and capacitors are in the branches and current sources and inductors are in the links (Fig. 3). Voltages of capacitors in the branches and currents of inductors in the links are state variables of the system. Derivative terminal equations are written as follows:

$$C \frac{d}{dt} v_c = i_C \qquad (1)$$

$$L \frac{d}{dt} i_L = v_L \qquad (2)$$

where all voltages and currents are instantaneous values. In these equations, the terms out of state variables are written in terms of state variables and source functions. By substituting eq.(1)and eq.(2) and rearranging, first-order differential equations are obtained. State variables of the system are 12. They are V_{C4}, V_{C7}, V_{C9}, i_{L27}, i_{L39}, i_{L57}, i_{L89}, i_{L69} , $i_{L14,}$ $i_{L8,}i_{L45}$and i_{L46} .

$$C_4 \frac{d}{dt} v_{C4} = i_{C4} \qquad (3)$$

$$i_{C4} - i_{L45} + i_{L14} + i_{L46} = 0 \qquad (4)$$

$$i_{C4} = i_{L45} - i_{L14} - i_{L46} \qquad (5)$$

$$C_4 \frac{d}{dt} v_{C4} = - i_{L14} + i_{L45} - i_{L46} \qquad (6)$$

Finally, first-order differential equations consisting of state variables (capacitor voltages) in the branches are obtained.

The State Equations of Inductors are also written as :

$$L_{27} \frac{d}{dt} i_{L27} = v_{L27} \qquad (7)$$

$$v_{L27} + v_{C7} - V_2 = 0 \qquad (8)$$

$$v_{L27} = -v_{C7} + V_2 \qquad (9)$$

$$L_{27} \frac{d}{dt} i_{L27} = -v_{C7} + V_2 \qquad (10)$$

All equations obtained may be written in Matrix form as shown in equation (11).

4. Conclusions:

The overall power system consists of multiple generating sources and several layers of transmission networks. This provides a high degree of structural redundancy that enables the system to withstand unusual contingencies and outages without service disruption to the consumers. The State-equations of an electric power system may be obtained step by step, using linear graph representation and proper tree technique, such as

$$\frac{d}{dt}\underset{\sim}{x}(t) = A\,\underset{\sim}{x}(t) + B\,\underset{\sim}{u}(t)$$

Variations of currents and voltages versus time, which are state variables of the system, may be obtained as y = Cx(t) + Du(t). Where A,B,C,D are coefficients-matrices and ~ denotes matrix representation.

$$\begin{vmatrix}
C_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_{27} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_{39} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & (L_{57}+L_{45}) & 0 & 0 & 0 & -L_{45} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (L_{89}+L_{78}) & 0 & 0 & 0 & 0 & L_{78} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (L_{69}+L_{46}) & 0 & 0 & -L_{46} & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{14} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -L_{45} & 0 & 0 & 0 & (L_5+L_{45}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -L_{46} & 0 & 0 & (L_6+L_{46}) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_{78} & 0 & 0 & 0 & 0 & (L_8+L_{78})
\end{vmatrix}
\begin{vmatrix} \dfrac{d}{dt} \end{vmatrix}
\begin{vmatrix} v_{C4} \\ v_{C7} \\ v_{C9} \\ i_{L27} \\ i_{L39} \\ i_{L57} \\ i_{L89} \\ i_{L69} \\ i_{L14} \\ i_{L5} \\ i_{L6} \\ i_{L8} \end{vmatrix}
=$$

$$\begin{vmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & -1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & -(R_{45}+R_{57}) & 0 & 0 & 0 & R_{45} & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & -(R_{78}+R_{89}) & 0 & 0 & 0 & 0 & -R_{78} \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & -(R_{46}+R_{69}) & 0 & 0 & R_{46} & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & R_{45} & 0 & 0 & 0 & -(R_{45}+R_5) & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & R_{46} & 0 & 0 & -(R_{46}+R_6) & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -R_{78} & 0 & 0 & 0 & 0 & -(R_{78}+R_8)
\end{vmatrix}
\begin{vmatrix} v_{C4} \\ v_{C7} \\ v_{C9} \\ i_{L27} \\ i_{L39} \\ i_{L57} \\ i_{L89} \\ i_{L69} \\ i_{L14} \\ i_{L5} \\ i_{L6} \\ i_{L8} \end{vmatrix}
+$$

$$\begin{vmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{vmatrix}
\begin{vmatrix} V_5 \\ V_3 \\ V_1 \end{vmatrix}
\qquad (11)$$

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